This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Terrestrial magnetism and atmospheric electricity

Louis Agricola Bauer, John Adam Fleming

Digitized by Google

50848

PHILLIPS LIBRARY

OF

HARVARD COLLEGE OBSERVATORY.

RESERVE LIBRARY

TRANSFERRED TO
CABOT SCIENCE LIBRARY

GODFREY LOWELL CABOT SCIENCE LIBRARY

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

TERRESTRIAL MAGNETISM
AND
ATMOSPHERIC ELECTRICITY

Digitized by Google

Digitized by Google

TERRESTRIAL MAGNETISM
AND
ATMOSPHERIC ELECTRICITY

AN INTERNATIONAL QUARTERLY JOURNAL

Conducted by

L. A. BAUER,

Department Terrestrial Magnetism, Carnegie Institution of Washington, D. C.

With the Assistance of

J. S. AMES
Johns Hopkins University, Baltimore

TH. MOUREAUX
Observatoire, Parc St. Maur

J. ELSTER AND H. GEITEL
Wolfenbüttel, Germany

H. F. REID
Johns Hopkins University, Baltimore

R. L. FARIS
Coast and Geodetic Survey, Washington

ADOLF SCHMIDT
Magnetisches Observatorium, Potsdam

J. A. FLEMING
Carnegie Institution of Washington

R. F. STUPART
Meteorological Service, Toronto

COUNCILORS

A. W. RÜCKER
England

A. ANGOT
France

M. RYKATCHEW
Russia

A. SCHUSTER
England

R. S. WOODWARD
United States, N. A.

VOLUME XVII

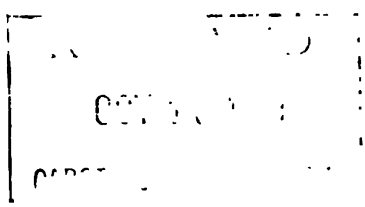
MARCH—DECEMBER, 1912

THE JOHNS HOPKINS PRESS

BALTIMORE, MARYLAND

1912

Digitized by Google



Digitized by Google

CONTRIBUTORS OF ARTICLES AND REVIEWS

L. A. BAUER,	Washington, District of Columbia.
F. G. BENEDICT,	Cambridge, Massachusetts.
F. BIDLINGMAIER,	Wilhelmshaven, Germany.
C. CHREE,	Richmond, Surrey, England.
W. G. CADY,	Middletown, Connecticut.
G. VAN DIJK,	De Bilt (Utrecht), Holland.
N. E. DORSEY,	Washington, District of Columbia.
E. VAN EVERDINGEN,	De Bilt (Utrecht), Holland.
R. L. FARIS,	Washington, District of Columbia.
H. W. FISK,	Washington, District of Columbia.
J. A. FLEMING,	Washington, District of Columbia.
R. H. GALT,	New York, New York.
A. GOCKEL,	Freiburg, Switzerland.
G. E. HALE,	Pasadena, California.
G. HARTNELL,	Cheltenham, Maryland.
D. L. HAZARD,	Washington, District of Columbia.
C. W. HEWLETT,	Washington, District of Columbia.
W. J. HUMPHREYS,	Washington, District of Columbia.
S. KALINOWSKI,	Warsaw, Poland.
B. MEYERMANN,	Tsingtau (Kiautschou), China.
J. M. MILLER,	Washington, District of Columbia.
N. A. F. MOOS,	Bombay, India.
H. MORIZE,	Rio Janeiro, Brazil.
A. NIPPOLDT,	Potsdam, Germany.
W. J. PETERS,	Washington, District of Columbia.
ADOLF SCHMIDT,	Potsdam, Germany.
D. STENQUIST,	Stockholm, Sweden.
W. W. STRONG,	Pittsburgh, Pennsylvania.
O. H. TITTMANN,	Washington, District of Columbia.

Digitized by Google

TABLE OF CONTENTS

MARCH, 1912

SPECIMEN VIEWS OF LAND MAGNETIC SURVEY EXPEDITIONS OF THE CARNEGIE INSTITUTION OF WASHINGTON, - - - - - - - - - - *Frontispiece* Pl. I

UEBER DEN ELEKTRISCHEN STROM ERDE-LUFT UND SEINEN ZUSAMMENHANG MIT DEN ERDSTRÖMEN UND DEN SCHWANKUNGEN DES ERDMAGNETISCHEN FELDES, - *A. Gockel* 1

RESULTS OF MAGNETIC OBSERVATIONS MADE BY THE UNITED STATES COAST AND GEODETIC SURVEY AT THE TIME OF THE SOLAR ECLIPSE OF APRIL 28, 1911, - *O. H. Tittmann* 21

MAGNETIC DECLINATIONS AND CHART CORRECTIONS IN THE INDIAN OCEAN CONTINUED, *L. A. Bauer and W. J. Peters* 31

DIE VERTEILUNG DER LEITFÄHIGKEIT DER ATMOSPHÄRE ÜBER DEM GROSSEN OCEAN NACH DEN BEOBACHTUNGEN DER *Galilee*, - - - - - - - - *A. Nippoldt* 33

DETERMINATION OF THE POLE DISTANCE OF A VERY SMALL MAGNET. (ABSTRACT), *J. M. Miller* 42

LETTERS TO EDITOR: Principal Magnetic Storms Recorded at the Cheltenham Magnetic Observatory, July-Dec., 1911, O. H. TITTMANN; Magnetische Störungen, Feb. 3–4, 1912, B. MEYERMANN, - - - - - - - - - - - - - 43

NOTES: Personalia, Work of Department Terrestrial Magnetism, Total Solar Eclipse October 10, 1912, Computed and Observed Moments of Inertia of Magnets, - 44

LIST OF RECENT PUBLICATIONS, - - - - - - - - - - - - - 46

JUNE, 1912

THE MAGNETIC OBSERVATORY AT ALIBAG, INDIA, - - - - - - *Frontispiece* Pl. II

THE PENETRATING RADIATION, - - - - - - - - - - *W. W. Strong* 49

RESULTS OF SOME LONG SERIES OF MAGNETIC OBSERVATORY OBSERVATIONS: COLABA (1846-1905), POLA (1847-1909), AND POTSDAM (1892-1900). *Reviewed by D. L. Hazard* 70

THE PHYSICAL THEORY OF THE EARTH'S MAGNETIC AND ELECTRIC PHENOMENA, No. V: ON THE FORMATION OF THE EARTH'S MAGNETIC FIELD, - - *L. A. Bauer* 79

MAGNETIC DECLINATIONS AND CHART CORRECTIONS IN THE ATLANTIC OCEAN ACCORDING TO THE OBSERVATIONS ON BOARD THE *Carnegie*, JUNE, 1910 TO MARCH, 1911. *L. A. Bauer and W. J. Peters* 97

LETTERS TO EDITOR: Circular Concerning Curves of Great Disturbance, E. VAN EVERDINGEN; Die Erdmagnetischen Elemente in Tsingtau, China, B. MEYERMANN; Request for Data, D. STENQUIST, - - - - - - - - - - - - - 102

NOTES: Personalia, New Magnetic Survey of Japan, British Admiralty Magnetic Chart, Magnetic Storms, Jan.-March, 1912, - - - - - - - - - - - - 104

ABSTRACTS AND REVIEWS: *K. Hoffmann*, Störungen der Niveauflächen, N. E. DORSEY; *D. Steiner*, Hysteresis Loss of the Heusler Alloys; *V. Hjort*, Comparison of Curves of Vertical Intensity, R. H. GALT, - - - - - - - - - - - - 104

LIST OF RECENT PUBLICATIONS, - - - - - - - - - - - - - 108

Digitized by Google

SEPTEMBER, 1912

A New Type of Compass Declinometer, - - - - - - - - *R. L. Faris* 109

The Physical Theory of the Earth's Magnetic and Electric Phenomena, No. VI: On the Origin of the Earth's Magnetic Field, - - - - *L. A. Bauer* 115

Magnetic Declinations and Chart Corrections Obtained by the *Carnegie* from Batavia to Manila, and Thence to Suva, Fiji, November, 1911, to June 5, 1912, - - - - - - - - - - *L. A. Bauer and W. J. Peters* 141

Resultate der Inklinationsbeobachtungen der Deutschen Südpolar Expedition 1901 bis 1903, - - - - - - - - - - - - - *Fr. Bidlingmaier* 145

Letters to Editor: Regarding Magnetic Records Obtained in Co-operation with Captain Scott's Antarctic Expedition, C. Chree; Magnetic Character of Days as Observed at the Cheltenham Magnetic Observatory, April 1 to June 30, 1912, G. M. T., O. H. Tittmann; The Magnetic Character of the Year 1911, G. van Dijk; Levé Magnétique du Bassin du Rio S. Francisco, H. Morize; Observation of the Magnetic Declination at Warsaw during the Solar Eclipse of April 17, 1912, S. Kalinowski; On the Moment of Inertia of Long Magnet H 26 at the Cheltenham Magnetic Observatory, R. L. Faris, - - - - - - - - - - 148

Abstracts and Reviews: *E. H. Kennard, C. G. Abbott, A. Angot, A. B. Chauveau, de Broglie, W. Knoche, H. Stade, G. C. Simpson and C. S. Wright, Magnetic Survey of Egypt, H. F. Skey* and *C. C. Farr, R. L. Faris, D. L. Hazard, V. F. Hess,* - - 159

Recent Publications, - - - - - - - - - - - - - - - 171

DECEMBER, 1912

Eminent Geographers at Washington, D. C., October 13, 1912, - - *Frontispiece* Pl. III

Preliminary Note on an Attempt to Detect the General Magnetic Field of the Sun, - - - - - - - - - - - - - - - - - *G. E. Hale* 173

Magnetic Declinations and Chart Corrections Obtained by the *Carnegie* from Suva, Fiji, to Papeete, Tahiti, July to September, 1912, *L. A. Bauer and W. J. Peters* 179

Ueber die Gegenseitige Einwirkung Zweier Magnete in Beliebiger Lage, *Adolf Schmidt* 181

Alibag Magnetic Observatory, - - - - - - - - - *N. A. F. Moos* 233

Magnetic Reaction Produced by a Copper Disk Rotating Between the Poles of a Magnet. (Abstract), - - - - *W. G. Cady and F. G. Benedict* 238

Principal Magnetic Storms Recorded at the Cheltenham Magnetic Observatory, *O. H. Tittmann* 239

Notes: Personalia; Lectures on Terrestrial Magnetism, 1912-13; Marine Earth Inductor; Total Solar Eclipse, October 10, 1912; Eminent Geographers at Washington, D. C., October 13, 1912, - - - - - - - - - - - - - - 239

Digitized by Google

Digitized by Google

Digitized by Google

SPECIMEN VIEWS

Digitized by Google

Terrestrial Magnetism

and

Atmospheric Electricity

VOLUME XVII MARCH, 1912 NUMBER 1

UEBER DEN ELEKTRISCHEN STROM ERDE-LUFT UND SEINEN ZUSAMMENHANG MIT DEN ERDSTRÖMEN UND DEN SCHWANKUNGEN DES ERDMAGNETISCHEN FELDES.

VON DR. A. GOCKEL, FREIBURG (SCHWEIZ).

Die Frage, ob ein Teil des magnetischen Erdfeldes seine Entstehung den in der Erde oder den zwischen der Erde und der Atmosphäre circulierenden elektrischen Strömen seine Entstehung verdankt, ist in neuerer Zeit vielfach behandelt worden.

Man hat nach Zusammenhängen zwischen den Schwankungen des Potentialgefälles[1] und denen der magnetischen Grössen gesucht. *Zölss*[2] hat auf einen solchen zwischen dem Gang der Leitfähigkeit und der Deklination aufmerksam gemacht. Es ist aber klar, dass, wenn ein ursächlicher Zusammenhang zwischen den elektrischen und magnetischen Elementen überhaupt existiert, er in erster Linie zwischen den letzteren und dem elektrischen Vertikalstrom und nicht dessen einzelnen Komponenten existieren muss. Leider lagen von der doch weitaus wichtigsten luftelektrischen Grösse, dem elektrischen Vertikalstrom, bis in die neueste Zeit herein nur vereinzelte Bestimmungen vor. Man hat sehr häufig das Potentialgefälle, in neuerer Zeit nicht viel weniger häufig die elektrische Leitfähigkeit oder den Ionengehalt der Atmosphäre gemessen, aber nur von sehr wenigen Observatorien sind Beobachtungen veröffentlicht, aus denen sich das Produkt von Potentialgefälle und Leitfähigkeit, der Vertikalstrom, berechnen lässt. Ganz vereinzelt nur wurden direkte Messungen dieses letzteren gemacht.

[1] Messerschmitt, Beitr. z. Geophysik **10**, 173, 1909.

[2] P. B. ZÖLSS, Wien. Ber. **112**, 2a, 1159, 1903.

Digitized by Google

Gerdien[5] hat zuerst gleichzeitige Messungen der Leitfähigkeit und des Potentialgefälles vorgenommen und aus diesen Grössen den Vertikalstrom berechnet, aber seine 50 Beobachtungen verteilen sich ganz unregelmässig über 15 Tage, so dass sie für eine Ableitung der täglichen Schwankungen des Vertikalstromes nicht zu gebrauchen sind. Nur die eine für unsere Zwecke wichtige Tatsache kann man ihnen entnehmen, dass nämlich das Maximum des Stromes auf die Mittagszeit und das Minimum, wie es scheint, auf die ersten Stunden nach Sonnenaufgang fällt.

P. Zölss hat ferner in der Zeit von Juli 1902—April 1903 eine grosse Anzahl Messungen der elektrischen Zerstreuung mit dem Elster und Geitelschen Zerstreuungsapparat und gleichzeitig auch Messungen des Potentialgefälles angestellt. Weitere Zerstreuungsmessungen sind in der Zeit vom 1. Mai 1903—31. August 1904 angestellt worden, gleichzeitige Potentialgefällmessungen wurden aber nicht veröffentlicht. Ich habe unter der nur in erster Annäherung richtigen Annahme, dass der mit dem Elster und Geitelschen Apparat gemessene Zerstreuungskoeffizient proportional der Leitfähigkeit ist, aus diesen Messungen den Gang des Vertikalstromes zu berechnen versucht. Einige Messungen sind nach dieser Methode auch von *Knoche*[4] in den Cordilleren ausgeführt worden.

Vereinzelte Messungen des Stromes als solchem wurden von *Ebert*[3] und von *Wilson*[6] mit in der äusseren Ausführung sehr verschiedenen, aber im Grunde auf demselben Prinzip beruhenden Apparaten vorgenommen. Dieses Prinzip besteht darin, die Elektrizitätsmengen zu messen, die eine bekannte Fläche in einer gegebenen Zeit verliert, wenn sie auf dem Potential der Erde gehalten wird.

Simpson[7] hat diese Methode zu einer Registriermethode umgestaltet. Seine Beobachtungen, die sich allerdings nur auf 10 Tage erstrecken, lassen wenigstens die tägliche Periode für den Monat November in Simla (Indien) deutlich erkennen.

In *Tortosa* (Spanien) wird das Potentialgefälle registriert, die Leitfähigkeit wird aber leider nur einmal täglich, 11*a*, bestimmt.

[5] H. Gerdien, Messungen des elektrischen Vertikalstromes in der Atmosphäre. Nachr. der K. Gesellsch. der Wissenschaften, Göttingen 1907.

[4] W. Knoche, Phys. Zt., **12**, 179, 1911.

[3] H. Ebert, Phys. Zt. **3**, 338, 1902.

[6] C. T. Wilson, Cambridge, Phil. Soc. Proc. **13**, 184 u. 364, 1906.

[7] G. Simpson, Phil. Mag. **19**, 715, 1910.

Digitized by Google

Ich konnte diese Beobachtungen immerhin für die Bestimmung der jährlichen Schwankungen des Vertikalstromes verwenden.

Ausführlichere Messungen des Vertikalstromes, die, soweit sie bis jetzt veröffentlicht vorliegen, sich allerdings auch nur über ein Jahr erstrecken, hat *K. Kähler*[8] ausgeführt. Da in Potsdam auch die magnetischen Elemente registriert werden, so eignen sich Kählers Messungen besonders gut zur Feststellung des Zusammenhanges zwischen elektrischem Vertikalstrom und dem magnetischen Felde. Die Kählerschen Messungen sind in der Weise ausgeführt, dass die Leitfähigkeit und das Potentialgefälle gesondert registriert wurden.

Nach derselben Methode hat *C. Dorno*[9] in Davos gearbeitet. Seine sehr sorgfältig ausgeführten Registrierungen erstrecken sich bereits über 2 Jahre.

Ueber die Resultate einer Anzahl von Messungen, die *Lutz*[10] in München mit einem nach dem Prinzip des Wilsonschen gebauten Apparat ausgeführt hat, ist bis jetzt nur kurz berichtet worden.

Ferner hat *Angenheister*[11] in Samoa Potentialgefälle und Leitfähigkeit der Atmosphäre registriert. Ich selbst habe hier in Freiburg (Schweiz) eine über längere Zeit sich erstreckende Reihe von Beobachtungen teils mit dem Wilsonschen Apparate, teils durch gesonderte Messungen der Leitfähigkeit und des Potentialgefälles ausgeführt.

Alle Beobachter kommen zu dem Resultate, dass die Schwankungen des Potentialgefälles und der Leitfähigkeit im entgegengesetzten Sinne verlaufen, so dass die Schwankungen des Stromes bedeutend kleiner sind als die der einzelnen Komponenten. Gerdien hat bei Ballonfahrten gefunden, dass der Strom auch längs einer Vertikalen ziemlich konstant bleibt. Im Mittel wurde für die Grösse des Stromes an niederschlagsfreien Tagen gefunden von:

Gerdien, Göttingen	2.67×10^{-16}	amp /cm^2
Kähler[8], Potsdam	2.22	" "
Dorno[9], Davos (Schweiz) (auf ebenes Feld reduziert)	2.23	" "
Lutz[10], München	2.04	" "
Daunderer, Aibling bei München	4.34	" "

[8] K. KÄHLER, Registrierungen des elektrischen Leitvermögens der Atmosphäre. Sonderabdruck aus Ergebnisse der Meteorolog. Beobachtungen in Potsdam. No. 223.

[9] C. DORNO, Studien über Licht und Luft des Hochgebirges. Braunschweig 1911.

[10] C. W. LUTZ, Münchener Sitzber. 1911, p. 305.

Digitized by Google

Wilson, Edinburg	2.22×10^{-16}	amp/cm²
Simpson, Simla	1.80	"
Angenheister[11], Samoa	2.5	"
Ansel[11], Island	3.0	"
Carse u. MacOwan,[12] Edinburg	2.86	"
Knoche,[13] Cordilleren	6.8	"

Für Tortosa lässt sich das Mittel der um 11 a gemachten Beobachtungen berechnen, man erhält dabei 4.1×10^{-16} amp/cm². Für Freiburg habe ich absolute Werte, wegen der Schwierigkeit der Reduktion des Potentialgefälles auf ebenes Feld, nicht berechnet. Man sieht aus der Tabelle, dass der Vertikalstrom an den verschiedensten Orten der Erde fast dieselbe Grösse besitzt.

Das von einem Strom erzeugte elektrische Feld besitzt an einem von dem Strom durchlaufenen Punkte kein Potential. Enthält das erdmagnetische Feld aber nur Kräfte, die ein Potential besitzen, so muss das längs einer geschlossenen Kurve auf der Erdoberfläche genommene Linienintegral $\int H \cos\theta \, dl = 0$ sein. (H, Horizontalintensität, θ Azimut des Kurvenstückes.) Ist dieses Integral von 0 verschieden, so kann ein elektrischer Strom existieren, der die Fläche der Kurve senkrecht durchsetzt. Seine Stärke, i, lässt sich berechnen, es muss sein $4\pi i = \int H \cos\theta \, dl$. Diese Integrale sind nun gebildet worden teils über ganze Breitegrade weg, teils über Trapeze, die aus Längen- und Breitegraden gebildet wurden, teils über Länder wie Oesterreich-Ungarn. Die gefundenen Resultate wichen teilweise nur so wenig von Null ab, dass unentschieden blieb, ob der Abweichung ein reeller Wert zukommt, oder ob sie nur auf Beobachtungsfehlern oder mangelhafter Reduktion der Beobachtungen auf dieselbe Epoche zurückzuführen ist. Bei der relativ geringen Anzahl der magnetischen Stationen ist auch eine Lösung der Frage auf diesem Weg sehr schwer. Nach Adolf Schmidt[13] berechnet sich aus den Neumayerschen magnetischen Karten für 1885 der zur Erzeugung des magnetischen Feldes nötige Strom durchschnittlich für die ganze Erde zu $\frac{1}{6}$ amp/km², oder 167×10^{-18} amp/cm². L. A. Bauer[14] aus neueren Berechnungen, wobei alle magnetische Karten 1840–'85 verwendet wurden, erhielt für den Erdteil 50° N bis

[11] H. Angenheister, Abh. d. K. Ges. d. Wissenschaften, Göttingen, Bd. IX, No. 2. 1911.

[12] Carse u. MacOwan, Proc. Roy. Soc. of Edinburgh. 30, 460, 1909-10.

[13] Adolf Schmidt. Mitteilungen über eine neue Berechnung des erdmagnetischen Potentials. Abh. d. II Kl. Bay. Ak. d. Wiss. XIX. Bd. I. Abth. p. 32, München, 1895.

[14] L. A. Bauer, Terr. Mag. 9, p. 127.

Digitized by Google

40° S, durchschnittlich, $\frac{1}{30}$ amp/km², oder 33×10^{-13} amp/cm²; dieser Strom wäre also immer noch 10,000 mal grösser als der wirklich beobachtete.

Die Frage, ob wenigstens ein Teil des magnetischen Erdfeldes von elektrischen Strömen herrührt, lässt sich aber auch dadurch lösen, dass man sich fragt, ob die Schwankungen des magnetischen Feldes zeitlich mit denen des elektrischen Stromes zusammenfallen. Ist dieses der Fall, so wird es erlaubt sein, auch auf einen ursächlichen Zusammenhang zu schliessen.

Die Betrachtung des vorliegenden Materials ergibt, dass der tägliche Gang des Vertikalstromes dem täglichen Gang des Potentialgefälles sich im allgemeinen anschliesst, wenn er auch weniger ausgeprägt ist als der des letzteren. Die Schwankungen des Potentialgefälles sind nämlich im allgemeinen bedeutend grösser als die der Leitfähigkeit. Es erklärt sich aus dieser Tatsache, dass Messerschmitt imstande war, auch Beziehungen zwischen dem Gang des Potentialgefälles und dem der magnetischen Elemente festzustellen. Nur bei dunstigem oder nebligem Wetter ist die Abnahme der Leitfähigkeit bedeutend grösser als die Zunahme des Potentialgefälles, so dass der Strom trotz hohem Potentialgefälle niedrige Werte annimmt.

Wie der des Potentialgefälles, schwankt auch der tägliche Verlauf des Vertikalstromes mit der Jahreszeit.

Der tägliche Gang des Vertikalstromes in Potsdam und Davos ist in den Figuren 1 und 2 wiedergegeben.

Man sieht, der Strom steigt von Mitternacht bis in die Vormittagsstunden an, nimmt dann gegen Mittag stark ab und steigt im Laufe des Nachmittags an bis zu seinem Maximum, das vor Mitternacht erreicht wird. Die Erniedrigung des Stromes um Mittag findet sich in Potsdam nicht im Winter; wie aber die Betrachtung der von Kähler gegebenen Kurven zeigt, greift diese Ausnahme nur Platz im Dezember, Januar und Februar, während im Oktober, November und März das mittägige Minimum sich findet, wenn auch schwächer ausgeprägt als im Sommer. Dass in Davos dieses Minimum auch im Winter vorhanden ist, dürfte damit zusammenhängen, dass in diesem sonnigen Hochgebirgstale im Winter die Sonnenstrahlung viel intensiver ist als in Potsdam, wo in den Wintermonaten trübe Tage die Regel bilden.

In Simla (Indien) ist der tägliche Gang des Vertikalstromes im November dem sommerlichen Gang in Potsdam ähnlich, jedoch

Digitized by Google

sind die Wendepunkte der Kurve um einige Stunden vorgerückt. Die Minima fallen auf 8 a und 4 p, die Maxima auf 1 und 9 p.

Auf Samoa folgt der Vertikalstrom dem bekannten Gang des Potentialgefälles mit Maxima am Vormittag und 10 p und Minima 4 a und 4 p.

Ein Blick auf die Figuren 1 und 2, in denen auch der tägliche Gang der magnetischen Vertikalintensität eingezeichnet ist, zeigt, wenn man von der Winterkurve in Potsdam absieht, die auffallende Uebereinstimmung in dem täglichen Gang der in Rede stehenden Elemente. Es ist zu berücksichtigen, dass die magnetischen Daten aus den Registrierungen gewonnen sind, die sich über die Jahre 1890—99 erstrecken[15], während die Werte des elektrischen Stromes in Potsdam und Davos nur je aus den Beobachtungen eines Jahres abgeleitet wurden.

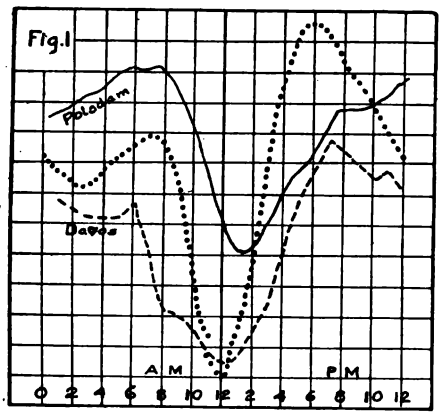
Was die Horizontalintensität betrifft, so stimmt der aus den Potsdamer Beobachtungen abgeleitete tägliche Gang sehr gut mit dem Gang des Vertikalstromes in Davos überein; gegenüber dem Gang in Potsdam zeigt sich im Sommer eine Verschiebung der Wendepunkte um einige Stunden. Der winterliche Gang des Stromes in Potsdam weicht, wie schon angegeben, überhaupt von dem der magnetischen Elemente ab.

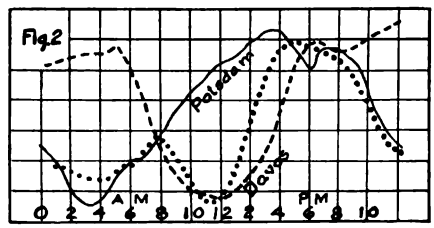
Fig. 3 stellt den täglichen Gang der Totalintensität im Sommer- und Winterhalbjahre dar. Man erkennt natürlich auch hier die Uebereinstimmung mit dem Gang des elektrischen Stromes. Ebenso wie das mittägige Minimum des letzteren ist auch das der magnetischen Feldstärke im Winter weniger ausgeprägt als im Sommer.

Die in Fig. 2 wiedergegebene Kurve des winterlichen Vertikalstromes in Potsdam kann nicht als die Darstellung einer vereinzelten Erscheinung angesehen werden. Berechnet man aus den von Zölss in Kremsmünster ausgeführten Beobachtungen unter der nur annähernd zutreffenden Voraussetzung, dass die mit dem Elster und Geitelschen Apparat mit Schutzdeckel gemessene Zerstreuung proportional der Leitfähigkeit ist, den täglichen Gang des Stromes in den Wintermonaten, so kommt man zu einer Kurve, die der Potsdamer sehr ähnlich ist. Bekannt ist ja auch, dass das Potentialgefälle, dessen Schwankungen in der Regel auch den Gang des Stromes bestimmen, im Winter fast überall nur eine einfache Periode mit einem Minimum um 4 a und einem Maximum

[15] G. Lüdeling, Ergebnisse zehnjähriger magnetischer Beobachtungen in Potsdam. Berlin, 1901.

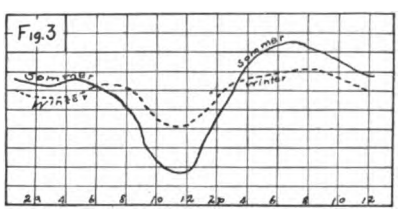
Digitized by Google

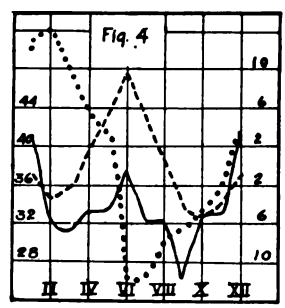




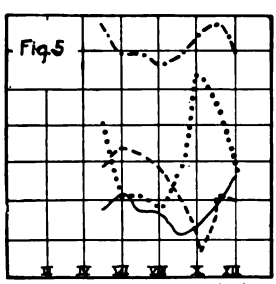
———— Potsd., 1 Thstr. = 2×10^{-8} E. S. E.
— — — Davos, 1 Thstr. = 8×10^{-8} E. S. E.

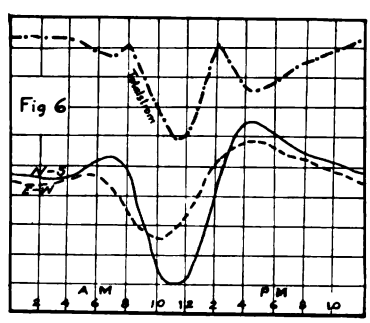
———— Strom. Potsdam, 1 Thstr. = 4×10^{-8} E. S. E.
— — — Strom. Davos, 1 Thstr. = 2×10^{-8} E. S. E.
·· ·· ·· ·· ·· Ver. Int. Potsd., 1 Thstr. = 2 γ.





———— Ver. Str., 1 Thstr. = 4×10^{-8} E. S. E.
— — — Hor. Int., 1 Thstr. = 20 γ
·· ·· ·· ·· ·· Ver. Int., 1 Thstr. = 40 γ





———— Vert. Str. Potsdam, 1909, 1 Thstr. = 2×10^{-17} Amp. / cm.²
— — — Hor. Int.
·· ·· ·· ·· ·· Ver. Int. } 1 Thstr. = 20 γ
—·—·— Tot. Int.

Digitized by Google

gegen Abend aufweist. In Freiburg (Schweiz) wurde sogar in dem heissen Sommer 1911 an manchen Tagen ein ähnlicher Verlauf des Stromes der täglichen Periode des vertikalen Erde-Luftstromes erhalten, und zwar an solchen Tagen, an denen in der Morgenfrühe das unterhalb des Beobachtungsortes gelegene, tief eingeschnittene Saanetal mit Dunst oder Nebel erfüllt war.

Da noch von keiner Station mehr als 1½ jährige Beobachtungen vorliegen, so lassen sich die jährlichen Schwankungen des Vertikalstromes überhaupt noch nicht mit genügender Sicherheit feststellen. In Fig. 4 ist der Verlauf in Potsdam und Davos berechnet, aus den Mittelwerten der normalen Tage wiedergegeben. Sieht man von der Störung in Potsdam in dem ziemlich kalten Juni 1909 ab, so kommt man zu dem Resultat, dass der Vertikalstrom ebenso wie das Potentialgefälle ein Maximum im Winter und ein Minimum im Sommer aufweist.

Zu demselben Resultate führen auch meine Beobachtungen in Freiburg. In Kremsmünster liegen aus den Sommermonaten so wenig Messungen vor, dass sich die jährliche Schwankung des Vertikalstromes daselbst nicht bestimmen lässt.

Auch die jährliche Schwankung der magnetischen Werte bietet solche Unregelmässigkeiten, dass man auf einen Vergleich mit dem Vertikalstrom verzichten muss. Fig. 4 gibt den Gang der Vertikal- und Horizontalintensität in Potsdam im Mittel der Jahre 1890—99 wieder. Das Maximum, das im Mittel in den Februar fällt, fiel aber z. B. 1899 in den Juli, 1892 in den Mai, das Minimum im Mittel in den Juni, 1899 aber in den November und 1892 in den Oktober. Man sieht, der Gang kann sich in einzelnen Jahren fast vollständig umkehren, und ein Vergleich mit dem elektrischen Strom ist nur dann möglich, wenn elektrische und magnetische Messungen aus demselben Jahre vorliegen.

In Fig. 5 sind die Monatsmittel der magnetischen Elemente in Potsdam für 1909 mit den für dieselbe Zeit berechneten Mitteln des elektrischen Vertikalstromes an diesem Ort zusammengestellt. Man sieht, dass der Gang der Horizontalintensität mit dem Maximum im Juni und dem Minimum im September sich gut dem Gang des elektrischen Stromes anschliesst. Weniger gut ist die Uebereinstimmung des Ganges der Totalkraft. Das Maximum der Vertikalintensität im Oktober korrespondiert ebenfalls mit einem Wendepunkt der Stromkurve.

Vergleicht man die Werte, wie sie sich aus den in Tortosa 11^h a gemachten Leitfähigkeits- und Potentialgefällmessungen berechnen,

Digitized by Google

so scheint allerdings nach Tabelle I ein Zusammenhang mit dem jährlichen Gang der magnetischen Feldintensität vorzuliegen,

TABELLE I.

Jährlicher Gang des mittägigen Erde-Luftstromes und der magnetischen Elemente in Tortosa 1910.

(Die Säkularvariation ist nicht ausgeschaltet.)

Monat	Strom	Horizontal-intensität	Vertikal-Intensität
Januar,	508	0.23229	0.37165
Februar,	521	226	154
März,	301	236	134
April,	332	241	175
Mai,	290	249	111
Juni,	348	252	064
Juli,	262	244	094
August,	328	258	147
September,	327	289	175
Oktober,	401	249	169
November,	440	265	172
Dezember,	519	274	181

wobei aber, wie dieses auch in Samoa beobachtet wurde, das Maximum des Vertikalstromes mit dem Minimum der Horizontalintensität zusammenfällt. Aber aus den oben angegebenen Gründen möchte ich auf den Zusammenhang, der sich aus diesen Kurven ergibt, keinen besonderen Wert legen. Ferner bleibt, solange nicht eine über viele Jahre ausgedehnte Reihe von Beobachtungen vorliegt, eine Schwierigkeit. Wenn man nämlich diejenigen Tage aussondert, welche magnetisch oder elektrisch gestört sind, so bleiben in jedem Monat nur 5—10 Tage übrig, welche zum Vergleich herangezogen werden können. Wie sich aus den Beobachtungen von Tortosa ergibt, sind elektrisch gestörte Tage nicht auch magnetisch gestörte.

Es ist mir nicht gelungen, irgend einen unmittelbaren Zusammenhang zwischen magnetischen Störungen und dem Verhalten der elektrischen Elemente, Potentialgefälle oder Leitfähigkeit zu finden. Ein solcher Zusammenhang ist von vorneherein auch gar nicht wahrscheinlich, denn magnetische Störungen pflanzen sich innerhalb weniger Sekunden über ganze Länder weg, der elektrische Vertikalstrom und seine Komponenten hängen von lokalen, meteorologischen Faktoren ab. Die magnetischen Störungen sind also offenbar, wie Schuster, Birkeland u. a. annehmen, von elektrischen Strömen in den höchsten Schichten der Atmosphäre und nicht von dem Vertikalstrom in den dem Erd-

Digitized by Google

boden anliegenden Schichten hervorgerufen. Man kann sich aber die Frage vorlegen: existieren nicht ausser den genannten Störungen, die sich auf grosse Gebiete erstrecken, lokale Abweichungen der magnetischen Elemente vom mittleren Gang, die sich auf elektrische Ströme zurückführen lassen? Leider sind noch keine Beobachtungen veröffentlicht, welche eine strenge Prüfung dieser Frage erlauben. Die Registrierung des Potentialgefälles und die gleichzeitige Registrierung der magnetischen Elemente in Tortosa lassen aber einen solchen Zusammenhang als sehr unwahrscheinlich erscheinen. An Tagen, an denen das Potentialgefälle, infolge dessen also auch der Strom, stundenlang negativ war, zeichnen die magnetischen Registrierinstrumente ruhig ihren regelmässigen Gang auf. Allerdings kann es nach den Beobachtungen von Kähler vorkommen, dass das P. G. an vertikal übereinander gelegenen Stellen verschiedenes Vorzeichen hat. In Potsdam sind eine grosse Reihe magnetisch gestörter Tage auch elektrisch gestört; an anderen magnetisch gestörten zeigt die Stromkurve zu gewissen Stunden Unregelmässigkeiten, da aber in der betreffenden Veröffentlichung[16] nur die Daten der gestörten Tage, nicht aber auch die Stunden, zu denen die Störung eintrat, angegeben sind, ist ein genauer Vergleich nicht möglich. Umgekehrt sind die elektrischen Messungen nur von den "normalen" (im ganzen höchstens ¼ aller Tage) veröffentlicht. Ob an den elektrisch nicht normalen Tagen kleine magnetische Störungen auftraten, lässt sich nicht ersehen, da nur die Daten der stark gestörten Tage (Charakter der Kurve 2) angegeben sind.

Nach Zölss zeigt der tägliche und jährliche Gang der elektrischen Zerstreuung nicht geringe Aehnlichkeit mit dem der Deklination, auch ist die Amplitude der Deklination grösser an Tagen mit grösserer Zerstreuung. Die Uebereinstimmung der beiden Faktoren im jährlichen Gang ist allerdings auch nach den Beobachtungen an anderen Stationen vorhanden. Eine Uebereinstimmung im täglichen Gang der Leitfähigkeit und der Deklination ist dagegen nach den Potsdamer Registrierungen nicht vorhanden. Die Deklination verharrt während der ganzen Nacht auf einem tiefen Stand, das Minimum wird in der Regel erst 7 a, in manchen Monaten aber auch schon 10 p erreicht, das Maximum tritt dagegen stets gegen 1 p ein. Die Leitfähigkeit zeigt aber zu letzterer Stunde in keinem einzigen Monat einen Wendepunkt, ihr Maximum fällt auf 4 a, ihr Minimum wenigstens in der Regel in

[16] Veröffentlichung des K. Preuss. Meteorol. Instit. No. 216, page 140, 1909.

Digitized by Google

die späten Abendstunden. Im Sommer bildet sich eine doppelte Welle aus, mit einer Einsenkung um die Mittagstunde. Im allgemeinen ist der tägliche Gang der Leitfähigkeit in Potsdam ein sehr unregelmässiger, die Kurven der einzelnen Monate weichen sehr stark von einander ab, während die Form der Kurve der Deklination in allen Monaten dieselbe bleibt und nur die Amplitude im Sommer sich vergrössert. Da die Deklinationswerte in Potsdam für die Zeit, aus der Leitfähigkeitsmessungen vorliegen, im einzelnen noch nicht veröffentlicht sind, so lässt sich nicht prüfen, ob Abweichungen des Ganges der Deklination von dem normalen auch von ebensolchen der Leitfähigkeit begleitet sind.

Dagegen entspricht der Gang der Leitfähigkeit in Davos mit einem während des ganzen Jahres ziemlich konstant bleibenden Minimum um 9 a und einem wenigstens sekundären Maximum um 4 p viel besser dem Gang der Deklination in Potsdam. Die zweistündige Phasenverschiebung lässt sich durch den Umstand erklären, dass in dem Hochtale von Davos die Sonne einige Stunden später aufgeht als an dem gleichen Datum in Potsdam. Aber auch in Davos fällt das Maximum der Leitfähigkeit auf 4 a, welcher Zeitpunkt an keinem Orte ein ausgezeichneter Punkt in der Kurve der Deklination ist. Die von Zölss beobachtete tägliche Schwankung der Zerstreuung mit einem Minimum um 4 a und einem Maximum um 3 p schliesst sich allerdings der Deklinationskurve recht gut an, desgleichen die von mir hier mit dem Apparat von Ebert bestimmte Kurve der Ionisation. In dem trockenen Sommer 1905 hat aber auch Zölss eine Kurve der Zerstreuung beobachtet, die sich mehr dem Gang der Leitfähigkeit in Davos nähert; es tritt ein Maximum der Zerstreuung 8 a auf und ein abendliches Minimum um Sonnenuntergang; das Morgenminimum fehlt, offenbar wegen der Abwesenheit der Taubildung, das mittägige Maximum ist durch die Staubbildung unterdrückt.

An anderen Orten hat man wieder andere Kurven beobachtet, es scheint demnach, als ob der tägliche Gang der Leitfähigkeit mit der Zeit und dem Ort stark wechselt. Da aber der tägliche Gang der Deklination innerhalb weiter Länderstrecken derselbe ist, und zwar nicht nur im Mittel, sondern auch in den Einzelwerten (südlich vom magnetischen Aequator tritt natürlich die entsprechende Richtungsänderung ein), so erscheint mir ein ursächlicher Zusammenhang zwischen dem Gang der Leitfähigkeit und dem der Deklination nicht möglich, und ich nehme an, dass die

Digitized by Google

von Zölss gefundene Aehnlichkeit der beiden Kurven nur eine zufällige ist.

Der jährliche Gang der Leitfähigkeit, soweit sich derselbe aus den Registrierungen eines Jahres entnehmen lässt, stimmt allerdings, wie auch Zölss in Kremsmünster gefunden hat, mit dem der Deklination insofern überein, als das Maximum der beiden Elemente in der Regel in den Sommer, das Minimum in den Winter fällt.

Die Beobachtungen von Zölss, dass mit der Zerstreuung auch die tägliche Amplitude der Deklination wächst, lässt sich in Tortosa wenigstens für die Zerstreuung um Mittag prüfen. Man findet, dass im Mittel die angegebene Beziehung allerdings vorhanden ist. Es scheint mir aber, dass sich auch in dieser Beziehung nur der Umstand ausspricht, dass sowohl Zerstreuung als Amplitude der Deklination vom Winter zum Sommer wachsen. Die Vermutung liegt nahe, dass sowohl die Deklination als auch die tägliche Amplitude derselben parallel der Temperatur gehen. Die Veröffentlichungen von Tortosa, die Temperatur and Deklination für jede Stunde des Tages angeben, erlauben eine Prüfung dieser Annahme. Es ergibt sich aber, dass nur im allgemeinen Gang, aber nicht in den Einzelwerten eine Uebereinstimmung zwischen den genannten Faktoren vorhanden ist; es scheint demnach, dass die Deklination nur vom Sonnenstand, aber nicht von der Temperatur am Beobachtungsort abhängig ist.

Da der Gang des Potentialgefälles dem des Vertikalstromes mehr oder minder parallel verläuft, so muss natürlich auch eine Aehnlichkeit zwischen diesem und der täglichen Schwankung der magnetischen Horizontal- und Vertikalintensität bestehen. Es erschien mir deshalb von Interesse, zu untersuchen, ob auch eine Beziehung zwischen den Einzelwerten der genannten Elemente besteht. Wie die Registrierung der sämtlichen in Frage kommenden Elemente in Tortosa zeigt, existiert eine solche Beziehung nicht.

Die Uebereinstimmung zwischen dem täglichen und jährlichen Gang des Potentialgefälles und dem der Vertikalintensität, hebt, wie schon bemerkt, auch *Angenheister* hervor. Nachdem er aber gezeigt, dass sowohl der elektrische Leitungs- als auch der Konvectionsstrom zu schwach ist, um die Variationen des magnetischen Feldes zu erklären, sucht er nach der Ursache eines indirekten Zusammenhanges zwischen Vertikalstrom und magnetischer Feldänderung und glaubt einen solchen in den Luftdruckänderungen, die

Digitized by Google

sich sowohl am Boden als auch in den höchsten Schichten der Atmosphäre geltend machen müssen, finden zu können. Er zeigt aber selbst, dass sich Schwierigkeiten ergeben, sobald man einen Zusammenhang zwischen den in den höchsten Schichten verlaufenden Elektronenströmen, welche als Ursache der magnetischen Feldänderung angesehen werden, und den der Beobachtung allein zugänglichen Vertikalströmen in den untersten Schichten der Atmosphäre rechnerisch verfolgen will. Es scheint mir aber auch aus anderen Gründen nicht wahrscheinlich, dass die Luftdruckschwankungen den Zusammenhang zwischen den täglichen und jährlichen Schwankungen des elektrischen und des magnetischen Feldes vermitteln. Wäre dieses nämlich der Fall, so müssten nach meiner Meinung die Schwankungen des elektrischen Feldes in viel stärkerem Masse, als dieses tatsächlich der Fall ist, von den Luftdruckschwankungen, besonders aber von dem Verhältnis der Amplituden der ganztägigen und der halbtägigen Welle abhängig sein. Während aber die Luftdruckschwankungen nach Phase und Amplitude an den einzelnen Orten sehr verschieden sein können, weisen die Schwankungen des magnetischen Feldes eine ganz überraschende Konstanz auf, dagegen erscheint an allen Orten die Abhängigkeit der magnetischen Variation vom Sonnenstand so deutlich ausgeprägt, dass es mir als das nächstliegende erscheint, die Aenderung des Sonnenstandes sowohl als die Ursache der Schwankungen des magnetischen Feldes als auch der des elektrischen an heiteren Tagen aufzufassen. Da die wechselnde Sonnenstrahlung auch die Ursache der Luftdruckschwankungen ist, so erklärt sich durch diese Auffassung auch ungezwungen der manchmal gefundene Zusammenhang der Luftdruckschwankungen mit denen des elektrischen und magnetischen Feldes. Es spricht sogar viel dafür, dass der Einfluss der Sonne auf das elektrische Feld durch die von der Sonnenstrahlung hervorgerufene Luftströmung zustande kommt. Der Einfluss der Sonne auf das magnetische Feld scheint mir aber ein direkterer und zum mindesten nicht durch die Bewegung der Luft in den untersten Schichten der Atmosphäre hervorgerufen zu sein. Welcher Art er ist, lässt sich zur Zeit noch nicht sagen.

Es ist aber noch eine andere Art des Zusammenhanges zwischen den Schwankungen des magnetischen und elektrischen Erdfeldes denkbar, nämlich durch die Vermittlung der Erdströme. Wie wir im Nachfolgenden im einzelnen sehen werden, zeigen auch diese eine regelmässige tägliche und jährliche Periode, die mit der der

Digitized by Google

Erde-Luftströme parallel geht, also ebenfalls eine Beziehung zum Sonnenstand zeigt. Wir sahen, dass der elektrische Leitungsstrom gegen Mittag ein Minimum, gegen Abend ein Maximum erreicht, die Stellen, an denen stärkeres oder schwächeres Einströmen von positiver Elektrizität in den Boden stattfindet, ziehen also mit der Sonne über die Erde weg. Die Strömung innerhalb der Erde muss also einen ähnlichen Gang einschlagen, wobei natürlich die Verteilung von Wasser und Land, Ebene und Gebirge mannigfaltige Aenderungen in der Richtung und Stärke des Erdstromes bewirken muss. Mit dieser Annahme und der Beobachtung, dass der Luft-Erdestrom im Laufe des Tages schwächer ist als in der Nacht, steht in guter Uebereinstimmung die Erscheinung, dass der Erdstrom überall ungefähr von *W* nach *E* läuft.

Der Erdstrom seinerseits muss aber, wie dieses schon längst vermutet wurde, die magnetischen Elemente beeinflussen. Ist die oben ausgesprochene Vermutung richtig, so ergibt sich sofort folgendes: Da der Erdstrom nicht von dem Einströmen der Elektrizität in den Boden an einem einzigen Ort, sondern von dem Strom über einem grösseren Gebiet abhängig ist, so können lokale Aenderungen des elektrischen Erde-Luftstromes weder den Erdstrom noch das von ihm abhängige magnetische Feld beeinflussen, dagegen müssen beide Elemente im grossen und ganzen dem Gang des elektrischen Stromes folgen. Bezüglich des magnetischen Feldes haben wir diese Art von Abhängigkeit vom elektrischen Strome bereits festgestellt. Mit dem Erdstrom werden wir uns im Nachstehenden beschäftigen. Auf die grosse Regelmässigkeit der täglichen Periode desselben hat schon Weinstein[17] hingewiesen. Bezüglich des magnetischen Feldes sei hier nur noch folgendes bemerkt: Wenn auch im grossen und ganzen die magnetischen Elemente an den verschiedensten Orten der Erde einen übereinstimmenden Gang aufweisen, so sind doch, wie auch Weinstein bemerkt, lokale Einflüsse vorhanden. Er nimmt an, dass, wenn ein äusseres Kräftesystem die erdmagnetischen Variationen hervorbringt, doch dessen Wirkung bedeutende lokale Aenderungen erfährt, so dass die ganze Variation eine Resultante mindestens von zwei Kräftesystemen ist, von denen eines ein äusseres, aber jedenfalls auch Variationen unterliegendes sein kann, während das andere, vielleicht ebenso wichtige, lokaler Natur ist, was selbstverständlich nicht hindert, dass es für grosse Gebiete sich gleich bleibt.

[17] B. Weinstein, Die Erdströme im deutschen Reichstelegraphengebiet. Braunschweig, 1900.

Digitized by Google

Die nachstehenden Ausführungen sollen beweisen, oder besser gesagt, wahrscheinlich machen, dass die Bewegung der magnetischen Variometer, wie dies auch Weinstein annimmt, ganz oder wenigstens zum Teil durch die Aenderung in Lage and Stärke der Erdströme verursacht werden. Häufig hat man auch schon die entgegengesetzte Annahme gemacht, dass nämlich die Erdströme, oder das, was wir beobachten, nämlich die aus der Erde abgeleiteten Ströme, Induktionsströme sind, die durch die Variationen der *Vertikalintensitat* des Erdmagnetismus verursacht werden. Die Unrichtigkeit dieser Annahme hat schon Weinstein nachgewiesen. Es müssten nämlich in diesem Falle den Maximis und Minimis der Geschwindigkeit der Aenderung der Vertikalintensität Maxima und Minima der Stärke der Erdströme entsprechen. Wie Weinstein zeigen konnte, ist dieses aber nicht der Fall. Zu demselben Resultate führten die Betrachtungen der Tortosaer Registrierungen. Auch dort ändert sich die Vertikalintensität am raschesten um 10 a und 3 p. Nun fällt zwar im Laufe des Vormittags in Tortosa ein Maximum der Entwickelung des Erdstromes etwa auf die angegebene Zeit. Führt man aber den Vergleich für die einzelnen Monate durch, so bemerkt man, dass das Strommaximum nur ausnahmsweise mit der grössten Geschwindigkeitsänderung der Vertikalintensität zusammen, in der Regel aber eine Stunde früher oder später, fällt; 3 p aber ist weder für den totalen Erdstrom noch einen seiner gemessenen Komponenten ein ausgezeichneter Punkt, der Strom ist um diese Zeit in der Abnahme begriffen und 4 p tritt ein Minimum ein, das sogar manchmal noch mit einem Maximum in der Aenderungsgeschwindigkeit der Vertikalintensität zusammenfällt. Dass die Beziehungen andere werden, wenn man nicht die Vertikalintensität, sondern die einzelnen Komponenten der Horizontalintensität ins Auge fasst, werden wir unten sehen.

Die Fig. 6 gibt den täglichen Gang des Erdstromes auf langen Linien Dresden-Berlin (S-N) und Thorn-Berlin (E-W) im Mittel der 4 Jahre 1884—87 nach Weinstein wieder. Ein Vergleich mit der Fig. 3 zeigt sofort die grosse Aehnlichkeit mit dem täglichen Gang der Totalintensität und damit auch die Vertikalintensität des Erdmagnetismus. Selbst die kleine Erniedrigung der Intensität 3^h morgens findet sich in den Kurven wieder. Dasselbe gilt auch für den Totalstrom.

Fig. 7a gibt den täglichen Gang der Stärke des totalen Erdstromes, wie er sich aus den beiden gemessenen Komponenten

Digitized by Google

berechnet, für Tortosa wieder, und zwar repräsentiert die ausgezogene Kurve den Gang im Sommer (Juni und September mussten bei der Berechnung ausgelassen werden), die gestrichelte Kurve den im Winter. Die beiden Leitungen in Tortosa sind nur je 1.8 Km. lang, um so auffallender ist, dass auch in diesen kurzen Leitungen, wie auch aus den Figuren 8 und 9 hervorgeht, der tägliche Gang des Stromes dem auf den langen Linien, wie er von Weinstein festgestellt wurde, parallel geht, wenn auch die Schwankungen in den kurzen Linien relativ bedeutend geringer sind. Auch das Verhältnis der beiden Komponenten in Bezug auf den jährlichen und täglichen Gang ist in Tortosa dasselbe wie in Norddeutschland, und es scheint mir, dass der Unterschied, den Nippoldt[18] zwischen Erdströmen in langen und kurzen Leitungen macht, nur davon herrührt, dass die von ihm in Betracht gezogenen kurzen Linien sämtlich im hohen Norden lagen.

Da der Erde-Luftstrom, wie wir oben auseinander gesetzt haben, dem Gang der Vertikalintensität folgt, so muss sich auch zwischen ihm und dem reinen Erdstrom eine Beziehung herausstellen.

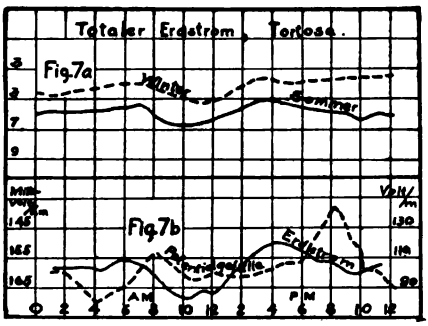
In Tortosa lässt sich leider, weil dort der Erde-Luftstrom nicht fortdauernd gemessen wird, ein Vergleich des Erdstromes nur mit dem Verlauf des Potentialgefälles durchführen. Wie Fig. 7b erkennen lässt, zeigen die beiden Elemente eine Aehnlichkeit, doch entspricht dem abendlichen Maximum des Potentialgefälles kein Wendepunkt in der Erdstromkurve. Störungen und negativen Werten des Potentialgefälles in Tortosa entsprechen aber immer Störungen des Erdstromes, aber nicht umgekehrt. Störungen des Erdstromes sind häufiger als die des Potentialgefälles, weil eben der Erdstrom vom Zufluss der Elektrizität über einem grossen Gebiet abhängen muss. Raschen Aenderungen des Potentialgefälles entsprechen auch rasche Aenderungen des Erdstromes, weitere Uebereinstimmungen im Gang der beiden Elemente finden sich aber nicht, insbesondere tritt der oben angegebene parallele Verlauf nur in den Mittel-, aber nicht in den Einzelwerten hervor. Die Bemerkung Nippoldts, dass Regen immer die Intensität des Erdstromes vermindert, findet in den Beobachtungen von Tortosa keine Bestätigung. Aehnliches gilt auch für Sodankylä[19].

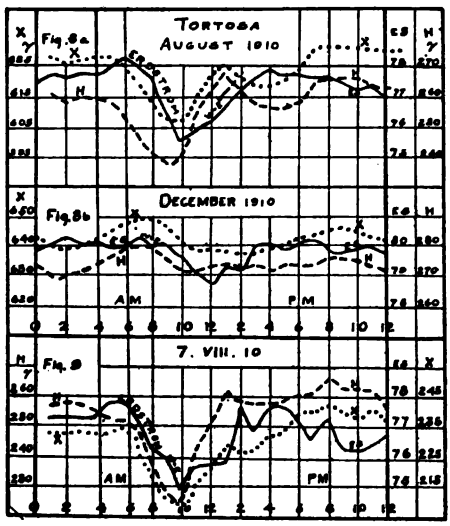
Die Uebereinstimmung im Gang der Vertikalintensität des Erdfeldes und des Erdstromes, die sich aus den obigen Figuren

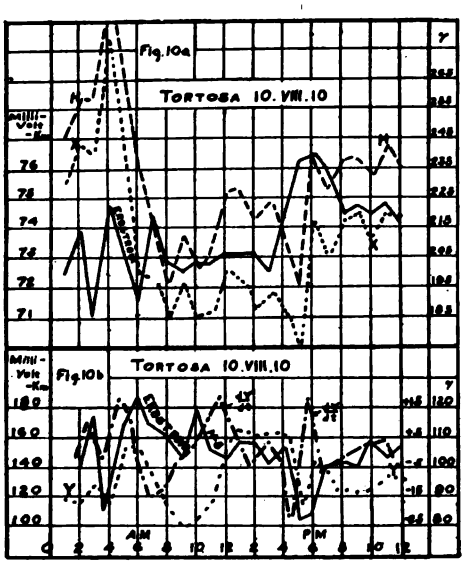
[18] A. NIPPOLDT, Meteorol. Zt. XXVIII, 244, 1911.

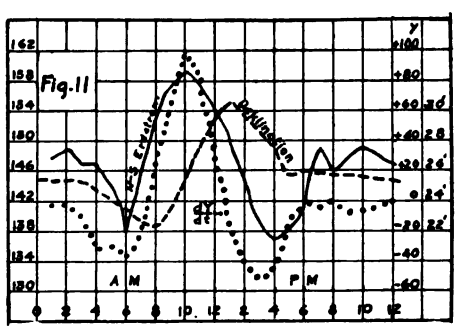
[19] Expedition Polaire Finlandaise, Tome III, Helsingfors, 1898.

Digitized by Google









Digitized by Google

für Potsdam und Tortosa ergibt, und die Weinstein durch den Vergleich mit dem Gang der Vertikalintensität an einer Reihe anderer Stationen feststellen konnte, wurde von *Battelli* bei seinen Erdstrommessungen in Italien nicht wieder gefunden, dagegen fand er, dass der Gang der ost-westlichen Komponente parallel verläuft dem der Horizontalintensität, der der nord-südlichen dem der Deklination. Die erstere Beziehung bestätigt sich auch durch die Registrierung von Tortosa, wie sich z. B. aus den Fig. 8a und 8b, den Gang der beiden Elemente im Monat August resp. Dezember darstellend, ergibt. Besser ist dagegen die Uebereinstimmung mit dem Gange der nord-südlichen (X) Komponente der Horizontalintensität, wie dieses auch schon Steiner[20] für lange Linien nachgewiesen hat. Da die Schwankungen der W-E Komponente relativ bedeutend kleiner sind als die des Gesamtstromes, so ist in Fig. 8 und 9 für die Ordinaten des Erdstromes ein 20mal grösserer Masstab gewählt worden als in Fig. 7.

Fig. 9 zeigt den Gang der Horizontalintensität und ihrer N-S Komponente sowie den der W-E Komponente des Erdstromes an einem ungestörten Tage im August 1910 in Tortosa. Man sieht, die Uebereinstimmung ist hier vor allem in den Abendstunden nicht vorhanden. Dieselbe bezieht sich also nur auf die Mittel- und nicht auch auf die Einzelwerte.

Fig. 10a und 10b gibt den Gang der beiden Elemente an einem stark gestörten Tag. Im grossen und ganzen bleibt auch hier der Gang derselbe, Zacken des einen Elementes entsprechen nicht auch solchen des anderen; doch ist zu berücksichtigen, dass die in den Tabellen veröffentlichten Zahlen nicht Momentanwerte angeben, sondern der mittleren Ordinate eines über eine Stunde sich erstreckenden Kurvenstückes entsprechen. Zwar veröffentlicht das Observatorium in seinen monatlichen Bulletins in dankenswerter Weise auch die registrierten Kurven selbst, doch ist der gewählte Masstab zu klein, um die Frage zu entscheiden, ob einer jeden Zacke der Stromkomponente auch eine solche des Wertes der Horizontalintensität entspricht; dagegen lässt der Anblick dieser Kurven erkennen, dass starken Aenderungen der Horizontalintensität zwar in der Regel, wenn nicht immer, Aenderungen der westöstlichen Stromkomponente in demselben Sinne entsprechen; umgekehrt entsprechen, mit ganz wenigen Ausnahmen, den Ausbuchtungen der Stromkurve immer auch solche der magnetischen Kurve. Im allgemeinen erscheint nach den Aufzeichnungen in

[20] Steiner, Terr. Magn. 13, 57, 1908.

Digitized by Google

Tortosa die Erdstromkomponente häufiger gestört als die Horizontalintensität. Man muss dabei berücksichtigen, dass solche Stromstörungen auch von lokalen Einflüssen, Widerstandänderungen an der Elektrode, Durchnässung des Bodens, Induktionen in der oberirdischen Leitung herrühren können. Vereinzelt finden sich auch Tage, die zwar magnetisch gestört sind, an denen aber der Erdstrom keine Störung zeigt.

Fig. 11 gibt den täglichen Gang der S-N Komponente des Erdstromes und der Deklination für die Sommermonate in Tortosa wieder. Trotzdem der Masstab für den Erdstrom nur halb so gross gewählt ist als in den vorhergehenden Figuren, erscheinen die Ausbuchtungen der Stromkomponente immer noch grösser als die der E-W Komponente. Die von Battelli behauptete Identität des Ganges der Stromkomponente und der Deklination lässt sich hier nicht wiederfinden. Wohl verlaufen die täglichen Schwankungen der beiden Elemente annähernd parallel, aber die Wendepunkte der Kurven sind um 1—3 Stunden verschoben. Auch in den Einzelheiten der in den Bulletins des Observatoriums veröffentlichten Kurven kann ich eine Uebereinstimmung nicht finden, obwohl Aehnlichkeiten an einigen Tagen nicht zu verkennen sind.

Dagegen ist, wie man aus der Figur ersieht, eine gute Uebereinstimmung im Gange dieser Erdstromkomponente und der Derivierten der west-östlichen Komponente Y der Horizontalintensität nach der Zeit vorhanden. Für lange Linien hat die Existenz einer solchen Beziehung schon Steiner nachgewiesen. Auch hier findet sich die Uebereinstimmung stets in den Mittel-, fehlt aber manchmal in den Einzelwerten (vgl. Fig. 10b).

In Bezug auf die jährliche Aenderung des Erdstromes gibt Weinstein folgendes an: "Das Hauptmaximum der Stromentwickelung fällt offenbar auf die Mitte zwischen März und April, ungefähr auf das Frühlingsäquinoctium, darauf folgt ein sekundäres Maximum zwischen Juni und Juli, also etwa um die Sommersonnenwende, ein weiteres, noch geringeres Maximum zwischen September und Oktober, vielleicht mit der Herbsttag- und Nachtgleiche zusammenfallend. Das Hauptminimum findet sich im Dezember, wohl um die Wintersonnenwende, und zwar ist dieses Minimum auffallend geringfügig, die grösste Stromstärke beträgt kaum den dritten Teil der im Februar herrschenden, ein zweites sekundäres Minimum trifft wohl zwischen Mai und Juni und ein drittes zwischen August und September ein. Im einzelnen sind offenbar bedeutende Abweichungen vorhanden, die jedoch zum Teil nur scheinbar sind, weil der Strom auch in der Richtung einen Gang im Jahre hat."

Die Beobachtungen in Tortosa sind zur Feststellung der jährlichen Veränderungen noch kaum ausreichend, weil für die W-E Komponente zwei Monate fehlen und in einigen anderen Monaten die Zahl der ungestörten, zur Mittelbildung verwendbaren Tage eine sehr geringe ist. Immerhin zeigt Tabelle II, dass der

Digitized by Google

TABELLE II.

Jährlicher Gang des Erdstromes in Tortosa.

(Das Azimut ist von Norden aus gezählt.)

Monat	Erdstrom			Vertikal-intensität	Potential-gefälle
	N-S Komponente	Total-strom	Azimut		
	$\frac{\text{Millivolt}}{\text{Km.}}$		°		$\frac{V}{m}$
Januar,	121	136	52.1	0.37165	132
Februar,	120	135	52.8	154	126
März,	131	147	52.3	134	122
April,	135	154	54.0	175	108
Mai,	134	152	53.5	111	108
Juni,	142	...	....	064	107
Juli,	147	164	51.8	094	101
August,	149	165	50.6	147	97
September,	160	...	....	175	97
Oktober,	138	150	48.7	169	108
November,	132	148	52.3	172	99
Dezember,	123	144	56.4	181	147

Gang auch auf den kurzen Linien in Tortosa ungefähr dem auf den langen Linien in Norddeutschland entspricht. Dass die Veränderungen weniger ausgeprägt, dass insbesondere das winterliche Minimum in Tortosa bedeutend weniger tief ist als in Norddeutschland, dürfte mit dem Umstand zusammenhängen, dass auch der Unterschied der Jahreszeiten in dem in der Nähe des Meeres in 41° n. B. gelegenen Tortosa weniger ausgeprägt ist als mitten in der norddeutschen Tiefebene.

Eine Uebereinstimmung im jährlichen Gang des Erdstromes mit der Vertikalintensität ist nicht vorhanden, mit dem Potentialgefälle und dem Erde-Luftstrom nur insofern, als den höheren Werten des Potentialgefälles und des Vertikalstromes im Winter niedere Werte, den niederen Werten im Sommer höhere Werte des Erdstromes entsprechen.

Der jährliche Gang des Vertikalstromes in Potsdam zeigt keine Aehnlichkeit mit der des Erdstromes auf langen Linien. Ein Einfluss von Erdbeben auf den Erdstrom scheint nach den Aufzeichnungen von Tortosa nicht vorhanden zu sein.

An die Untersuchung des Zusammenhanges kosmischer Vorgänge (Mondperiode, Sonnenflecken) mit dem reinen Erd- oder Erde-Luftstrom wird man erst denken können, wenn einmal Registrierungen vorliegen, die sich über eine Reihe von Jahren erstrecken.

Digitized by Google

RESULTS OF MAGNETIC OBSERVATIONS MADE BY THE UNITED STATES COAST AND GEODETIC SURVEY AT THE TIME OF THE SOLAR ECLIPSE OF APRIL 28, 1911.

By O. H. Tittmann, *Superintendent.*

With a view to furnishing additional data for the study of the effect of an eclipse of the Sun upon the Earth's magnetism, the observers in charge of the magnetic observatories of the Coast and Geodetic Survey were directed to be sure that the magnetographs were in good working order at the time of the solar eclipse of April 28, 1911, and to make declination observations every fifteen minutes for a period of six hours, including the eclipse. The observations were made in response to a request for cooperative work received from Dr. L. A. Bauer.

The observatories are all supplied with magnetographs of the Eschenhagen type. The instrument rooms are so well insulated that the temperature seldom changes more than two or three-tenths of a degree Centigrade in the course of a day, and the effect of change of temperature on the magnets could therefore be neglected for the period covered by the eclipse.

The location of the observatories and necessary data regarding the variometers are given below.

Location of the magnetic observatories.

Station	State	Lat.	Long. W. of Gr.	Observer in charge
		° ′		
Vieques,	Porto Rico	18 08.8	65°26.'9	G. Hartnell
Cheltenham,	Maryland	38 44.0	76 50.5	J. E. Burbank
Tucson,	Arizona	32 14.8	110 50.1	L. W. Weed
Sitka,	Alaska	57 03.0	135 20.1	F. L. Adams
Honolulu,	Hawaii	21 19.2	158 03.8	O. H. Gaarden

Magnetic elements and scale values.

Observatory	Approximate Mag. Elements			Scale Values of Mag'graph		
	D	H	Z	D	H	Z
	° ′	γ	γ	′	γ	γ
Porto Rico,	2 28W	28775	34285	1.01	2.77	5.72
Cheltenham,	5 45W	19775	56225	1.02	1.68	5.45
Tucson,	13 29 E	27390	46100	1.00	2.22	3.51
Sitka,	30 18 E	15595	56315	1.01	3.42	4.63
Honolulu,	9 32 E	29145	24220	1.00	2.57	2.51

In the following tables are given the values of the declination, horizontal intensity, and vertical intensity for every five minutes Greenwich mean civil time, from 19ʰ April 28 to 1ʰ April 29, based on the photographic records of the magnetographs. In reading

Digitized by Google

the ordinates from the magnetograms a graphical interpolation was attempted, so that a tabular result is approximately the average value for the five-minute period of which the tabular time is in the middle.

There are also given tables showing the normal diurnal variation of D, H and Z for the latter part of April, based in most cases upon three or four quiet days early in April and four or five quiet days early in May.

TABLE I.

Declination, April 28, 1911.

G. M. T. h m	Vieques ° ′	Cheltenham ° ′	Tucson ° ′	Sitka ° ′	Honolulu ° ′
19 00	2 30.6W	5 49.2W	13 28.3 E	30 20.8 E	9 34.4 E
05	30.7	49.2	28.3	20.3	34.5
10	30.8	49.0	28.3	20.4	34.5
15	30.8	49.0	28.3	20.4	34.5
20	30.7	48.8	28.3	20.6	34.4
25	30.6	48.8	28.3	19.6	34.2
30	30.6	48.8	28.2	19.2	34.1
35	30.6	48.8	28.2	18.8	34.0
40	30.6	48.7	28.1	18.1	33.9
45	30.6	48.6	28.1	17.8	33.8
50	30.5	48.5	28.2	17.6	33.7
55	30.4	48.5	28.2	17.6	33.6
20 00	30.4	48.4	28.2	17.1	33.5
05	30.3	48.3	28.2	16.8	33.5
10	30.2	48.2	28.1	16.8	33.4
15	30.2	48.0	28.1	16.6	33.3
20	30.0	47.8	28.2	16.5	33.2
25	29.9	47.7	28.3	15.6	33.0
30	....	47.7	28.2	15.7	33.0
35	....	47.6	28.2	15.7	32.9
40	29.8	47.4	28.2	15.8	32.8
45	29.8	47.2	28.1	16.2	32.8
50	29.8	47.1	28.2	16.0	32.7
55	29.7	47.0	28.1	16.5	32.7
21 00	29.6	47.0	28.2	15.8	32.6
05	29.5	46.9	28.2	15.6	32.5
10	29.4	46.8	28.2	15.3	32.4
15	29.3	46.6	28.2	15.2	32.4
20	29.3	46.5	28.2	14.9	32.3
25	29.2	46.3	28.2	14.8	32.3
30	29.2	....	28.2	14.7	32.2
35	29.2	46.0	28.3	13.7	32.1
40	29.2	46.0	28.2	14.8	32.1
45	29.1	45.9	28.2	14.4	31.9
50	29.1	45.8	28.2	14.5	31.7
55	29.1	45.7	28.3	14.1	31.5
22 00	29.1	45.6	28.3	14.0	31.4
05	29.1	45.6	28.4	13.8	31.3
10	29.1	45.5	28.4	13.7	31.1
15	29.1	45.3	28.4	13.5	30.9
20	29.1	45.2	28.4	13.8	30.7
25	29.1	45.2	28.4	13.6	30.6
30	29.0	45.1	28.5	13.6	30.6

Digitized by Google

TABLE I.—Continued.

Declination, April 28, 1911.

G. M. T. h m	Vieques ° ′	Cheltenham ° ′	Tucson ° ′	Sitka ° ′	Honolulu ° ′
22 35	2 28.9W	5 44.9W	13 28.5 E	30 13.6 E	9 30.5 E
40	28.9	44.8	28.6	13.2	30.3
45	28.9	44.8	28.6	12.8	30.2
50	28.9	44.9	28.6	12.7	30.1
55	28.9	45.0	28.6	12.6	29.9
23 00	28.9	45.0	28.6	12.5	29.7
05	28.9	44.9	28.7	12.5	29.6
10	28.8	44.8	28.7	12.7	29.5
15	28.8	44.8	28.8	12.6	29.5
20	28.8	44.6	28.8	12.8	29.4
25	28.7	44.5	28.8	12.6	29.3
30	28.6	44.0	28.9	12.3	29.2
35	28.5	43.7	28.9	12.4	29.1
40	28.5	43.6	28.9	12.2	29.0
45	28.6	44.1	28.8	12.3	29.0
50	28.6	44.2	28.8	12.3	28.9
55	28.5	43.8	28.8	11.7	28.8
24 00	28.4	43.8	29.0	10.8	28.7
05	28.3	43.8	28.9	10.3	28.7
10	28.3	44.0	28.7	11.2	28.8
15	28.4	44.5	28.7	11.9	28.9
20	28.4	44.8	28.7	12.1	29.0
25	28.5	44.7	28.8	12.5	29.1
30	28.5	44.8	28.9	12.7	29.2
35	28.4	44.5	29.1	12.6	29.3
40	28.4	44.1	29.3	12.4	29.4
45	28.4	43.6	29.4	12.3	29.5
50	28.4	43.8	29.5	12.9	29.6
55	28.4	43.7	29.5	13.0	29.6
25 00	28.4	43.8	29.4	12.8	29.7

TABLE II.

Horizontal Intensity, April 28, 1911

G. M. T. h m	Vieques γ	Cheltenham γ	Tucson γ	Sitka γ	Honolulu γ
19 00	28782	19783	27381	15593	29148
05	82	84	81	88	47
10	81	81	80	87	47
15	81	80	80	85	47
20	80	79	80	88	47
25	79	81	83	93	47
30	79	81	83	91	48
35	79	80	84	94	48
40	79	78	84	95	47
45	78	78	83	92	47
50	78	78	83	92	46
55	78	78	84	91	46
20 00	77	77	85	93	45
05	77	77	85	92	45
10	77	78	85	91	45
15	77	77	83	91	44
20	74	71	83	90	42
25	73	69	81	90	41

Digitized by Google

TABLE II—Continued.

Horizontal Intensity, April 28, 1911.

G. M. T.	Vieques	Cheltenham	Tucson	Sitka	Honolulu
h m	γ	γ	γ	γ	γ
20 30	287..	19772	27383	15590	29141
35	..	74	84	89	41
40	75	75	85	89	42
45	75	77	86	88	42
50	76	78	87	88	43
55	76	79	87	88	44
21 00	76	77	87	88	44
05	76	77	88	88	45
10	76	78	88	88	45
15	77	79	88	89	46
20	77	79	89	90	46
25	77	78	88	89	47
30	78	..	87	90	47
35	78	77	87	88	48
40	79	81	89	85	49
45	80	81	90	93	51
50	80	81	89	88	51
55	80	80	88	87	52
22 00	81	80	88	87	53
05	81	79	88	588	54
10	81	79	88	589	55
15	82	80	88	591	56
20	83	82	89	593	57
25	83	83	89	593	58
30	83	83	89	593	58
35	83	82	88	595	58
40	83	81	87	595	59
45	83	80	87	594	59
50	83	79	86	593	59
55	83	78	86	592	60
23 00	83	78	85	592	61
05	83	78	85	591	61
10	84	79	86	595	62
15	84	81	86	597	63
20	84	81	86	598	64
25	85	83	86	600	65
30	85	83	85	606	65
35	84	83	85	606	65
40	84	81	83	602	65
45	84	79	83	596	65
50	84	80	83	598	66
55	84	83	84	608	66
24 00	83	81	83	616	65
05	82	80	81	613	64
10	81	76	80	600	64
15	81	76	80	593	63
20	81	74	80	590	63
25	81	75	79	589	62
30	82	74	79	590	62
35	81	74	79	594	61
40	81	75	79	600	59
45	81	77	78	603	58
50	81	77	79	600	57
55	82	80	81	603	57
25 00	82	80	81	603	57

Digitized by Google

TABLE III.

Vertical Intensity, April 28, 1911.

G. M. T. h m	Vieques γ	Cheltenham γ	Tucson γ	Sitka γ	Honolulu γ
19 00	34296	56219	46083	56306	24227
05	96	19	83	06	27
10	96	20	83	05	27
15	96	20	83	06	27
20	96	20	83	05	27
25	96	20	83	06	26
30	96	21	83	07	26
35	96	21	83	07	25
40	96	21	83	07	24
45	96	21	83	07	24
50	96	22	83	07	23
55	96	22	83	07	23
20 00	96	23	83	08	23
05	97	23	83	08	23
10	97	24	84	08	22
15	97	24	84	08	22
20	94	24	84	07	21
25	94	25	84	08	20
30	..	25	84	08	20
35	..	26	84	07	20
40	96	26	84	07	20
45	96	26	84	06	20
50	97	26	84	06	19
55	97	27	84	06	19
21 00	97	27	84	06	19
05	97	26	84	06	19
10	97	26	84	07	18
15	97	27	84	07	18
20	97	27	84	07	18
25	97	27	83	08	18
30	97	..	83	08	18
35	97	26	83	09	18
40	98	26	84	08	18
45	98	27	84	08	18
50	98	27	85	08	17
55	98	27	85	08	17
22 00	298	27	85	08	17
05	298	26	86	08	16
10	299	26	86	09	16
15	300	26	87	09	16
20	300	26	88	08	15
25	300	27	89	08	15
30	300	27	89	08	15
35	300	28	90	08	15
40	300	28	90	09	15
45	300	27	90	10	15
50	300	27	90	11	14
55	300	27	91	11	15
23 00	300	27	91	11	15
05	300	27	91	11	15
10	299	27	91	10	15
15	299	27	91	11	15
20	299	27	91	12	16
25	298	27	91	12	16
30	298	27	91	14	16

Digitized by Google

TABLE III—Continued.

Vertical Intensity, April 28, 1911.

G. M. T.	Vieques	Cheltenham	Tucson	Sitka	Honolulu
h m	γ	γ	γ	γ	γ
23 35	34297	56227	46091	56315	24216
40	297	27	91	15	16
45	297	27	91	15	17
50	297	27	91	15	17
55	297	27	91	16	17
24 00	297	28	91	18	17
05	296	28	91	20	17
10	296	28	91	18	17
15	296	28	90	16	18
20	296	28	90	16	18
25	296	28	90	15	18
30	296	28	90	15	19
35	296	27	89	15	19
40	296	27	89	16	19
45	296	27	90	17	20
50	296	27	90	17	20
55	296	27	90	18	21
25 00	296	27	90	18	21

TABLE IV.

Diurnal variation of declination, April, 1911.

L. M. T.	Vieques	Cheltenham	Tucson	Sitka	Honolulu
h	′	′	′	′	′
1	+0.1	+0.3	+0.4	—0.5	+0.1
2	+0.3	+0.2	+0.5	—0.5	+0.3
3	+0.5	0.0	+0.7	—0.2	+0.7
4	+0.7	+0.7	+1.2	+0.3	+0.9
5	+0.9	+1.5	+1.6	+1.8	+1.1
6	+1.5	+2.5	+2.1	+3.0	+1.6
7	+2.1	+2.9	+3.6	+4.0	+2.6
8	+1.9	+3.5	+2.9	+4.8	+2.8
9	+1.0	+2.8	+1.1	+4.4	+2.1
10	+0.4	+0.7	—1.3	+3.6	+0.7
11	—0.4	—1.9	—3.0	+1.5	—1.2
12	—1.2	—4.0	—3.0	—0.6	—2.4
13	—1.4	—4.6	—2.4	—2.2	—2.8
14	—1.5	—4.2	—2.2	—3.7	—2.3
15	—1.1	—2.8	—1.8	—4.2	—1.3
16	—0.8	—1.7	—1.1	—4.2	—0.7
17	—0.6	—0.3	—0.4	—3.4	—0.5
18	—0.5	+0.4	—0.1	—1.8	—0.5
19	—0.4	+0.4	0.0	—0.6	—0.4
20	—0.3	+0.4	+0.1	—0.5	—0.3
21	—0.3	+0.6	+0.1	—0.2	—0.2
22	—0.3	+0.5	+0.2	—0.2	—0.1
23	—0.3	+0.6	+0.3	—0.3	—0.1
24	—0.1	+0.4	+0.4	—0.5	0.0

Digitized by Google

TABLE V.

Diurnal variation of horizontal intensity, April, 1911.

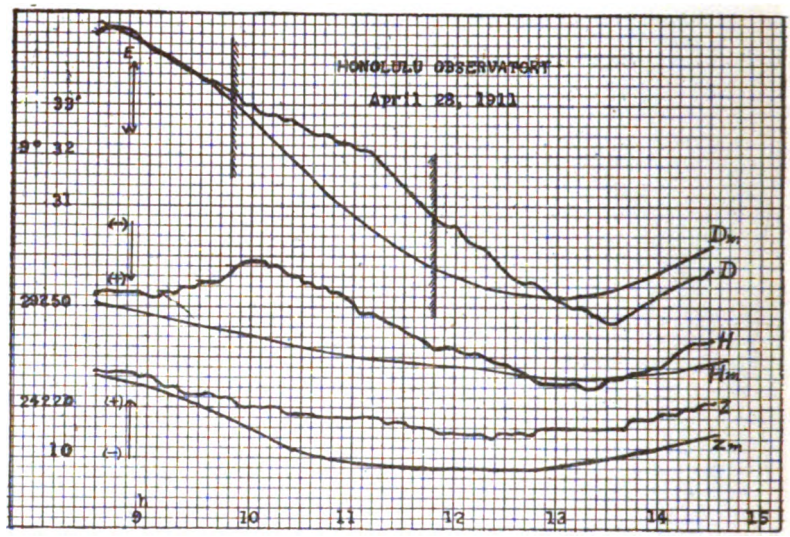
L. M. T.	Vieques	Cheltenham	Tucson	Sitka	Honolulu
h	γ	γ	γ	γ	γ
1	— 3	+ 1	+ 3	+ 5	— 5
2	— 1	+ 3	+ 3	+ 7	— 3
3	— 1	+ 3	+ 3	+ 8	— 3
4	0	+ 3	+ 3	+ 9	— 3
5	+ 1	+ 5	+ 6	+ 8	— 2
6	+ 1	+ 6	+ 8	+ 8	— 2
7	0	+ 6	+ 6	+ 8	— 3
8	— 1	0	0	+ 1	— 3
9	+ 3	—11	— 3	— 4	+ 2
10	+ 8	—20	— 6	— 8	+ 6
11	+10	—21	—11	—11	+10
12	+11	—11	—12	—15	+12
13	+10	— 5	— 7	—19	+15
14	+ 6	+ 4	— 2	—15	+13
15	— 1	+ 7	+ 2	— 6	+ 8
16	— 6	+ 8	+ 5	+ 1	+ 3
17	— 9	+ 7	+ 5	+ 4	— 3
18	— 7	+ 6	0	+ 4	— 5
19	— 4	+ 5	0	+ 3	— 5
20	— 5	+ 3	0	+ 2	— 9
21	— 5	+ 1	0	+ 1	— 7
22	— 3	+ 2	— 2	+ 2	— 6
23	— 2	+ 1	— 1	+ 3	— 6
24	— 2	0	+ 1	+ 4	— 5

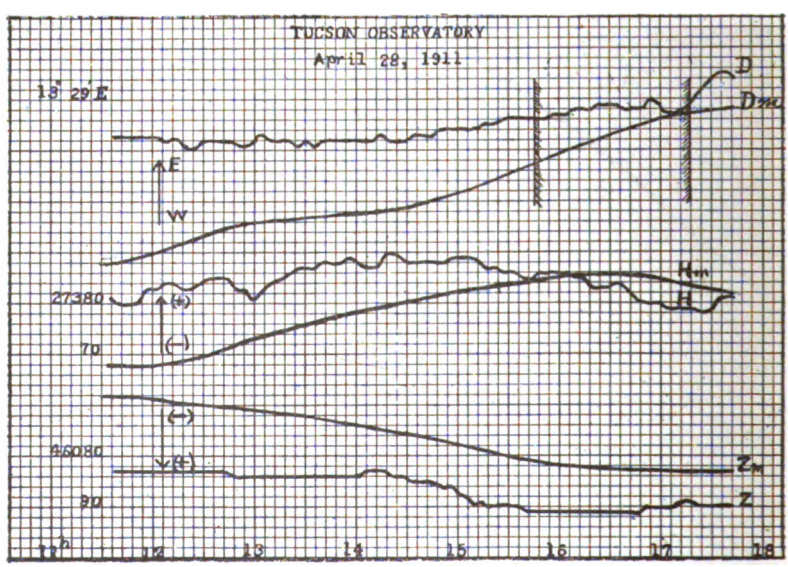
TABLE VI.

Diurnal variation of vertical intensity, April, 1911.

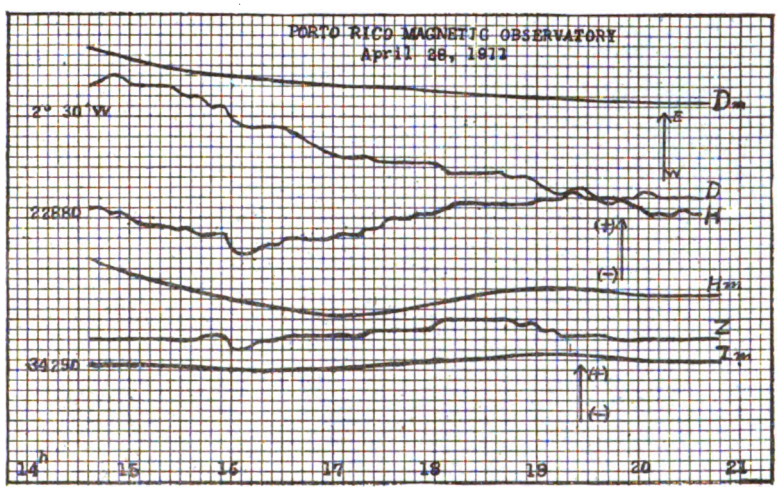
L. M. T.	Vieques	Cheltenham	Tucson	Sitka	Honolulu
h	γ	γ	γ	γ	γ
1	— 1	0	+ 3	+ 1	+ 3
2	— 1	0	+ 3	0	+ 3
3	0	— 1	+ 3	0	+ 4
4	0	— 1	+ 3	— 1	+ 4
5	+ 1	+ 2	+ 3	— 2	+ 4
6	+ 2	+ 2	+ 5	— 2	+ 8
7	0	+ 2	+ 6	— 3	+10
8	— 1	+ 2	+ 5	— 3	+ 8
9	— 2	— 1	0	— 4	+ 4
10	— 2	— 5	— 6	— 4	— 4
11	— 2	—10	—11	— 5	—11
12	— 2	—11	—11	— 5	—12
13	— 1	— 5	— 9	— 5	—12
14	+ 1	0	— 6	— 2	— 8
15	+ 1	+ 4	— 2	+ 3	— 3
16	0	+ 5	+ 2	+ 4	+ 1
17	0	+ 6	+ 3	+ 5	— 1
18	+ 2	+ 4	+ 3	+ 5	— 2
19	+ 3	+ 2	+ 2	+ 5	— 1
20	+ 2	+ 2	+ 2	+ 3	0
21	+ 1	+ 2	+ 1	+ 3	+ 1
22	+ 1	+ 1	+ 1	+ 2	+ 2
23	0	— 1	+ 2	+ 2	+ 2
24	— 1	0	+ 3	+ 1	+ 2

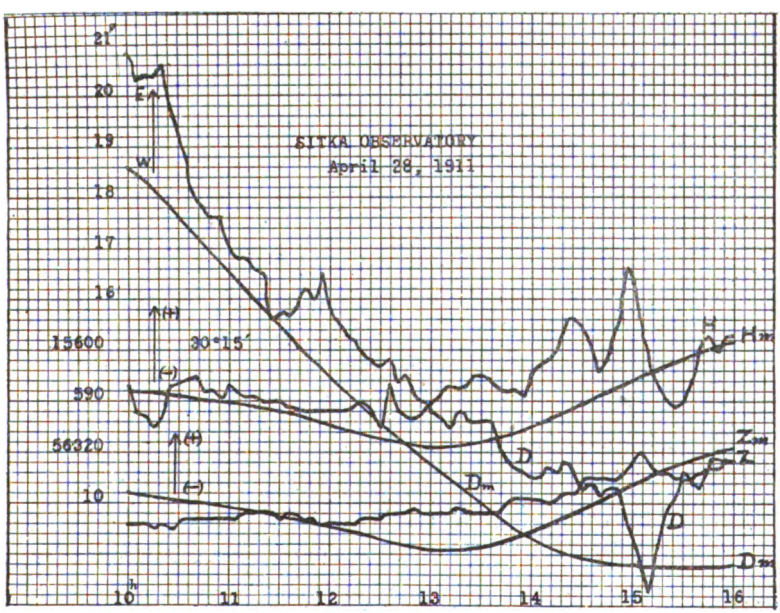
Digitized by Google



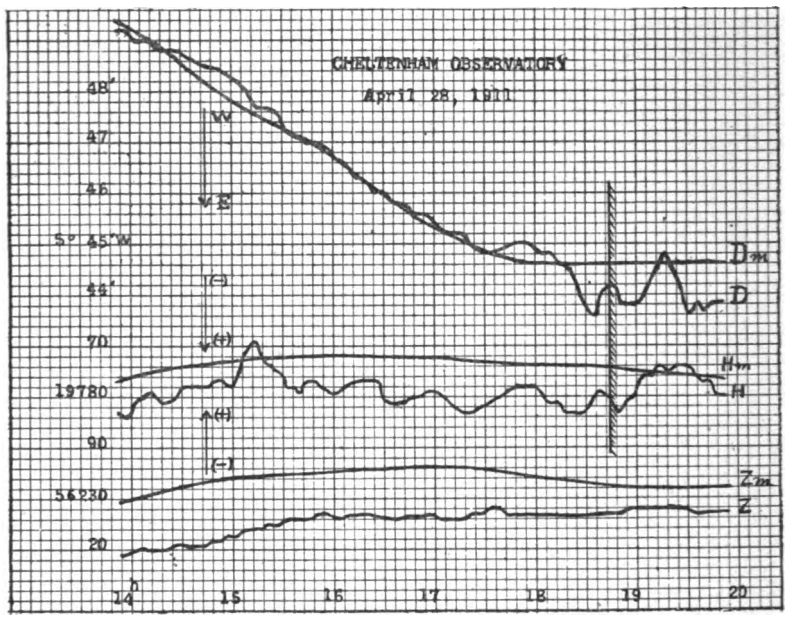


Digitized by Google





Digitized by Google



Curves of Eclipse Magnetic Observations, April 28, 1911

The curves shown in the illustrations for each of the five magnetic observatories are the graphical representation of the numerical data given in the tabulations of the magnetic elements. The curves marked D_m, H_m, Z_m show the normal diurnal variation of D, H, and Z for the latter part of April, 1911; the curves marked D, H, and Z show the variation in declination, horizontal intensity and vertical intensity on April 28, 1911, from 19 hours to 25 hours Greenwich civil mean time, which covers the period of the eclipse at the observatories. The vertical hachured lines on the Honolulu, Tucson, and Cheltenham curves show the approximate time of beginning and ending of the eclipse. The short arrows show the direction of motion corresponding to increasing or decreasing magnetic force. On the left side of the illustration the full value of the magnetic element is written on the line which was taken as the zero line in constructing the normal diurnal variation curves (curves marked D, H, and Z).

United States Coast and Geodetic Survey,
Washington, D. C.

Digitized by Google

MAGNETIC DECLINATIONS AND CHART CORRECTIONS IN THE INDIAN OCEAN.—CONTINUED.

By L. A. Bauer and W. J. Peters.

The present tables, Nos. 4 and 5, contain the results for magnetic declination obtained on board the *Carnegie* from Mauritius

TABLE IV.

Magnetic Declinations and Chart Corrections, Mauritius to Colombo, August to September, 1911.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	East Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1911	° ′	° ′	°	°	°	°	°	°	°
Aug. 16	20 05 S	57 27	—9.8	—8.8	—9.1	—8.3	—1.0	—0.7	—1.5
17	18 58 S	56 52	—8.6	—8.0	—8.0	—7.7	—0.6	—0.6	—0.9
18	16 26 S	55 37	—6.9	—6.8	—7.0	—6.6	—0.1	+0.1	—0.3
20	9 09 S	53 14	—3.8	—4.0	—4.6	—4.4	+0.2	+0.8	+0.6
21	7 06 S	52 03	—3.1	—3.5	—4.3	—4.0	+0.4	+1.2	+0.9
21	6 22 S	51 46	—3.3	—3.5	—4.2	—3.9	+0.2	+0.9	+0.6
22	6 04 S	51 29	—3.0	—3.5	—4.2	—3.9	+0.5	+1.2	+0.9
23	3 37 S	51 44	—2.1	—2.8	—3.7	—3.6	+0.7	+1.6	+1.5
24	0 55 S	52 19	—1.3	—2.3	—3.2	—3.0	+1.0	+1.9	+1.7
25	1 28N	52 58	—1.2	—1.8	—2.8	—2.6	+0.6	+1.6	+1.4
26	2 54N	53 26	—0.9	—1.6	—2.4	—2.3	+0.7	+1.5	+1.4
27	4 23N	54 26	—0.7	—1.2	—2.0	—1.9	+0.5	+1.3	+1.2
28	6 29N	54 47	—0.4	—0.8	—1.7	—1.5	+0.4	+1.3	+1.1
29	9 34N	55 43	0.0	—0.2	—1.0	—0.9	+0.2	+1.0	+0.9
29	10 54N	57 04	+0.1	+0.1	—0.7	—0.5	0.0	+0.8	+0.6
30	11 30N	58 36	+0.3	+0.2	—0.6	—0.4	+0.1	+0.9	+0.7
30	11 22N	60 02	0.0	+0.2	—0.6	—0.4	—0.2	+0.6	+0.4
31	10 59N	62 10	—0.2	+0.1	—0.6	—0.4	—0.3	+0.4	+0.2
31	10 35N	63 48	—0.3	0.0	—0.5	—0.5	—0.3	+0.2	+0.2
Sep. 1	10 18N	65 20	—0.4	0.0	—0.5	—0.6	—0.4	+0.1	+0.2
1	10 40N	66 00	—0.6	0.0	—0.5	—0.6	—0.6	—0.1	0.0
2	12 04N	66 40	—0.4	+0.2	—0.2	—0.4	—0.6	—0.2	0.0
2	13 34N	66 56	—0.3	+0.3	—0.1	—0.1	—0.6	—0.2	—0.2
3	15 31N	67 07	+0.2	+0.6	+0.1	0.0	—0.4	+0.1	+0.2
3	15 12N	67 40	—0.1	+0.5	+0.1	0.0	—0.6	—0.2	—0.1
4	13 42N	68 47	—0.3	+0.3	—0.1	—0.2	—0.6	—0.2	—0.1
4	12 30N	69 36	—0.6	+0.1	—0.2	—0.3	—0.7	—0.4	—0.3
5	11 02N	70 32	—0.9	—0.1	—0.3	—0.5	—0.8	—0.6	—0.4
5	10 10N	71 21	—1.0	—0.4	—0.3	—0.6	—0.6	—0.7	—0.4
6	9 30N	72 13	—1.3	—0.6	—0.5	—0.7	—0.7	—0.8	—0.6
6	8 59N	73 10	—1.1	—0.8	—0.5	—0.7	—0.3	—0.6	—0.4
7	8 26N	74 09	—1.5	—0.9	—0.6	—0.8	—0.6	—0.9	—0.7
7	8 04N	75 00	—1.8	—1.0	—0.6	—0.8	—0.8	—1.2	—1.0
8	7 48N	76 06	—1.4	—1.0	—0.7	—0.8	—0.4	—0.7	—0.6
8	7 33N	76 56	—1.5	—1.0	—0.6	—0.9	—0.5	—0.9	—0.6
9	7 20N	77 57	—1.7	—1.0	—0.6	—0.8	—0.7	—1.1	—0.9
9	7 11N	78 43	—1.9	—1.0	—0.6	—0.8	—0.9	—1.3	—1.1
10	6 57N	79 49	—1.8	—1.0	—0.6	—0.8	—0.8	—1.2	—1.0

Digitized by Google

to Colombo, August 16 to September 10, 1911, and from Colombo to Batavia, September 16 to October 26, 1911. The same statements and conventions apply as for the previous tables, Nos. 1, 2, and 3 in the September, 1911, issue of the Journal (Vol. XVI, p. 133), except that instead of the word "errors" used before, the more proper term "corrections" is now applied, thus conforming with the earlier article (Vol. XV, p. 143).

The present results will assist materially in locating correctly the agonic line in the Arabian Sea and in the Bay of Bengal.

TABLE V.

Magnetic Declinations and Chart Corrections, Colombo to Batavia, September to October, 1911.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	East Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1911	° ′	° ′	°	°	°	°	°	°	°
Sep. 16	7 21N	79 26	—1.6	—1.0	—0.5	—0.7	—0.6	—1.1	—0.9
17	7 34N	82 54	—1.5	—0.7	—0.5	—0.6	—0.8	—1.0	—0.9
18	8 08N	83 22	—1.6	—0.6	—0.3	—0.4	—1.0	—1.3	—1.2
18	9 05N	84 00	—1.4	—0.3	—0.2	—0.3	—1.1	—1.2	—1.1
19	10 12N	84 16	—1.0	—0.1	—0.2	—0.2	—0.9	—0.8	—0.8
19	11 16N	84 42	—0.9	0.0	0.0	0.0	—0.9	—0.9	—0.9
21	14 53N	85 06	—0.3	+0.3	+0.3	+0.3	—0.6	—0.6	—0.6
23	12 57N	87 41	—0.3	+0.2	+0.1	+0.2	—0.5	—0.4	—0.5
24	12 01N	88 54	—0.6	+0.2	+0.2	+0.2	—0.8	—0.8	—0.8
25	10 30N	91 08	—0.5	+0.2	+0.2	+0.1	—0.7	—0.7	—0.6
26	9 35N	92 51	—0.5	+0.2	+0.2	+0.2	—0.7	—0.7	—0.7
26	9 14N	93 31	—0.4	+0.2	+0.2	+0.2	—0.6	—0.6	—0.6
27	9 01N	93 51	—0.4	+0.2	+0.2	+0.2	—0.6	—0.6	—0.6
28	8 28N	95 14	—0.4	+0.3	+0.3	+0.3	—0.7	—0.7	—0.7
29	6 48N	97 35	—0.1	+0.6	+0.4	+0.5	—0.7	—0.5	—0.6
29	6 30N	98 05	—0.4	+0.6	+0.5	+0.6	—1.0	—0.9	—1.0
30	6 31N	98 24	—0.4	+0.7	+0.7	+0.6	—1.1	—1.1	—1.0
30	6 23N	98 43	0.0	+0.7	+0.7	+0.7	—0.7	—0.7	—0.7
Oct. 1	5 25N	99 48	+0.2	+0.9	+0.6	+0.7	—0.7	—0.4	—0.5
2	4 02N	100 09	+0.2	+0.9	+0.6	+0.7	—0.7	—0.4	—0.5
3	3 08N	100 40	+0.3	+0.9	+0.5	+0.6	—0.6	—0.2	—0.3
4	2 17N	101 51	+0.5	+1.0	+0.6	+0.8	—0.5	—0.1	—0.3
5	1 45N	102 41	+0.6	+1.0	+0.7	+0.9	—0.4	—0.1	—0.3
6	1 10N	103 42	+0.8	+1.1	+0.8	+1.0	—0.3	0.0	—0.2
6	1 14N	104 01	+0.7	+1.1	+0.8	+1.0	—0.4	—0.1	—0.3
9	1 10N	104 10	+0.8	+1.1	+0.8	+1.0	—0.3	0.0	—0.2
10	0 59N	104 11	+0.8	+1.1	+0.8	+1.0	—0.3	0.0	—0.2
11	0 46N	104 23	+0.8	+1.1	+0.8	+1.0	—0.3	0.0	—0.2
12	0 44N	104 23	+0.8	+1.1	+0.8	+1.0	—0.3	0.0	—0.2
13	0 42N	104 35	+0.8	+1.1	+0.9	+1.1	—0.3	—0.1	—0.3
14	0 33N	104 37	+0.9	+1.1	+0.9	+1.1	—0.3	—0.1	—0.3
16	0 04N	105 00	+0.9	+1.1	+1.0	+1.1	—0.2	—0.1	—0.2
23	2 57 S	106 11	+0.9	+1.0	+0.8	+1.0	—0.1	+0.1	—0.1

Digitized by Google

DIE VERTEILUNG DER LEITFAEHIGKEIT DER ATMOSPHAERE UEBER DEM GROSSEN OZEAN, NACH DEN BEOBACHTUNGEN DER *"GALILEE."*

VON DR. A. NIPPOLDT.

Von Mitte August 1907 bis Mitte Mai 1908 hat der Observator P. H. Dike des Department Terrestrial Magnetism an der Carnegie Institution eine grosse Zahl von Messungen der luftelektrischen Leitfähigkeit der Atmosphäre über dem grossen Ozean auf dem damaligen Vermessungsschiffe—die *Galilee*—angestellt. Er berichtet darüber in dieser Zeitschrift in der Septembernummer 1908, Seite 119 bis 128. Die Ergebnisse werden besonders auf den Einfluss der meteorologischen Elemente untersucht; es lassen sich jedoch noch weitere Schlüsse daraus ziehen, und zwar vor allem auf die Verteilung der Leitfähigkeit über den Ozean hin. Hierbei scheint sich trotz der zum Teil sehr weiten Entfernung zum Lande die Leitfähigkeit dennoch als durch die Landverteilung beeinflusst herauszustellen. Dies wäre ein Ergebnis von fundamentaler Bedeutung für die moderne Theorie der Luftelektrizität.

Die Beobachtungen geschahen mit einem Gerdienschen Apparat zur Bestimmung der absoluten Leitfähigkeit, dessen innerer Durchmesser der übliche von 16 cm, dessen Länge mit 35 cm um 21 cm hinter jener des endgültigen Modells zurückstand. Das Instrument hat stets gut gearbeitet, vor allem auch bezüglich der Isolation. Da damals ein anderes Elektroskop noch nicht bestand, kam eines der üblichen Aluminiumblattinstrumente zur Anwendung. Dies abzulesen wird bekanntlich durch die Schiffsbewegungen ungemein erschwert, doch versichert Dike, dass er trotzdem befriedigende Zahlen erhalten habe. Die Ebene der Ableseskala stand während des Ablesens stets der Mittschiffslinie parallel, so dass die rollende Bewegung eliminiert wurde. Dazu ruhte das Instrument auf einem Schlingertisch. Es sind zwei Aufstellungsörter benutzt worden. Der Wechsel spricht sich in den Beobachtungen nicht aus. Wellenspritzer, die ja vermöge des Effektes der Wasserfallelektrizität sehr gestört hätten, sind nicht vorgekommen, da bei solcher Wetterlage nicht beobachtet wurde.

Die Messungen geschahen gelegentlich einer der grossen Vermessungsreisen, welche unter Leitung L. A. Bauers von der Carnegie-Institution unternommen wurden, um die Verteilung der erdmagnetischen Elemente auf dem grossen Ozean kennen zu lernen.

Digitized by Google

Sie ging aus von dem magnetischen Observatorium zu Sitka, einem Küstenorte in Alaska, und führte, wie die Kartenbeilage zeigt, über die Hawaiischen Inseln, die Mitway-Inseln nach den Marschallinseln, von da südwärts längs des 170. Grads östlicher Länge nach Neuseeland, dann im allgemeinen östlich, später nordöstlich nach Callao, hierauf nordwestlich bis auf die Höhe der amerikanisch-mexikanischen Grenze unter 138° W und dann nach San Franzisko.

Das Instrument ist dreimal geeicht worden, das erste Mal in Sitka, ein zweites Mal im Canterbury College in Christchurch auf Neuseeland und ein drittes Mal im United States Bureau of Standards in Washington, D.C. Der ersten Messung wird nicht so grosses Gewicht beigelegt, wie den anderen beiden. In einer anderen Arbeit[1] berichtet Dike noch etwas ausführlicher über den Gang der Beobachtungen und erwähnt insbesondere, dass die Empfindlichkeit von Sitka an stetig zugenommen habe bis Christchurch, von da an aber konstant geblieben sei, und zwar innerhalb von 3%. Es trifft sich aber günstig, dass von 30° nördlicher Breite bis in die südlichsten annähernd gleich viele Beobachtungen mit der etwas veränderlichen wie mit der konstanten Empfindlichkeit vorliegen und dass das Verteilungsgesetz in beiden Beobachtungsreihen dasselbe ist. Von Sitka bis 30° n. Br. setzt es sich offensichtlich ebenfalls fort, wenn auch zwei etwas herausfallende Werte auftreten. Dike erwähnt, dass er auf diesem Teil der Reise vielleicht noch nicht die Uebung besass, wie später. Es schien dem Verfasser aber nicht notwendig, deshalb jene Werte zu verwerfen. Erwähnt sei noch, dass während der ganzen Reise dieselben Aluminiumblätter zur Verwendung kamen.

Die Originalbeobachtungen wurden um nicht mehr als drei Stunden Abstand vom Mittag vorgenommen und werden a. a. O. schon im Mittel zusammengefasst. Als Ort wird der Schiffsort um Mittag angegeben. Die tägliche Variation, an sich schon nicht gross, fällt durch die annähernd gleiche Beobachtungszeit und mehr noch durch nachstehende Mittelbildung so weit heraus, dass die Gesetze der räumlichen Verteilung davon nicht merklich beeinflusst werden.

Dikes Originaltabellen enthalten 63 solcher Tagesmittel. Es werden die spezifischen Leitfähigkeiten für beide Ladungen in 10^{-4}[2] elektrostatische Einheiten gegeben. Zugrunde gelegt sind die totalen spezifischen Leitfähigkeiten, also der Betrag, $\lambda = \lambda_n + \lambda_p$. Die beiliegende Routenkarte (Fig. 1) enthält neben dem Schiffsort die Be-

[1] Journal of *Science*, 27. März 1909, p. 197–209; siehe besonders p. 201.

[2] Die Tabelle hat versehentlich 10^4.

Digitized by Google

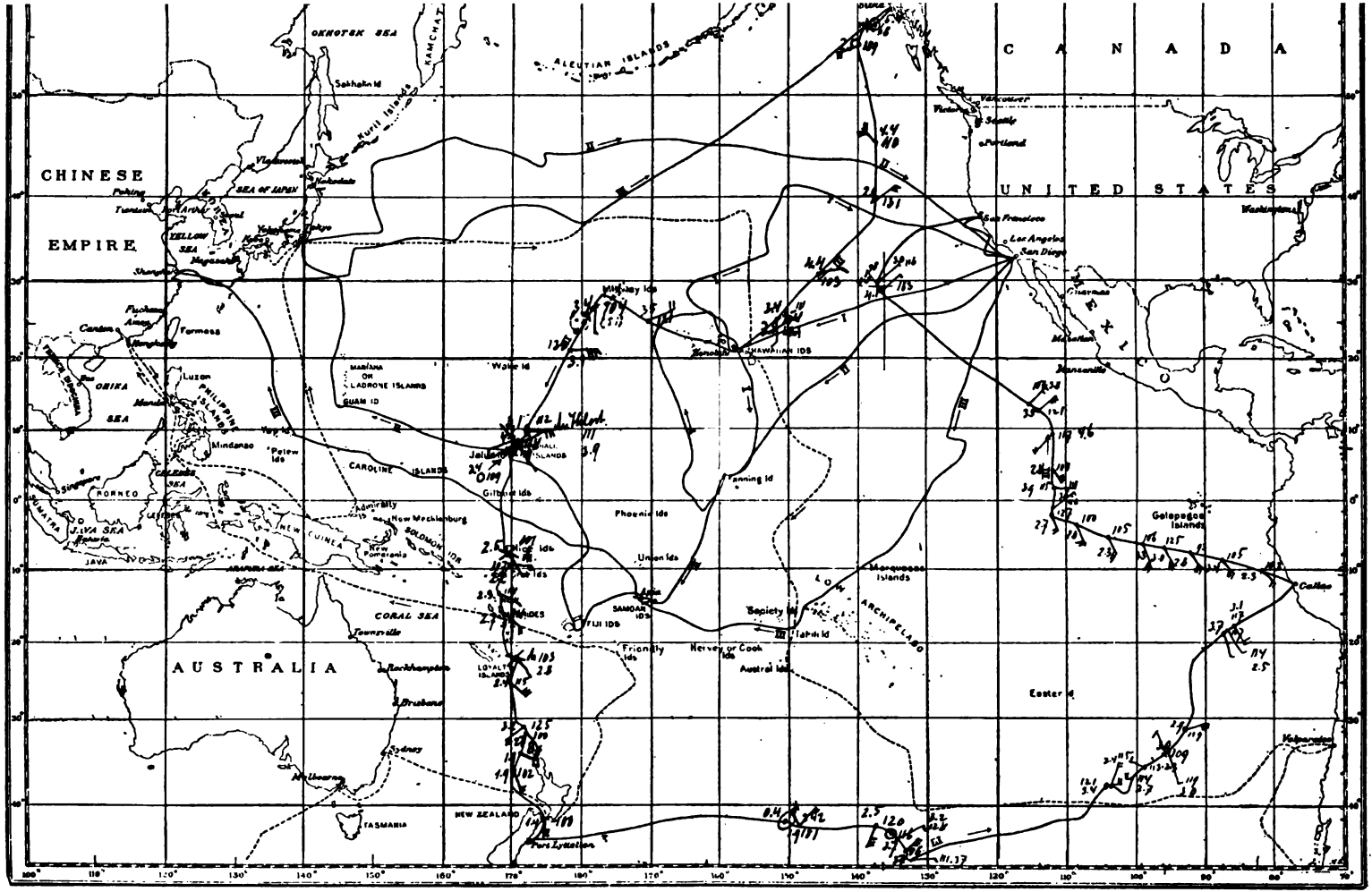


FIG. 1. ROUTENKARTE DES VERMESSUNGSSCHIFFES, "GALILEE," 1905–08 (DIE VOLLGEZOGENEN LINIEN).

Digitized by Google

träge dieser λ in Ganzen und Zehnern und das Verhältnis λ_p / λ_n auf Hundertstel; daneben ist die Windrichtung und Stärke eingetragen, der Pfeil mit dem Winde fliegend, die Befiederung nach Dikes Stärkeangaben, die Windstille durch einen Kreis gekennzeichnet.

Wir teilen das Gebiet durch die Längen- und Breitengrade in Felder von je 10 Graden nach beiden Richtungen und vereinen alle in je ein solches Feld fallende zum Mittel. Tabelle I enthält die so entstandene Uebersicht für die totale Leitfähigkeit. Neben jeder Zahl gibt eine kleinere die Anzahl der Einzelwerte an. Hierauf bilden wir Mittel längs der Meridional- und längs der Breitenstreifen.

Diese Uebersicht zeigt, dass in Länge, d. h. von Ost nach West, kein erkennbares Gesetz besteht, alle Werte bis auf einen einzelnen sind von dem Gesamtmittel 3.1 nicht erheblich entfernt. *Dagegen ist in Breite sehr wohl eine Abnahme von Nord nach Süd vorhanden.*

Unterscheidet man zwischen östlicher und westlicher Gruppe, getrennt durch den 140. Grad westlicher Länge, so findet man die meridionale Verteilung:

westl. v. 140°W	4.0	4.4	3.5	3.7	3.1	3.7	2.4	2.5	2.7	2.4	1.2	aus 31 Werten.
östl. v. 140°W	4.0	4.4	3.0	4.2	3.6	3.8	2.9	2.9	—	2.9	—	aus 30 Werten.

Mithin drückt das Gesetz in Breite sich in beiden Beobachtungshälften aus, was sehr für seine Realität spricht. In der folgenden Berechnung sind aber nur die Gesamtmittel benutzt worden.

Das Gesetz der Abnahme der Leitfähigkeit von Nord nach Süd kann nur einer physikalischen Ursache zugeschrieben werden, welche ebenfalls von Nord nach Süd in ihrer Wirkung abnimmt. Als solche bot sich dem Verfasser vor allem die Abnahme der Landbedeckung der Erde in dieser Richtung, denn von allen Ionisatoren, die luftelektrisch von erheblichem Einfluss sind, ist die von der Bodenluft ausgehende Dissociirung die wichtigste. Die durchdringende Strahlung des Meeresbodens kann die gewaltige Wassermasse des besonders tiefen Grossen Ozeans nicht durchdringen. Die Radioaktivität des Meerwassers ist gegenüber jener der Bodenluft äusserst gering. Gerade aus dem grossen Ozean sind jüngst durch W. Knoche Messungen des Emanationsgehaltes zwischen Chile und der Osterinsel veröffentlicht worden.[3] Sie ergaben 0.05 Mache-Einheiten mit einem Maximum von 0.20 M. E., während v. d. Borne[4] in Luft aus Alluvialerde 0.63, aus Diluvialsand 0.95 und

[3] *Phys. Zeitschr.* 13, 112–115, 1912.

[4] *Habilitations*schrift. Breslau 1905.

Digitized by Google

Tabelle I.

	160–170 E	170–180 E	180–170W	170–160W	160–150W	150–140W	140–130W	130–120W	120–110W	110–100W	100–90 W	90-80 W	Mittel
° °													
60–50 N	.....	.....	.....	.....	.....	.....	4.0 (2)	.....	.....	.....	.....	.....	4.0 (2)
50–40 N	.....	.....	.....	.....	.....	.....	4.4 (1)	.....	.....	.....	.....	.....	4.4 (1)
40–30 N	.....	.....	.....	.....	.....	4.4 (1)	3.0 (2)	.....	.....	.....	.....	.....	3.5 (3)
30–20 N	.....	3.1 (1)	3.4 (1)	.....	.....	.....	4.2 (2)	.....	.....	.....	.....	.....	3.7 (4)
20–10 N	.....	3.1 (1)	.....	.....	.....	.....	.....	.....	3.6 (2)	.....	.....	.....	3.4 (3)
10– 0 N	3.7 (7)	.....	.....	.....	.....	.....	.....	.....	3.8 (3)	.....	.....	.....	3.7 (10)
0–10 S	2.4 (4)	.....	.....	.....	.....	.....	.....	.....	3.2 (2)	2.2 (2)	3.0 (3)	3.4 (1)	2.7 (12)
10–20 S	2.5 (2)	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2.9 (4)	2.8 (6)
20–30 S	2.4 (1)	2.8 (3)	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	2.7 (4)
30–40 S	.....	2.4 (4)	.....	.....	.....	.....	.....	.....	.....	3.0 (2)	2.8 (4)	.....	2.7 (10)
40–50 S	.....	1.4 (1)	.....	.....	0.4 (1)	1.9 (1)	3.4 (5)	.....	.....	.....	.....	.....	2.6 (8)
Mittel...	3.1 (14)	2.6 (10)	3.4 (1)	.....	0.4 (1)	3.2 (2)	3.6 (12)	.....	3.6 (7)	2.6 (4)	2.9 (7)	3.0 (5)	.....

Digitized by Google

aus tiefen Grubenwettern 2.5 bis 35 M. E. feststellte. Man sagt nun, die Lebensdauer von Ra-Ems sei zu kurz, um vom Kontinente aus auf so grosse Entfernungen hin noch wirksam zu bleiben. Allein man hält sich da oft an die Halbzeit von 3.7 Tagen. In dieser Zeit ist aber die Wirksamkeit nicht verschwunden, sondern nur auf die Hälfte gesunken.

Gerade während der Niederschrift dieser Zeilen erscheint die Fortsetzung der eben erwähnten Arbeit W. Knoches[5] und knüpft an diesen Punkt an. Sie berechnet eine Schar Curie'scher Abfallskurven für die Ra-Emanation, deren Parameter Windwege in 24 Stunden sind. Danach lässt sich vor allem finden, wie viel Emanation in einer gegebenen Entfernung vom Lande noch vorhanden ist, ausgedrückt in Prozenten der vom Lande mitgebrachten Anfangsmenge. Unter zu Grundelegung seiner eigenen Messungen zwischen Chile und der Osterinsel findet er unter der tatsächlich beobachteten Windstärke für diese Ausgangsmenge durchaus mögliche Aktivierungszahlen.

Für das Gesetz der Landabnahme von Nord nach Süd wurden die Zahlen von Heiderich benutzt, welche Supan auf Seite 26 seiner "Grundzüge der physischen Erdkunde"[6] veröffentlicht. Sie geben für Gürtel von 10° Breite die Landbedeckung in Prozenten der ganzen Fläche zwischen den Breitenkreisen, und zwar für die westliche, die östliche Hälfte und die ganze Erde. Die Zahlen sind:

	Nord						Süd					
	° ° 60–50	° ° 50–40	° ° 40–30	° ° 30–20	° ° 20–10	° ° 10–0	° ° 0–10	° ° 10–20	° ° 20–30	° ° 30–40	° ° 40–50	° ° 50–60
Westl. Hemisphäre..	40.9	33.8	27.2	17.1	15.6	16.4	23.7	20.5	13.4	9.1	4.8	2.1
Oestl. Hemisphäre...	73.2	70.7	59.7	57.5	37.7	29.5	21.9	24.5	32.5	11.2	1.7	0.0
Ganze Erde.....	57.0	52.2	43.5	37.3	26.7	23.0	22.8	22.5	22.8	10.1	3.3	1.0

Die östliche Hälfte, die im wesentlichen die alte Welt umfasst, kommt für den Grossen Ozean nicht in Betracht. Die Ausgleichung unserer Zahlen der Leitfähigkeit gab, unter Berücksichtigung der verschiedenen Gewichte der Mittelwerte je nach der Anzahl der Einzelwerte:

für die ganze Erde: $\lambda = 3.15 + 0.013\ (P - 26.8)$,
für die westl. Hälfte: $\lambda = 3.15 + 0.041\ (P - 18.7)$.

[5] *Phys. Zeitschr.* **13**, 152–157, 1912.

[6] *Bei Veit u. Co.*, Leipzig, 1903.

Digitized by Google

Die Rückberechnung zeigt, dass die berechneten Werte auf der nördlichen Halbkugel zu klein, auf der südlichen zu gross sind. Die ganze Erde liefert als Summe der Fehlerquadrate 221, die westliche Hälfte 196; der mittlere Fehler einer einzelnen Darstellung ist für die ganze Erde ±0.47, für die westliche Hälfte ±0.44ESE.

Dies Ergebnis ermutigte zu einer näheren Erforschung der Frage. Hierzu erschien es zunächst erforderlich, die Heiderichschen Zahlen dem vorliegenden Beobachtungsgebiet etwas mehr anzupassen, d. h. statt seiner westlichen Greenwicher Hälfte die Erdhälfte einzuführen, deren Mittelpunkt in 150° westl. Länge auf dem Aequator liegt. Es wurde eine Lambertsche flächentreue Projection zu grunde gelegt und durch Ausmessen der neu hinzutretenden und der nunmehr wegfallenden Landgebiete die Heiderichschen Zahlen dem Probleme angepasst. Weiterhin wurden nicht mehr die Originalzahlen der Leitfähigkeit benutzt, sondern ausgeglichene Werte, wobei gerechnet wurde nach

$$\frac{g_{n-1}\,\lambda_{n-1} + 2g_n\,\lambda_n + g_{n+1}\,\lambda_{n+1}}{g_{n-1} + 2g_n + g_{n+1}},$$

worin die g die der Zahl von Beobachtungen entsprechenden Gewichte sind. Es ergeben sich so folgende zugeordnete Wertereihen:

	Nord					Süd			
	° ° 50-40	° ° 40-30	° ° 30-20	° ° 20-10	° ° 10-0	° ° 0-10	° ° 10-20	° ° 20-30	° ° 30-40
Landbedeckung....	44	32	18	16	14	17	18	14	16
Leitfähigkeit.....	3.90	3.66	3.60	3.61	3.33	2.87	2.74	2.72	2.53

Hierbei ist allerdings die isolierte Gruppe hoher Werte unter 140-130 Westl. Länge und 40-50° Südl. Breite nicht mitbenutzt worden.

Das Resultat ist:

$$\lambda = 3.22 + 0.032\ (P - 21)\ ,$$

Die Summe der Fehlerquadrate ist nur noch 124, der mittlere Fehler der Darstellung eines Wertes ±0.39 ESE.

Vergleicht man das Ergebnis der Rückberechnung mit den ausgeglichenen beobachteten Werten, etwa nach nebenstehender Figur, so ist festzustellen, dass noch systematische Abweichungen vorhanden sind. Sie besagen: auf der nördlichen Halbkugel ist das wirksame Land zu klein, auf der südlichen zu gross oder, was richtiger

Digitized by Google

sein wird: im Norden wirkt das Land intensiver (weil es näher ist) als im Süden.

Wollte man diese Frage weiter verfolgen, so müsste man für die einzelne Beobachtung feststellen, welche Entfernung von der Küste gerade bestand, und zwar dürfte man diese Entfernung nicht etwa geradlinig messen, sondern längs der Windbahn rückwärts

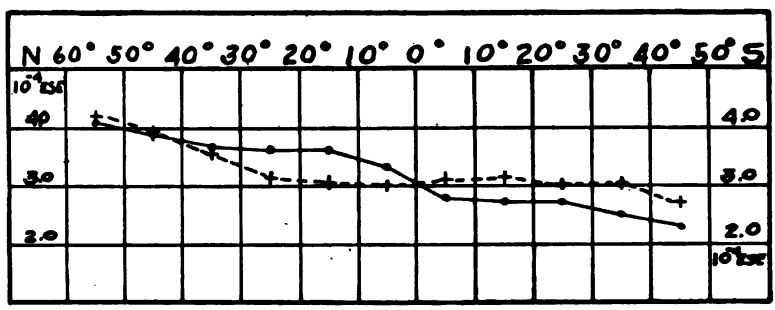


FIG. 2.

——— Beobachtete Leitfähigkeit.
+— —+ Berechnete Leitfahigkeit.

verfolgen. Dies führt zu umfangreichen synoptischen Witterungsstudien, die der Verfasser aus äusseren Gründen zur Zeit nicht vornehmen kann. Nur ganz im allgemeinen sei darauf hingewiesen, dass die unmittelbar am Orte festgestellte Windrichtung—also die letzte Tangente an die Strömungsbahn der Luft—überwiegend von der Meeresseite zur Landseite orientiert war, wie die Uebersichtskarte zeigt. Es ist also anzunehmen, dass reine Landwinde auch rechnerisch ein glatteres Ergebnis erzielt hätten.

Der lineare Abfall ist jedenfalls so gut parallel der Abnahme der Landbedeckung gefunden worden, dass man sagen kann: *die Leitfähigkeit der Atmosphäre über dem Grossen Ozean ist der Hauptsache nach durch die Landbedeckung der betreffenden Breitenlage gegeben.*

Kein anderes Gebiet ist so geeignet, unsere Frage zu lösen, als gerade der Grosse Ozean, erstens da seine Küsten annähernd auf einem grössten Kreise liegen, zweitens da er nördliche und südliche Breiten durchzieht und zuletzt, da er allein ein vollkommen ausgebildetes Windsystem besitzt. Alle Punkte im Atlantischen und die nördlichen im Indischen Ozean sind gar zu küstennah. So konnte der Verfasser aus einer Durchsicht der zahlreichen Messungen der Leitfähigkeit von der zweiten Kreuzfahrt der *Carnegie*[1] kein ent-

[1] *Diese Zeitschrift.* **16**, 237 *u. ff.*, 1911.

Digitized by Google

sprechendes Gesetz für diese beiden Weltmeere finden. Schon die absoluten Werte sind teilweise viel grösser, so namentlich im Indischen Ozean, offenbar weil gar zu gewaltige Landmassen ihn von drei Seiten umgeben. Ebenso liessen die zahlreichen Leitfähigkeits- oder Radioaktivitätsmessungen über den verschiedenen Meeren, die Abhängigkeit vom Land nur mehr oder weniger vermuten.

Es wäre also zu wünschen, dass noch einmal auf dem Stillen Ozean das nordsüdliche Gesetz der Abnahme der Leitfähigkeit ermittelt werde.*

Zum Schluss sei noch darauf hingewiesen, dass noch eine weitere Frage durch Schiffsbeobachtungen zu lösen wäre, nämlich die, ob irgend ein Anteil der Leitfähigkeit auf Rechnung der Dissociierung der oberen Luftschichten durch die Kathodenstrahlung seitens der Sonne gesetzt werden kann. Im allgemeinen dringt diese Ionisierung bekanntlich nicht in die Troposphäre ein, wohl aber könnte dies in den Gegenden tiefster Durchdringung, den Nachbargebieten der magnetischen Pole der Fall sein. Am nördlichen magnetischen Pol wird der nahe Landeinfluss sich mit dem anderen mischen, dagegen müsste der Südpol reinere Verhältnisse geben, da hier fast überall alles Land mit Inlandeis überdeckt, die Wirksamkeit der Erdbodenemanation daher sehr gering ist. Es käme also eine Umsegelung der Antarktis etwa zwischen 50 und 60° südlicher Breite in Frage. Dikes Messungen zwischen 120 und 140° w. L. und 40 bis 46° südl. Br. geben für diese Breitenlage hohe Werte, die mit einem von der Antarktis stammenden Wind einsetzen, dessen Luft radioaktiver ist und das Schiff nun einige Tage mit verschiedenen Richtungen begleitet.

Wenn also in Herrn Bauers grosszügigem Plane der magnetischen Vermessungen der Weltmeere noch Platz für eine Umsegelung der Antarktis in eisfreien Breiten ist, so möchte Verfasser die gleichzeitige Ausführung von Bestimmungen der Leitfähigkeit angelegentlich empfohlen haben.

* The cruise of the *Carnegie* in the Pacific Ocean during 1912, viz., Manila, Suva, San Diego, Tahiti, Coronel and around the Horn will doubtless afford further opportunity for testing the deduction which Nippoldt draws above from the *Galilee* observations, viz., as to the decrease of the electrical conductivity of the atmosphere over the Pacific Ocean in going from north to south.—L. A. B.

Digitized by Google

DETERMINATION OF THE POLE DISTANCE OF A VERY SMALL MAGNET.[1]

By J. M. Miller.

(Abstract)

If two circular coils, which are linear single turns, are adjusted so that they are coplanar and concentric, and if two currents are sent in opposite sense around the coils, the two magnetic fields will annul each other at the common center if the ratio of the two currents is equal to the ratio of the radii of the two coils. This gives a method of comparing the radii of coils experimentally. In such a comparison the neutralization of the two fields is usually indicated by a short magnetic needle which is suspended at the common center of the two coils and lies in their common plane.

On account of the finite length of the magnet an error is introduced into the comparison equal to $\frac{1}{4} K$ where K is a known function of the pole distance of the magnet and the radii of the coils. If, however, the needle is rotated by torsion or otherwise, out of the plane of the coils, the ratio of the neutralizing currents is changed by a factor $\frac{1}{4} K \sin^2 a$ where a is the angle through which the magnet has been rotated. Hence by measuring the angle a and also the change in the ratio of the neutralizing currents, K can be determined, and from this the pole distance of the magnetic needle can be calculated.

A piece of steel of only 2.314 mm. length was used as the magnetic needle and the pole distance for this very short magnet was determined to an accuracy of about three per cent. The resulting value for the pole distance was 1.956 mm.

Kohlrausch's modulus which is the ratio of the pole distance to the total length of the magnet is, therefore, equal to 0.845. Thus in the case of this extremely short magnet the poles are situated approximately one-twelfth of the total length from the ends, which agree closely with the results obtained by Kohlrausch and others for long magnets.

United States Bureau of Standards,
Washington, D. C.

[1] Presented before the Philosophical Society of Washington, Jan. 13, 1912. Jour. Wash. Acad. Sc. v. II, p. 140.

Digitized by Google

LETTERS TO EDITOR

PRINCIPAL MAGNETIC STORMS RECORDED AT THE CHELTENHAM MAGNETIC OBSERVATORY.

JULY 1 TO DECEMBER 31, 1911.

Latitude 38° 44'.0N; Longitude 76° 50'.5, or 5h 07'.4 W. of Greenwich.

GREENWICH MEAN TIME						RANGE		
Beginning			Ending			D (Declination)	H (Hor'l. Int.)	Z (Vert'l. Int.)
	h	m			h	′	γ	γ
June	30,	21 46	July	2,	15	23.8	123	44
July	10,	21 39	July	11,	20	12.0	92	34
July	17,	20	July	19,	22	23.2	86	72
August	23,	8 07	August	27,	24	42.6	154	105
Sept.	19,	18	Sept.	20,	20	36.8	126	59
October	10,	4 00	October	11,	6	37.0	152	130
Nov.	12,	22 00	Nov.	14,	19	23.0	73	36
Dec.	10,	17 57	Dec.	12,	1	39.1	105	102

GEORGE HARTNELL, *Observer-in-Charge.*

O. H. TITTMANN, *Superintendent,*
Coast and Geodetic Survey.

MAGNETISCHE STÖRUNGEN, FEB. 3-4, 1912.

Der Magnetograph des Observatoriums in Tsingtau verzeichnete am 3. und 4. Februar 1912 zwei Störungen, die eine auffallende Aehnlichkeit miteinander besitzen und auch dadurch von Interesse sind, dass sie fast zur gleichen Tageszeit auftraten. Charakteristisch sind bei beiden die Pulsationen, die etwa 1 Stunde vor Beginn des Anwachsens der H. I. auftreten, dann wieder verschwinden und erst wieder mit der betrachteten Störung einsetzen. Diese selbst zeigen ein rasches Anwachsen der H. I. und ein langsameres Abnehmen. Sie gleichen einer am 5. Oktober 1911 hier registrierten Störung, die jedoch nicht von Pulsationen begleitet ist. Es treten aber auch bei ihr ganz schwache Pulsationen etwa 1 Stunde vor dem Anstiege auf, zudem fällt die Störung auch auf etwa die gleiche Tageszeit. (Auch bezüglich der Mondphase ist eine gewisse Aehnlichkeit vorhanden.)

Digitized by Google

In Declination und Vertikal-Intensität sind die Störungen kaum bemerkbar, jedoch auch von Pulsationen begleitet, besonders in Vertikal-

TSINGTAU. HORIZONTAL INTENSITÄT.

[1 mm. ordinate = 3.6 γ; H. I. zunehmend nach oben].

Intensität. Die letzteren traten im Februar häufig auf, besonders bei der Horizontal-Intensität, und am bemerkbarsten um Mitternacht, meist mit Perioden von 2-3 Minuten und Amplituden bis 2 γ. Sie sind häufig mit einem Anwachsen von H. I. verbunden.

Den Herren Collegen wäre ich sehr dankbar, wenn sie Kopieen der (gleichen) Störungen vom 3. u. 4. Februar 1912 und vom 5. Okt. 1911 hier veröffentlichen oder mir zuschicken wollten.

Tsingtau, China, 17. Febr. 1912. B. MEYERMANN.

NOTES

1. Personalia. Captain Eduard Körber has been appointed director of the K. u. k. Hydrographisches Amt in succession to Rear Admiral Adolf Sobieczky, retired.

Professor Hellmann, director of the Prussian Meteorological Institute, was recently elected a member of the Royal Academy of Sciences of Berlin. Professor Adolf Schmidt has been made a Geheimer Regierungsrat.

2. Work of the Department of Terrestrial Magnetism. Dr. L. A. Bauer returned to Washington December 24th last after a nine and a half months' trip of inspection of the magnetic work being conducted, under his direction, on board the *Carnegie* and by land expeditions in Australia and Asia. *En route* he visited the following magnetic observatories: Apia, Christchurch, Melbourne, Mauritius, Kodaikanal, Bombay and Alibag, Dehra Dun, Barrackpore, Toungoo, Batavia and Buiten-

Digitized by Google

zorg, Hongkong, Manila and Antipolo, Zi-ka-wei, Tsingtau, Tokio, Honolulu and Tucson, Arizona. He also conferred with various government officials and scientific organizations and made arrangements regarding co-operative magnetic survey work. He observed the total solar eclipse of April 28, 1911, at Tau Island of the Manua group, having been favored with clear weather.

The *Carnegie*, in command of W. J. Peters, arrived at Manila early in February and is now en route to Suva, Fiji Islands. From thence she will proceed to San Diego, California, instead of Acupulco, Mexico, as originally announced. Besides Mr. Peters, the present scientific personnel consists of Dr. H. M. W. Edmonds and Messrs. H. D. Frary and H. F. Johnston.

Dr. N. E. Dorsey (Ph. D. Johns Hopkins, 1897), has resigned his position as associate physicist in the United States Bureau of Standards, having been appointed Research Associate in the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. He will have charge of special experimental and theoretical work, in which he will be assisted by Dr. R. H. Galt.

Mr. E. Kidson continues the general magnetic survey of Australia.

The following have been appointed magnetic observers: D. W. Berky for work in Northwest Africa with Mr. W. H. Sligh; Donald Mackenzie and H. R. Schmitt, members of Mr. J. P. Ault's party in Peru, Bolivia, Paraguay and Uruguay; C. W. Hewlett for ocean observational work aboard the *Carnegie*, beginning at San Diego, and A. D. Power. Prof. H. D. Harradon (A. B. Bates College, 1906) has been appointed translator and librarian.

3. Total Solar Eclipse in South America, October 10, 1912. Arrangements will be made to place as many as possible of the South American magnetic parties of the Carnegie Institution of Washington along the belt of totality beginning near Esmeraldas, Ecuador, and emerging about seventy miles south of Rio de Janeiro. *Co-operation of other institutions and persons in magnetic, electric, and allied observations during this period is solicited.*

4. Computed and observed moments of inertia of magnets. At the Berlin meeting of the Commission on Terrestrial Magnetism of the International Meteorological Committee (*cf.* T. M. v. 16, pp. 182-183) Dr. Liznar called attention to his proposal made some years ago "to determine the moment of inertia of the magnets used in oscillations directly from the dimensions and mass instead of observationally by means of other less homogeneous bodies, *e. g.*, brass cylinders." It will, therefore, be of interest to give here briefly the preliminary results of some comparative observations using both methods.

The standard magnetometer of the Coast and Geodetic Survey at the Cheltenham Magnetic Observatory is of the large Wild-Edelmann type,

Digitized by Google

the design of magnet and suspension purposely having been made very simple so as to admit of carrying out Wild's recommendation that the moment of inertia be computed rather than determined experimentally. As the result of careful determinations of the moment of inertia of the long magnet in 1902 and 1909, using various inertia rings, the value of 527.21 C. G. S. units was found at 20°C, whereas the equally carefully determined value as computed from the dimensions and mass of the magnet was 528.90 or about 3-1000 greater. If the observed value be used it would make this magnetometer accord well with the International Magnetic Standard of the Department of Terrestrial Magnetism, as dependent upon its international series of comparisons. The computed moment of inertia gives intensity results by about 0.0015H too high as based upon comparisons with about twenty magnetometers of various design and construction.

Similar experience was had at the Mount Weather Observatory with an instrument of the same type as before. The observed moment of inertia was 526.90, whereas the computed one was 527.34, or nearly 1-1000 larger than the former.

So also for the small French magnetic survey type of magnetometer, possessed by the Coast and Geodetic Survey, it was found that for long magnet L_1, the observed moment of inertia was 26.07, the computed one 27.42, or 1-20 larger, and for long magnet L_2, the observed value was 26.15 and the computed one 27.57, or again about 1-20 larger.

Thus in four cases which have come to our notice thus far the computed moment of inertia of a magnet is greater than the observed value. In each case the magnet house of the magnetometer used was of brass so that any effect due to impurity or to damping would enter into the experimentally determined moments of inertia. For the last case the discrepancies between observed and computed values can thus be largely explained. The matter is being investigated further.

LIST OF RECENT PUBLICATIONS

Terrestrial Magnetism.

Association Internationale des Academies. Procès-verbaux des seances de la Commission pour le Levé Magnétique le long d'un Paralléle le l'Association Internationale des Academies, le 21-23 Septembre 1910, à Berlin. St. Petersburg, Imprimerie Acad. Imp. Sci. 1911 (36 with 1 plate) 30½ cm. [*Cf.* Terr. Mag., v. XV, 177–180. The plate shows status of magnetic survey of the Globe. The next meeting of the International Association of Academies will be held in St. Petersburg in 1913.]

Bauer, L. A. Annual Report of the Director, 1911, Dept. of Terrestrial Magnetism, Carnegie Institution of Washington. Repr. from Year Book No. 10, pp. 198–207. [Plate gives status of work 1905–11.]

Digitized by Google

BIRKELAND, R. Mouvement d'une particule électrisée dans un champ magnétique. Arch. Sci. Phys., Genève, Jan. 15, 1912 (32).

BOCHUM. Ergebnisse der Deklinations-Beobachtungen in Bochum im Jahre 1909. (Erdmagnetisches Observatorium der westfälischen Berggewerkschaftskasse zu Bochum.) Bochum, Schaefers & Ecken, 1911 (20 with 1 pl.) 28½ cm. P.

BOCHUM. Ergebnisse der Deklinations-Beobachtungen in Bochum im Jahre 1910. (Erdmagnetisches Observatorium der westfälischen Berggewerkschaftskasse zu Bochum.) Bochum, Schaefers & Ecken, 1912 (20) 28½ cm. P.

BRESTER Jz., A. Essai d' explication de l'oscillation diurne de l'aiguille aimantée. Ciel et Terre, Bruxelles, v. 32, No. 12, Dec. 1911 (411-418).

BULGAKOV, N. The Gaussian theory of terrestrial magnetism and its significance in contemporary science. St. Petersburg, 1911, (122) 27 cm. P.

FARIS, R. L. Results of magnetic observations made by the Coast and Geodetic Survey between July 1, 1910, and June 30, 1911. Washington, D. C., Dept. Com. & Lab., U. S. Coast & Geod. Surv., 1912. Rep. for 1911, App. 3 (77-158) 29 cm.

FARIS, R. L. Distribution of the magnetic declination in the United States for January 1, 1910 with isogonic chart and secular change tables. Washington, D. C., U. S. Dept. Com. & Lab., Coast & Geod. Surv. Spec. Pub., No. 9, 1911 (14 with 1 chart) 18½ cm.

FARIS, R. L. Terrestrial magnetism. D. Appleton & Co., New York. Amer. Year Book, 1911 (596–597).

LÜDEMANN, K. Ablesevorrichtung für Magnetnadeln und Nonien von Max Hildebrand in Freiberg i. S. [Illus.] Zs. Instrumentenk., Berlin, v. 32, No. 1, Jan., 1912 (18-19).

NANSEN, FRIDTJOF. Die magnetische Abweichung im Anfang des 16. Jahrhunderts. Extract—Petermanns Geogr. Mitt., Gotha, Jan., 1912 (8-12) 28½ cm. P.

SCHMIDT, AD. Ergebnisse der magnetischen Beobachtungen in Potsdam und Seddin im Jahre 1910. Veröffentlichungen des Königlich preussischen meteorologischen Instituts, herausgegeben durch dessen Direktor, G. Hellmann. Berlin, Behrend & Co., 1911, No. 241 (24 and 4 pl.) 33½ cm. P.

SCHÜCK, A. Der Kompass. Hamburg, 1911 (18 with 46 pl.) 36 cm.

STANKIEWITCH, B. W., AND IOTCHIDLOWSKY, I. J. Annuaire de l'Observatoire Météorologique et Magnétique de l'Université Impériale à Odessa. 1910. Odessa, 1911 (222) 25½ cm. P.

STÖRMER, CARL. Sur les trajectoires des corpuscules électrisés dans l'espace sous l'action du magnétisme terrestre avec application aux aurores boréales, etc. Second memoire. Arch. Sci. Phys., Genève, Jan. 15, 1912 (51).

UNITED STATES COAST AND GEODETIC SURVEY. Report of the Superintendent of the Coast and Geodetic Survey showing the progress of the work from July 1, 1910 to June 30, 1911. (O. H. Tittmann, Superintendent.) Contains—Results of magnetic observations made by the Coast and Geodetic Survey between July 1, 1910 and June 30, 1911. By R. L. Faris—. Washington, D. C., Dept. Com. & Lab., U. S. Coast & Geod. Surv., 1912 (584 with 9 charts in pocket) 30 cm.

VEGARD, L. On the properties of the rays producing Aurora Borealis. Phil. Mag., London, Feb., 1912 (211). [Starting from the view that aurorae are caused by electric solar radiations, the author attempts to show that certain forms, especially the draperies, as regards position, occurrence, structure, and form are to be explained by rays possessing the properties rather of the α type than as assumed by his colleague Birkeland, of the β type which, he says, "meets with great difficulties." No new facts were at the author's disposal. He does not consider the combined effect of both kinds of rays, the β type, as known, being necessary for Birkeland's theory of magnetic storms.]

Digitized by Google

Wolff. Deviationsschwankungen infolge der plötzlichen Erwärmung des Schiffskörpers durch Sonnenstrahlen. Seefahrt, 1911, No. 21.

Zi-ka-wei Observatoire. Observations magnétiques faites a l'Observatoire de Lu-kia-pang, Vol. I, 1908. Shang-hai, Catholic Mission, 1911 (76 with 6 pl. and 2 charts) 31 cm. P.

Atmospheric Electricity.

Bergwitz, K. Die Registrierung des atmosphärischen Potentialgefälles mit Hilfe des Einfaden- und Zweifaden-Elektrometers nach Wulf. Physik. Zs., Leipzig, v. 13, No. 1, Jan. 1, 1912 (28-30).

Celestial Electric Phenomena. Review—Professor Birkeland of Christiana University has been making researches on the nature of electricity, and also endeavoring to explain a number of celestial phenomena upon an electrical basis. Elec. Rev., Chicago, Ill., v. 59, No. 25, Dec. 16, 1911 (1221)

Dieckmann, M. Das Elektrometerwehr. Physik. Zs., Leipzig, v. 13, No. 3, Feb. 1, 1912 (108-112).

Dorno, C. Studie über Licht und Luft des Hochgebirges. Braunschweig, Friedr. Vieweg & Son, 1911 (VIII + 154 with 78 tables and 30 figs.) 33½ cm. Bd.

Hess. Ueber Messungen der durchdringenden Strahlung bei zwei Freiballonfahrten. Abstr.—Weltall, Berlin, v. 12, No. 6, Dec., 1911 (85).

Ichinohe, R., Kinoshita, M., and Kimura, M. Electrical conductivity of the air. Coll. Sci. and Engin., Mem., Kyoto, v. 3, July, 1911 (155-169).

Joly, J., and Smyth, L. B. Amount of radium emanation in the soil and its escape into the atmosphere. Dublin, Sci. Proc. R. Soc., v. 13, No. 11, Aug., 1911 (148-161) Abstr. 1504. Sci. Abstr., London, v. 14, Pt. 11, No. 167, Nov., 25 1911 (506).

Kinoshita, S., Nishikawa, S., and Ono, S. Amount of radio-active products present in the atmosphere. Tokyo, Su. Buts. Kw. K., v. 6, No. 6, June, 1911 (92-111).

Knoche, Walter. Bestimmungen des Emanationsgehalts im Meerwasser und der induzierten Aktivität der Luft zwischen der chilenischen Küste und der Osterinsel. I. Physik. Zs., Leipzig, v. 13, No. 3, Feb. 1, 1912 (112-115).

Miscellaneous.

Poincaré, Henri. The trajectory and mass of comets. The forces that mold the "Bearded Meteor" and its path. Sci. Amer. Sup., New York, N. Y., v. 72, No. 1878, Dec. 30, 1911 (426–427).

Trabert, W. Beobachten wir wirklich zu viel? Met. Z. S. Jan., 1912, p. 7.

Righi, A. Sur la décharge à étincelle dans un gaz raréfié et sur sa transformation en faisceau de rayons magnétiques (Université de Bologne. Laboratoire de Physique). Radium, Paris, v. 9, No. 1, Jan., 1912 (24-32 with 12 figs.)

Runge, C. Zur astronomischen Ortsbestimmung auf See- und Luftschiffen. Repr. Göttingen, Nachr. Ges., Wiss., 1911 (11 with 4 text figs.) 24 cm. P.

Steiner, Desider. Die Hysteresisverluste der Ferromagnetisierbaren Manganaluminium-Bronzen in Abhängigkeit von der Frequenz des Wechselfeldes. Von der Technischen Hochschule zu Darmstadt zur Erlangung der Würde eines Doktor-Ingenieurs genehmigte Dissertation. Leipzig, Johann A. Barth, 1911 (50 with 5 pl.) 23 cm.

Williams, S. R. A model of the elementary magnet. Physic. Rev., Lancaster, Pa., v. 34, No. 1, Jan., 1912 (40-47 with 9 figs.)

Zi-ka-wei, Observatoire de. Calendrier-annuaire pour 1911. 3e année de Siuen-Tong (9e année). Shanghai, Imprim. Mission Catholique, 1910 (IV + 68) 15 cm.

Zilius, Jonas. Periodizität der tellurischen Erscheinungen und ihr Zusammenhang mit den Sonnenflecken. (33.33jährige Periodizität des Erdklimas). Weltall, Berlin, v. 12, No. 3, Nov., 1911 (33-42).

Digitized by Google

Digitized by Google

Fig. 1. The Absolute Building

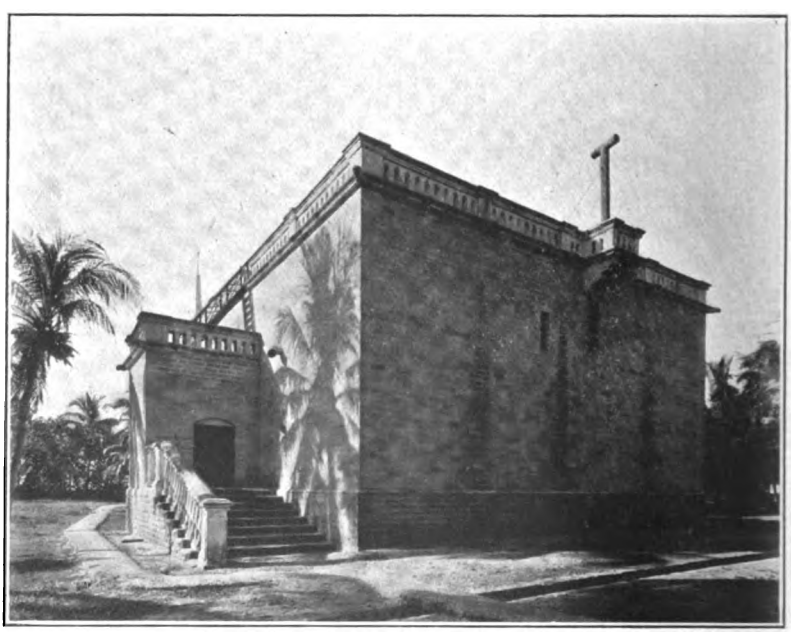


Fig. 2. The Variation Building

THE MAGNETIC OBSERVATORY AT ALIBAG, INDIA

Digitized by Google

Terrestrial Magnetism

and

Atmospheric Electricity

LUME XVII JUNE, 1912 NUMBER 2

THE PENETRATING RADIATION.

By W. W. STRONG,

University of Pittsburgh.

Outline.

(1) Its discovery.
(2) Daily and annual variations.
(3) Variations characteristic of the locality.
(4) The penetrating radiation at different altitudes above the ground.
(5) The penetrating radiation over ocean and lake surfaces.
(6) The γ radiation due to radioactive products in the ground.
(7) The γ radiation due to radium products in the air.
(8) The probable source of the penetrating radiation.
(9) Some problems relating to the penetrating radiation.

ITS DISCOVERY.

The announcement of the discovery of the penetrating radiation was made simultaneously by Rutherford and Cooke[1] and McClennan and Burton[2] in 1902. Investigations on radioactive substances had shown that some of these emit a very penetrating radiation, the γ radiation, resembling hard X-rays. "Since the excited activity obtained from the atmosphere is very similar in character to the excited radiations from thorium and radium, Rutherford and Cooke thought it possible that some penetrating rays might be given off from the surface of the Earth and the walls of rooms on which excited activity from air is distributed." In order to test for the presence of this kind of a radiation, the ionization in a one liter electroscope was measured in lead screens of different thicknesses. A screen 2mm. thick had very little effect on the ionization while a lead screen 50 mm. thick or a

[1] RUTHERFORD and COOKE, *Phy. Rev.*, 16, p. 183, 1903; COOKE, *Phil. Mag.*, 5, p. 403, 1903.
[2] MCCLENNAN AND BURTON, *Phy. Rev.*, 16. p. 184, 1903.

Digitized by Google

water screen 70 cm. thick reduced the rate of discharge about 30 per cent. Increasing the amount of lead in the screens to a weight of 5 tons did not cause any further decrease in the rate of leak. The removal of the screen resulted in the ionization current returning to its previous value. The radiation seemed to come equally from all directions, and Rutherford and Cooke conclude that its source may be due to the excited radioactivity upon the surfaces of the room. Similar effects, produced on an electroscope placed on frozen ground, showed that there is a penetrating radiation in the open air. In a brass vessel they give the rate of discharge as corresponding to the production of 10 ions per cu. cm. per sec., this being reduced to about 6 ions when the penetrating radiation is screened off. [The potential gradient used in Cooke's work was about 20 volts per cm.] Cooke[3] suggests that the penetrating radiation has its origin in the radioactive matter that is distributed throughout the earth and atmosphere.

McClennan and Burton used a cylinder 125 cm. long and 25 cm. in diameter with an electrode placed in the center charged to a potential of 165 volts. Filling the cylinder with air at a pressure of 400 cm. of mercury and surrounding it with a screen of 25 cm. of water reduced the ionization from 21.1 to 13.3, the units being arbitrary.

These and the investigations of other workers prove the universal existence of a penetrating radiation over all land surfaces. A small part of the ionization in the open air is produced by this radiation, which seems to possess properties that are the same as those of the γ rays from radium. Whether the penetrating radiation is the cause of any other terrestrial phenomena is not known.

DAILY AND ANNUAL VARIATIONS.

The earlier investigators discovered that the ionization in closed vessels was subject to various kinds of changes. The earliest to observe any periodic variation was G. Jaffé[4]. Jaffé found the natural ionization in nickel carbonyl to be about 5.1 times the ionization in air. The variations in the ionization in small vessels was much larger than the variations in large vessels, and amounted in certain instances to 40 per cent. [The electroscope used was of the Wilson type, *Proc. Roy. Soc.*, 68, p. 155; 69, p. 277, charged to about 80 volts.] Electroscopes placed near

[3] *Phil. Mag.*, 5, p. 411, 1903.
[4] *Phil. Mag.*, 8, p. 556, 1904.

Digitized by Google

together showed similar variations, whereas when placed in different rooms they showed very slight agreement, the deviations being sometimes in opposite directions. Large variations were common to both. A slight diurnal variation was noted, possessing a minimum between noon and 2 P. M. Large vessels of 3 liters capacity were found to show variations of about 10 per cent. Jaffé concluded that the variations depend upon the amount of surface of the ionization chamber rather than upon its volume. Borgmann[5] found a minimum of ionization to occur at 3 P. M.

The first to discover the diurnal period of the ionization in closed vessels was A. Wood[6] and N. R. Campbell. Vessels of 6 liters capacity were used with a central electrode charged to a high potential (potential not given). In most cases a gradual increase in the ionization was noted for the first ten or twelve days. McClennan and Burton, using smaller vessels, had observed a rapid increase at first, followed by a slow increase to a steady value. The increase was noticed for lead, no matter what the kind of gas used. In the case of a lead vessel in coal gas, the ionization current doubled in the course of seven days. Zinc vessels did not show this initial increase of the ionization current. Superimposed upon this permanent increase of the ionization current, Wood and Campbell discovered a periodic variation having two maxima and two minima daily. The most marked maximum appeared at about 11 P. M., and the most marked minimum, at 2 P. M.; the secondary maximum, at 8 A. M., and the secondary minimum, at about 4.30 A. M. The diurnal variation was found to take place independently of the kind of vessel and the nature of the gas. Curves are given showing that the range of the diurnal changes frequently amount to 15 or 20 per cent of the total ionization produced in the vessel. No relation was found between these variations and those of temperature. The atmospheric potential curves were shown to be somewhat similar and to possess two maxima and two minima at about the same time as those of the ionization curves. That temperature should not effect the ionization is to be expected since Clo[7] has shown that the ionization due to γ rays from radium is independent of the temperature between 20° and 615° C.

The experiments of Wulf[8] agree with those of Wood and Camp-

[5] *Nature*, **70**, p. 80, 1904.
[6] Woop, *Nature*, **73**, p. 583, Apr. 19, 1906; Wood and Campbell, *Phil. Mag.*, **13**, p. 265, 1907.
[7] *Astrophys. Journ.*, **33**, p. 115, 1911.
[8] *L'Electrometre Bifilaire*, p. 62, Louvain, 1910.

Digitized by Google

bell in showing a close relationship between the variation of the penetrating radiation and the potential gradient.

The writer[9] showed that the ionization in a closed vessel was subject to quite large variations. These variations were considered to be of three types: one due to meteorological changes, such as rain, snow, etc.; the second was of the same type as that discovered by Wood and Campbell, having a diurnal period; the third due probably to some effect on the apparatus such as would affect the nature of the ions formed, the radiometric effect on the gold leaf and possibly the leakage of the electric charge into the insulating supports. The electroscope had a capacity of about 2.6 liters and the active electrode was usually charged to a potential of from 1,500 to 2,000 volts. This was for the curved electrode[10] type of electroscope. Small electroscopes showed variations of a different type.

(*a*) The ionization in a "curved electrode" instrument was found to be decreased about 25 per cent in value by a fall of snow. This effect appeared to be due to the products emitting the γ radiation in the air being swept down by the fall of snow. The recovery of the ionization shortly after the fall of snow was explained as being due to the γ ray products of the air returning to their equilibrium value. Unfortunately the writer has not been permitted the opportunity to repeat these experiments.

(*b*) The diurnal variation showed maxima at 9 A. M. and 10 P. M., and minima at 7 A. M. and 6 P. M.

(*c*) Screening the electroscope was found to make the ionization current[11] quite constant and from this result it was concluded that the variation of the ionization in closed vessels was therefore due to a variation in the intensity of the penetrating radiation.

(*d*) The maximum at 9 A. M. and the minimum at 6 P. M. seem to be somewhat peculiar in their nature. The 6 P. M. minimum was found to be very marked, and its appearance was so sudden that it was considered to be due in part to a condition in the ionization chamber.

(*e*) The writer[12] concluded that a considerable part of the penetrating radiation came from radioactive products in the air, and that variable meteorological conditions such as the existence of cyclones and anti-cyclones were responsible for some of the variations observed in the ionization currents in closed vessels.

[9] *Science*, p. 52, July 12, 1907.
[10] *Phy. Rev.*, **27**, p. 41, 1908.
[11] STRONG, *Science*, **29**, p. 470, 1909.
[12] *Terr. Mag.*, **12**, p. 145, 1907.

Digitized by Google

(*f*) Small vessels were found to show a maximum rate of leak during the middle of the day, and this was shown to be due to the effect of illumination.[13]

McKeon[14] has studied the leak between concentric cylinders by insulating the inner one and measuring the charge that it acquired and the variation of this charge after having reached the steady state. The outer cylinder was 120 cm. long and 24 cm. in diameter; the inner one, 110 cm. long and 19 cm. in diameter. On connecting the insulated inner cylinder to the electrometer, the quadrants gradually acquire a charge which reaches a maximum in a few hours and then falls off without dying away altogether or ever changing sign. After some twenty hours there appears a progressive change in the charge—which may be either an increase or a decrease—and a diurnal periodic change—principal maximum at 10.30 P. M.; secondary maximum 11 A. M.; principal minimum 6 P. M.; and secondary minimum 5 A. M.

Pacini[15] found the maxima for the ionization in a closed vessel to appear at 2-3 A. M. and 9-10 A. M., the minima occurring at 7-8 A. M. and 1 P. M. The greatest production of ions per cu. cm. per sec. was found to be 30 and the minimum 6.3. The ionization as given by an Ebert instrument seemed to vary inversely as the ionization in a closed vessel. Mache[16] concludes from observations made at Innsbruck that the portion of the penetrating radiation coming from radioactive products in the air or existing as an active deposit on the surface of the ground may be more than four times as great as the quantity coming from the ground itself. Mache and Rimmer[17] consider that the active deposit on the surface of the ground is due largely to the Earth's being at a negative potential with reference to the air.

Mache[18] has measured the rate of discharge of an electrode placed in a box made of thin sheet zinc 1.5 by 1 by 1m., the box being supported on posts 2 m. high. The electrode was connected to an electroscope, the rate of discharge being measured by the time for the potential to fall from 220 to 212 volts, the reciprocal of this being taken as proportional to the intensity of the penetrating radiation. Observations were made at 8 A. M., 2 P. M., and 8 P. M. The following are the results: (1) The intensity of

[13] STRONG, *Phy. Rev.*, **27**, June, 1908.
[14] *Phy. Rev.*, Nov., 1907.
[15] *Rend. Lincei*, **18**, p. 123, 1909.
[16] *Sits d. K. Akad. d. Wis., Wien*, p. 119, 1910.
[17] *Phys. Zeit.*, **7**, p. 617, 1906.
[18] *Akad. Wiss. Wien, Sits. Ber.*, **119**, pp. 55-87, Jan. 1910.

Digitized by Google

the radiation shows a distinct annual period with a maximum in the autumn. The diurnal variation shows a maximum for the morning observation. For a whole year the figures are: 8 A. M., 81; 2 P. M., 67; 8 P. M., 68. (2) No conspicuous differences were noticed between radiation intensity and a rising, steady, or a falling barometer.

Gockel[28], expressing the rate of leak in terms of volts per hour, gives the following values: average in the winter for snow-free ground, 10.7; for snow-covered ground, 10.9. No definite yearly period was detected although there seemed to be a maximum from January to March and in July and August.

Important and very interesting experiments have been made by Jaffé[29] on the ionization of liquids such as hexane, using a potential gradient of about 200 volts per cm. The ionization current is about 12 times that of air. About two-thirds of the conductivity was shown to be due to the penetrating radiation.

Diurnal Period.

Observer	Variable	Maxima	Minima
Wood and Campbell	Ionization current; electric field strong; value not given.	11 P. M. 8 A. M.	2 P. M. 4.30 A. M.
Strong	Ionization current; electric field very strong.	10 P. M. 9 A. M.	6 P. M. 7 A. M.
McKeon	Radiation current? No electric field.	10.30 P. M. 11 A. M.	6 P. M. 5 A. M.
Pacini	Ionization Current	2.30 A. M. 10 A. M.	1 P. M. 7.30 A. M.
Wulf	Ionization current in Wulf electrometer.	9 P. M. 9 A. M.	1 P. M. 1 A. M. about
Börnstein	Barometric pressure in the ground.	10 P. M. 10 A. M.	4 P. M. 4 A. M.

[28] *Jahr. d. Rad. u. Elek.*, 9, p. 8, 1912.

[29] *Ann. d. Phys.*, 28, p. 326, 1909; 32, p. 148, 1910. *Phys. Zeit.*, 11, p. 571, 1910; *Le Radium*, 8, p. 293, 1911. *Compt. Rend.*, 151, p. 717, 1910.

Digitized by Google

The experimental work upon the variations of the ionization in closed vessels are of three types: temporary variations which disappear after the ionization chamber has been closed for some time; irregular variations whose cause is not known, one variation of this type, however, being sometimes attributed to the variation in the radioactive content of the air and the ground due to meteorological conditions; and periodic variations. The above description exemplifies how meagre is our knowledge of the exact nature of these variations and the experimental conditions existing in the ionization chamber during their measurement.

VARIATIONS CHARACTERISTIC OF THE LOCALITY.

If the penetrating radiation is due to radioactive matter, it follows that its intensity at any place will depend upon the amount of radioactive matter in the neighborhood. As the analyses of rocks have shown that their radium content is quite variable, it would be expected that the intensity of the penetrating radiation would depend upon the radioactive content of the surface soil and rocks and of the air. Indeed, the measurement of the intensity of the penetrating radiation might be made to serve as a test for the existence of radium and thorium existing in the surface rocks. A region containing rocks rich in uranium ought therefore be the source of an intense penetrating radiation.

At present little reliable data is at hand that gives any quantitative information of this kind, although there is considerable evidence of a qualitative nature. One reason for this is the fact that it is difficult to separate the effect of the ionization chamber upon the ionization currents measured. Among those who have made measurements of this kind have been Wulf[21], Bergwitz[22], Gockel[23], etc.

Gockel gives the following values of q, the number of ions produced per cu. cm. per sec. by the penetrating radiation, at different places: In a garden, 9.5; in a hole in the garden, 12.0; on a balcony, 13.5; in a room, 12.0; in a tunnel of granite, 33.5; in an ice chamber, 3.1.

According to Wulf[24] the walls of houses are the source of penetrating γ rays. In the following table from Wulf, the effect of the

[21] *Phys. Zeit*, **10**, p. 152, 1909.
[22] *Habilitationsschrift*, Braunschweig, 1910.
[23] *Jahr. d. Rad. u. Elek.* **9**, p. 4, 1912.
[24] *Le Radium*, t. 7, p. 173, 1910.

Digitized by Google

walls is the excess of the number of ions formed within them and the number formed in the open.

Place	Material of Construction	Age	Radiation in ions /cc. /sec.
Abbey of Maria-Laach, near Andernach-sur-Rhine	Volcanic tufa	50 yrs.	13.7
Fauquemont, College	Brick	15 "	5.7
Louvain, College	"	30 "	8.0
Namur, College	"	100 "	3.7
Wijnandsrade, Chateau	"	210 "	0.0

Wulf succeeded in obtaining the decay curves inside and outside of caves. Outside the decay curves indicated the presence of thorium, but inside only radium. The amount of deposit on an active wire was 150 times as great inside as outside the cave. It was found that the ionization in pools or small lakes in caves was much less than in the cave. The following table is due to Wulf:

	Matter	Meters below surface	Decrease of number of ions /cc.
Fauquemont	Water	20	10.0
Maria-Laach	Water	12	7.3
Hau-sur-Lesse	Stone	100	9.5
Charleroi	Coal-mine	983	2.2
Auvelais	"	200	0.7
Fleurus	Baryte	20	5.1
Namur	Water	2	6.5

Wulf has found a decrease in the ionization on the lake Maria-Laach. When on the lake, plunging the apparatus to a depth of 12 m. had very little effect in further decreasing the ionization. On the other hand, he finds that placing the apparatus in a hole 20 cm. deep the ionization was increased from 23.9 to 30.1 ions per cc. On the surface of a small lake the daily variation was found to be as great as on land.

Wulf[25] concludes from his observations that most of the penetrating radiation comes from radioactive matter in the surface of the ground not more than a meter deep. He thinks that the variation in the value of the penetrating radiation may be due to movements through the ground of air very rich in emanation.

In connection with the various problems of atmospheric ionization it is very important that observations be made of the active deposit, emanation content, intensity of ionization, intensity of the penetrating radiation, etc., at various altitudes in the atmosphere.

[25] *Annal. Soc. Sci. de Bruxelles*, **34**, p. 119, 1909–10.

Digitized by Google

THE PENETRATING RADIATION AT DIFFERENT ALTITUDES ABOVE THE GROUND.

A. Gockel[26] has measured the intensity of the penetrating radiation at altitudes of 4,000 m., using a Wulf electrometer. Large values of q are found even at these altitudes.

Wulf[27] has made a series of measurements of the number of ions produced by the penetrating radiation on the top of the Eiffel Tower (300 m.), and finds the value to be 3.5 ions per cu. cm. per sec., as compared with 6 ions at the foot of the tower and 10 at Valkenburg. If it is assumed that the γ rays are reduced to half their intensity in 80 m. of air, then the slow fall of the effect of the penetrating radiation can not be explained as coming entirely from the ground.

Gockel[28] gives the following values for the leak in a Wulf electrometer at different altitudes:

Starting	13.3	15.8
1,700 m	8.3	13.8
1,900 m	....	15.1
2,500 m	....	12.1
2,860 m	10.0	....
Landing	9.8	....

Hess[29] has obtained the following values of q, using a Wulf electrometer, with walls sufficiently thick to absorb all β rays. The value of the natural ionization when the instrument was placed in water was $q_0 = 25$. The coefficient of absorption of γ rays in air was found to be $0.45\ (10)^{-4}$.

Height	Ionization
Surface	32.3 ions per cu. cm. per sec.
150–440 m	28.1 " " " " " "
440–800 m	34.7 " " " " " "
800–900 m	34.3 " " " " " "
900–1000 m	35.5 " " " " " "
Landing	34.9 " " " " " "

The above ascent was made before noon. An ascent made during the night of October 12, 1911, showed no decrease in the value of q.

The general results of the observations on the intensity of the penetrating radiation at different altitudes are those of Gockel and Hess, who have found practically no decrease in the intensity with altitude; and those of Wulf on the Eiffel Tower, Bergwitz[30]

[26] *Phys. Zeit.*, **11**, p. 280, 1910.
[27] *Phys. Zeit.*, **11**, p. 811, 1910.
[28] *Phys. Zeit.*, **12**, p. 595, 1911.
[29] *Phys. Zeit.*, **12**, p. 1001, 1911; *Das Weltall*, **12**, p. 85, 1911.
[30] *Habilitationsschrift*, Braunschweig, 1910.

Digitized by Google

on a church tower, and McClennan and Macullum[31] on a tower—these observers finding a marked decrease in the intensity of the radiation; Wulf a decrease of 40 per cent for an altitude of 300 m.; Bergwitz a decrease of 50 per cent in 85 m.; and McClennan and Macullum a decrease of 25 per cent in 64 m.

THE PENETRATING RADIATION OVER OCEAN AND LAKE SURFACES.

Quite a number of measurements have been made of the radium content of ocean and lake waters, and the amount present has always been found to be very small—of the order of about $1.0\ (10)^{-15}$ grms. of radium per gram of sea water. From this poverty of the ocean and lake waters in radium it would follow that the emanation content of the air over mid-ocean should be very small. If, then, the penetrating radiation is due to radioactive products in the air and in the Earth's crust, its intensity far from land should be very small.

In general observers find much less active deposit far out at sea than is found over the land, although noted exceptions to this rule have been found. The exceptions may have been due to the wind currents being especially favorable for the carrying of radium emanation to sea, to the greater exposure of water surfaces to the winds or to the active deposit particles having a different mobility in ocean air from what they have in air over the land.

McClennan and Wright were among the first to take up measurements of the penetrating radiation above water surfaces. The following table gives the value of q as found by Wright:

	Lead	Zinc	Aluminium vessel
On Lake Ontario.....	6.4	4.4	4.8
On Land, Toronto.....	9.8	8.2	7.7
	3.4	3.8	2.9

The Canadian observers, such as Wright, Cline, Macallum, etc., have invariably found the evidence to indicate that the penetrating radiation, in the locality where they have experimented, is due to a γ radiation from the radioactive matter in the ground and that the value which they obtain on lake surfaces is the quantity q_0, the "natural" ionization of the vessel. Their conclusions seem to be in accord with the theoretical results that one obtains from the values of the radium content of the rocks and of the air in this locality. Then again these observers have not obtained any evidence of any diurnal periodicity in the intensity of the ionization of closed vessels.

[31] *Phil. Mag.*, 22, p. 639, 1911.

Digitized by Google

This is a condition that might be expected in the colder regions of the Earth's surface, although the work of Simpson in Norway does not support this view. One would expect that frozen ground covered with snow, would liberate much less emanation than ground in a warmer climate. The experiments of Pacini seem to support the view that in warmer climates a larger part of the penetrating radiation has its source in the radioactive products of the air.

Pacini[32] has extended his study of the penetrating radiation to include observations on the water near Livourne, Italy. He used two Wulf electrometers and obtained the same results with each. The value of the ionization per c. c. in closed vessels was about two-thirds of the value obtained on the shore (about 15 ions). The observations were made about 300 m. from land where the water was 4 m. deep. The variations of the penetrating radiation under these conditions seemed to be due to variations of the amount of radioactive matter carried by the winds from the land and the amount of the variations was as great as on land.

The observations of Wulf and Gockel are in more or less agreement with those of the Canadian observers. In general the value of q on lake surfaces, as measured by them, is small, while in holes, caves, and tunnels q may have quite a high value. Submerging the ionization apparatus below the lake surfaces does not usually result in any very considerable decrease of the ionization.

Kleinschmidt[33] has made measurements of the penetrating radiation from 300 to 500 m. above the surface of the sea without obtaining any evidence that the intensity was any different at this altitude from what it was at the surface.

Simpson and Wright[34] have observed the ionization in a closed 26 liter zinc vessel on the ocean. The ionization was found to increase as the coast was approached, and the phenomena are explained by them as being due to an active deposit formed upon the ship.

In general it may be concluded that the evidence obtained from observations of the penetrating radiation on lakes and the ocean favor the view that the radiation comes from the ground and is very weak over water surfaces. Pacini's results, however, indicate that a large part of the penetrating radiation comes from the air

[32] *Ann. dell' Uff. Centr. Meteor.*, **32**, part 1, 1910; *Le Radium*, **8**, p. 307, 1911; *Nuovo Cimento*, **6**, vol. 3, Feb. 1912.

[33] *Jahr. de Rad. u. Elek.*, **9**, p. 7, 1912.

[34] *Proc. Roy. Soc.*, **85**, p. 175, 1911.

Digitized by Google

and can be observed at distances sufficiently far from the coast that any radiation from the ground would be negligible.

THE γ RADIATION DUE TO RADIOACTIVE PRODUCTS IN THE GROUND.

The ionization due to the γ rays from radium and thorium in the ground can be calculated from the ordinary formula:

$$N = \int_0^\infty 2\pi r^2 \frac{N_0 Q}{r^2 e^{\lambda r}} dr = \frac{2\pi Q N_0}{\lambda}, \qquad (1)$$

N being the number of ions produced per cu. cm. per sec., λ the coefficient of absorption of the ground, Q the amount of radioactive matter per cu. cm. of the ground; N_0 is a constant giving the value of N when r = 1 cm. If $Q = 1.4\ (10)^{-12} \times 2.7$ and $\lambda = 0.34\ (2.7)$, Eve finds that $N = 0.8$ ion. On the sea this value would be about 0.006.

In calculating the intensity of the penetrating radiation for different heights above the ground one may make use of the analagous problem in optics of calculating the intensity of radiation from a source of light having a given volume distribution and given coefficients of absorption for each medium. Consider a point P in air whose coefficient of absorption is λ at a distance r_1 from a radioactive substance whose coefficient of absorption is κ and whose far surface is at a distance r_2 from P. The intensity of ionization per unit volume at P due to a density of radium distribution Q is,

$$I = \int Q C e^{-\lambda r_1} \left[\int_{r_1}^{r_2} e^{-\kappa (r - r_1)} dr \right] d\omega, \qquad (2)$$

C, being the intensity at unit distance, when there is no absorption, from the element of radioactive matter cut out by the solid angle dw. The integration is made to include the whole quantity of radioactive matter. If the radioactive matter is distributed uniformly throughout the region whose coefficient of absorption is κ, then,

$$I = \frac{QC}{\kappa} \int e^{-\lambda r_1} \left[1 - e^{-\kappa (r_1 - r_2)} \right] d\omega. \qquad (3)$$

This value of I leads to the result that the ionization per unit volume due to a distribution of Q grams of radium per unit volume throughout a mass having the surface S is the same as the total ionization throughout the volume bounded by S, due to Q grams of radium concentrated at P.

Digitized by Google

The problem of calculating the γ radiation from an infinite slab of thickness h containing a homogeneous distribution of radioactive material has been made by King[35]. Let z be the distance between P and the slab, ϕ the angle between PA and the normal from P to the slab, and ψ the azimuth angle which the plane

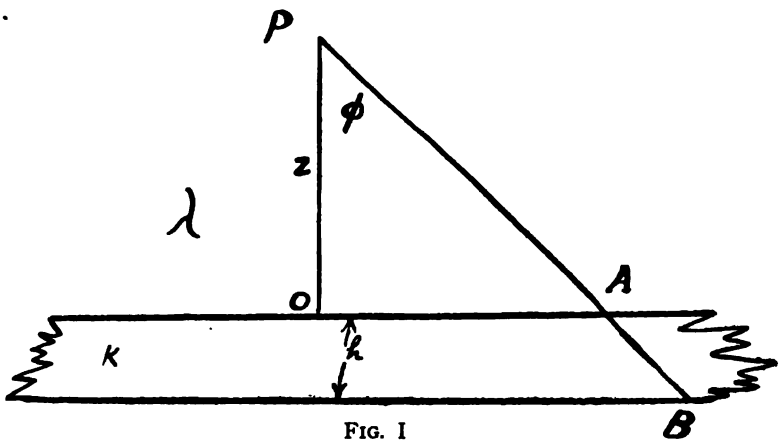


FIG. I

through PA and PO makes with a fixed direction in the plane of the slab. Then $AB = r_2 - r_1 = \frac{h}{\cos\phi}$. $PA = r_1 = \frac{z}{\cos\phi}$. $d\omega = \sin\phi\, d\phi\, d\psi$. The above equation on integrating with respect to ψ between 0 and 2π becomes:

$$I = \frac{2\pi QC}{\kappa}\int_0^{\frac{\pi}{2}} e^{-\lambda z \sec\phi}\left[1 - e^{\kappa h \sec\phi}\right]\sin\phi\, d\phi \quad . \qquad (4)$$

Letting $\cos\phi = \frac{x}{u}$, one has

$$\int_0^{\frac{\pi}{2}} e^{-x\sec\phi}\sin\phi\, d\phi = x\int_x^{\infty}\frac{e^{-u}du}{u^2} = e^{-x} - x\int_x^{\infty}\frac{e^{-u}du}{u} \qquad (5)$$

and[36]

$$\int_x^{\infty}\frac{e^{-u}du}{u} = -\int_{-\infty}^{-x}\frac{e^{u}du}{u} = Ei(-x) = -\frac{f(x)}{x} + \frac{e^{-u}}{x} \qquad (6)$$

From tables $f(x)$ can be calculated and this is done by King. When x is small and γ is Euler's constant 0.5772,

$$Ei(-x) = \gamma + \log x - x + \frac{x^2}{2.2!} - \frac{x^3}{3.3!} + \cdots \qquad (7)$$

[35] *Phil. Mag.*, 23, p. 244, 1912.

[36] GLAISHER, *Phil. Trans.*, p. 367, 1870; MILLER and ROSEBERG, *Trans. Roy. Soc. of Canada*, 9, p. 73, 1903.

Digitized by Google

and

$$f(x) = 1 + x \log x - (1 - \gamma)\, x - \frac{x^2}{2} - \text{- - - -} \quad . \tag{8}$$

When x is large the asymptotic expansion of $f(x)$ is obtained from (6) by successive partial integrations as

$$f(x) = \frac{e^{-x}}{x}\left[1 - \frac{2!}{x} + \frac{3!}{x^2} - \text{- - - -}\right]. \tag{9}$$

The intensity of ionization at a distance z is then

$$I = \frac{2\pi\, Q\, C}{\kappa}\left[F(\lambda z) - f(\lambda z + \kappa h)\right]. \tag{10}$$

This is in different notation the formula worked out by Eve.[11]

If n is the number of ions produced per sec. per unit volume of air by the γ radiation from the radium in the ground; Q is the amount of radium per unit volume in the ground (say 3.8 $(10)^{-12}$ grms.); κ is .034 × 2.7 cm.$^{-1}$ say; n_0 is the number of ions produced per sec. per unit volume in air under normal conditions by the γ rays from a curve of radium supposed concentrated at a point at a distance of 1 cm. from the place of ionization; and λ is about .44 $(10)^{-4}$ cm.$^{-1}$, then

$$n = \frac{2\pi\, Q n_0}{\kappa} \int_1^\infty e^{-\lambda z/v}\, dv \tag{11}$$

Taking the surface intensity at $z = 0$ as unity, King has calculated values of λz, κh, z and h for various values of z.

The curves show that even in the case of distribution of radioactive matter throughout an infinite thickness, practically the whole effect on the gradient is due to a layer about 10 cm. thick.

In the chapter upon the variation of the penetrating radiation with increase of altitude it was shown that the observations of some observers indicated a considerable decrease. In general, however, observations have indicated a much smaller diminution than theory would account for, while the observations of Hess and Gockel indicate no decrease at all.

THE γ RADIATION DUE TO RADIUM PRODUCTS IN THE AIR.

Knowing the amount of radium emanation in the air and assuming that there exists an equilibrium amount of radium C present, one can calculate the ionization produced per cu. cm. per second by the γ radiation from radium C.

[11] *Phil. Mag.*, **21**, p. 551, 1911.

Digitized by Google

In air at a distance r from a quantity of radium Q, the ionizing effect is $\frac{Q}{r^2 e^{\lambda r}}$, λ being the coefficient of absorption of the rays. Let N_0 be the number of ions produced per cu. cm. at a distance of 1 cm. from Q grms. of radium, then:

$$N = \frac{N_0 Q}{r^2 e^{\lambda r}} \quad . \qquad (12)$$

Curves of the functions I. $y = e^{-x}$.

II. $y = f(x) = e^{-x} + x \, Ei\,(-x)$.

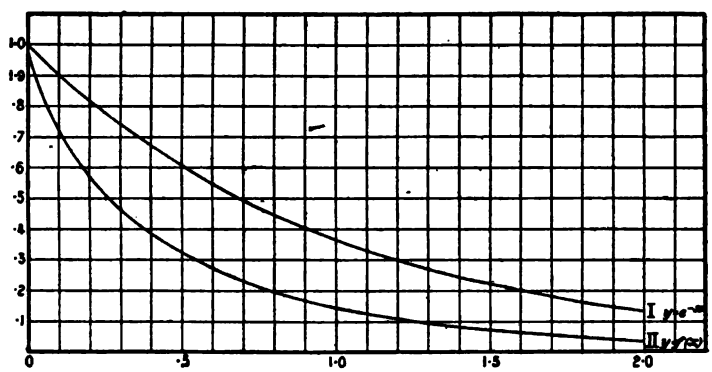


FIG. II

Decrease of intensity of penetrating radiation with height above the earth's surface. Intensity at surface is taken as unity. The various curves refer to intensities due to radioactive material in layers of different thickness.

Curve I. $h = .01$ cm. Curve III. $h = 1{\cdot}1$ cm.

Curve II. $h = {\cdot}11$ cm. Curve IV. $h = 11$ cm.

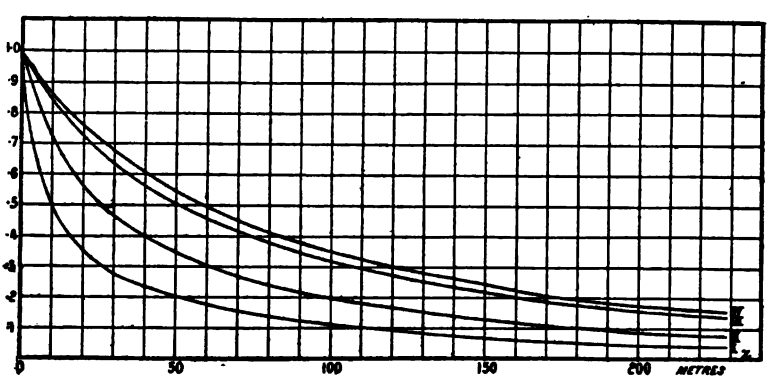


FIG. III

Digitized by Google

The ionization produced per cu. cm. due to a uniform distribution of radium is:

$$\int_0^\infty N_0 \, 4\pi r^2 \frac{Q}{r^2 e^{\lambda r}} \, dr = \frac{4\pi Q N_0}{\lambda} \quad . \tag{13}$$

Eve[38] has measured the value of N [3.1 (10)^{10}], and calculates the ionization in the open air arising from the action of the α, β and γ rays from the radium products in the air to be as follows:

Rays	Radium	Thorium
α	1.63	1.00
β	0.033	0.025
γ	0.035	0.025

Among those that have supported the view that a considerable portion of the penetrating radiation (and especially that portion which shows seasonal, meteorological and diurnal variations) is due to radioactive products in the air have been the author[39], Pacini[40], Mache[41], Gockel[42] (from balloon observations) etc. According to this view a very considerable portion of the radium emanation is liberated from the upper layer of soil and carried about in the atmosphere by the winds. This emanation would probably be carried to very considerable heights—at least as high as the height of cumulus and nimbus clouds. This view of the origin of the penetrating radiation would explain the balloon observations of Hess and Gockel.

The explanation of the diurnal period of the penetrating radiation depends upon the assumption that the barometric pressure in the ground shows a diurnal period. Theoretically the former should lag several hours behind the latter—the lag being the time required for the formation of the γ product, radium C. Data on the diurnal period of the penetrating radiation are not sufficiently exact to corroborate this lag.

Observations have been made by Börnstein[43] on the diurnal period of the barometric pressure at a depth of 1 m. The diurnal variation was found to be the same as on the surface, maxima at 10 A. M. and 10 P. M., and minima at 4 A. M. and 4 P. M., occur a little later than on the surface. If this periodicity of the barometric pressure is the cause of the diurnal periodicity of the

[38] *Le Radium*, **8**, p. 64, 1911.
[39] *Phy. Rev.*, **27**, p. 39, 1908; *Terr. Mag.*, Dec., 1907.
[40] *Nuovo Cimento*, **6**, vol. 3, Feb., 1912.
[41] *Wiener Ber.*, **119**, p. 55, 1910.
[42] *Phys. Zeit.*, **11**, p. 280, 1910; **12**, p. 595, 1909.
[43] *Phys. Zeit.*, **12**, p. 771, 1911.

Digitized by Google

penetrating radiation, this latter should have a phase lag of about 3 hours, due to the time required for radium emanation to come to an equilibrium value with radium *C*.

G. Wulf[44] explains the diurnal period of the barometric pressure (maxima in the evening and morning and minima at noon and midnight) as being due to an air tide and to the pressure of the sun's light.

THE PROBABLE SOURCE OF THE PENETRATING RADIATION.

As a result of the work of Cooke and the calculations of Eve, it was generally assumed at the time of Wood and Campbell's discovery of the diurnal period of the ionization in closed vessels, that the penetrating radiation came from radioactive products in the ground. No suggestion had been made as to the cause of the diurnal period of the ionization in closed vessels, although Wood and Campbell emphasize the similarity between these variations and those of the potential gradient of the air.

That the radiation does not come from an extraterrestrial source[45] seems to be thoroughly proved since the atmosphere (equivalent to 76 cm. of mercury) would completely absorb any such radiation. The experiments made to find the direction of the radiation have in general been inconclusive. The reason of this is probably due to the penetrating radiation being more or less diffuse and to its excitation of secondary radiation. It is quite probable that the constitution of the penetrating radiation is very complex. Recently it has been shown by Danysz[46] that the β radiation from various radioactive products is extremely complex. He finds at least 23 distinct types (depending upon their velocity) of β rays from the radium emanation. The general phenomena of vacuum tube discharges and radioactivity indicate that cathode and X rays and β and γ rays are very closely related. Cathode rays are the source of X rays whenever their acceleration is considerably changed. On the other hand, although a few cases have been noticed where β rays excite a γ radiation, yet in general it seems that the β and γ rays are both due to a common cause and if the β rays are very complex it is to be presumed that the γ rays are also.

The various elementary substances are known to possess characteristic fluorescent X radiations, the absorption coefficients

[44] *Journ. de la Soc. Physico-Chimique Russe*, p. 181, 1910.
[45] RICHARDSON, *Nature*, 73, p. 607, 1906; 74, p. 55, 1906.
[46] *Le Radium*, 9, p. 5, 1912.

Digitized by Google

of the radiations depending upon the nature of the secondary radiator. A substance only emits its characteristic X radiation when excited by a beam of X rays that is harder than its own radiation. Up to the present no corresponding types of fluorescent γ radiations have been found. If such existed, the penetrating radiation would consist of scattered primary γ rays and fluorescent γ radiations characteristic of the air, water, and the kind of soil and rock constituting the upper surface of the ground. Whether or not any effect of this kind takes place, it is probable that the penetrating radiation is composed of radiations of different coefficients of absorption and that the constitution of the radiation may be quite different in different localities.

The ionization in closed vessels is presumably due to external radiations, to secondary radiations from the walls of the vessel excited by these, to radioactive impurities in the gas itself or in the walls of the vessel and to any spontaneous ionization of the gas, if such exist. The smallest ionization that can be obtained in any way from vessels of given shape, size, and material, the enclosed gas being as pure as possible, will be called the "natural" ionization of the particular vessel and the intensity of this ionization will be called q_0, being given in ions produced per cu. cm. per sec.

The natural ionization of a vessel, depending upon the existence of a possible spontaneous ionization of the gas of the electroscope, to radioactive impurities of the walls or to a radiation that is not absorbed by thick lead screens, is presumably constant in quantity. (The writer[a] found the rate of leak of an electroscope surrounded by four feet of water in Sept., 1907, and in July, 1908. The value of the "natural" ionization was found to be the same in both cases.)

Variations in the ionization in closed vessels as usually measured by the ionization current may be due to several causes. (*a*) The gas or the walls may contain radioactive matter which slowly decays. For our purposes a variation of this type will not be considered. (*b*) The external penetrating radiation and the secondary radiation which it excites may undergo regular or irregular variations due to outside causes. As no method has as yet been devised for separating the effect of the direct and the secondary rays, these will be considered together. (*c*) The mobility or the rate of recombination of the ions may be modified by temperature,

[a] *Science*, p. 470, March 19, 1909.

Digitized by Google

illumination, etc., changes. Unfortunately very little is known concerning changes of this kind. Knoll[48] has shown that moisture and temperature have a very pronounced effect, and Gockel[49] has concluded that many of the variations attributed to changes of the intensity of the penetrating radiation due to variable meteorological conditions may be due to a change in the character of the ions formed in the closed vessel. Pacini[50] has found at times differences in the rate of leak of two Wulf electrometers placed under the same conditions, thus showing that there are certain instrumental errors that probably enter into the determinations of the ionization current.

The γ radiations coming from the products in the ground will vary from place to place according to the richness of the surface soils and rocks in radioactive matter. At any one place the radiation may vary to some extent depending on rain, snow, and the porosity of the surface soils. In very rocky regions any variation of this kind should be small.

The variation of the penetrating radiation coming from radioactive products in the air would be expected to vary very greatly under different meteorological conditions, such as rain, winds, changes of temperature, barometer, etc., since these products originate either directly or indirectly from the gaseous emanations of radium and thorium contained in the ground. The total radioactive content of the air over any place at any time depends upon the history of that air for several days before and would therefore depend on the nature of the prevailing winds.

In order to explain some variations in the rate of leak of electric charges in closed vessels the writer[51] considered that during cyclonic conditions the penetrating radiation was more intense than during anti-cyclonic conditions. During the former state of the winds part of the penetrating radiation comes from air that has been sweeping over the surface of the ground and that therefore ought to be rich in emanation. During anti-cyclonic conditions the air, coming from the higher strata of the atmosphere, would contain very little emanation and the penetrating radiation would then have a smaller value. Eve[52] finds that cyclonic winds cause the emanation content of the air to increase while anti-cyclonic winds are accompanied by a small emanation content.

[48] *Wien Ber.*, **115**, 2a, p. 164, 1906.
[49] *Jahr. d. Rad. u. Elektronik*, **9**, p. 2, 1912.
[50] *Annali dell' Ufficio Centr. Meteor e Geodin, Ital.*, **32**, Parte 1, 1910.
[51] *Phy. Rev.*, **27**, June, 1908.
[52] *Phil. Mag.*, **16**, p. 622, 1908.

Digitized by Google

The above are a few of the views that have been held as to the source of the penetrating variation. Experimental evidence seems to show that at places the penetrating radiation is nearly constant in value and probably comes almost entirely from the ground. At other places considerable variations are found in the intensity of the ionization currents in closed vessels, and the accepted explanation of these is that the penetrating radiation itself varies greatly in intensity. The simplest way of explaining these variations is that of assuming their source to be radioactive products in the air.

SOME PROBLEMS RELATING TO THE PENETRATING RADIATION

The problems concerned with the solution of the causes of the ionization in closed vessels include the problems of the intensity and the nature of the penetrating radiation and any secondary radiations which it may excite; the composition and the nature of radioactive matter distributed in ordinary substances; the question whether the common elements themselves are radioactive or not; the problem as to whether a gas is spontaneously ionized or not; and the nature of the phenomena of ionization itself. These are problems which the earliest workers upon the subject set out to answer, and as yet the results are more or less conflicting as to the nature of the penetrating radiation while the other problems have not been answered to any degree of certainty. One very good method of attack would be as follows:

(*a*) The study of the ionization in closed vessels under as simple conditions as possible. The work should be done in an underground laboratory as free from radioactive walls, emanations, etc., as possible, with all of the penetrating radiation cut off by screens. The temperature should be constant and the light used for illumination should be as constant and weak as possible, it being filtered of its heat rays. Variations of the following nature could be studied:

> Effect of the nature of the walls of the ionization chamber at different (1) temperatures; for different (2) sizes of the chamber; for different kinds of (3) gases and liquids in the chamber; for the gases at different (4) pressures; for the gases and liquids subjected to (5) ionization agents that produce ions of known mobility and diffusion constants;

(*b*) Under the above simplified conditions a thorough study should be made of the effect of various kinds of nuclei, dust, etc., upon the ionization currents. The action of light, heat rays, and ionizing agents could be used to produce and modify these nuclei. One effect of this nature is the radiometer effect of the light used for illumination purposes.

(*c*) Under the above simplified conditions a thorough study should be made of the ionization currents for electric fields of

Digitized by Google

different and known intensities and as much knowledge should be gained concerning the nature of the "natural ions" of gases as possible.

(*d*) The manner by means of which electric charges leak into the well-known insulators, such as sulphur, and the effect of illumination and temperature on this leakage should be carefully studied.

(*e*) It is well known that slight impurities in some gases produce very great effects upon the nature of the ions formed. In the study of natural ionization little care of this kind has been taken by most observers. The effect of different degrees of humidity should be carefully studied.

(*f*) After a thorough study of the "natural" ionization in closed vessels, a similar study should be made of the ionization produced by the penetrating radiation and by the γ rays of radium, thorium, and uranium.

(*g*) The various ionization effects obtained in closed vessels could be applied to the explanation of observations made of dispersion, the measurements of the ionic content of the air, etc. It is well known that measurements of this kind at present have little value on account of the complexity of the ionization phenomena as they take place in the open air.

(*h*) Simultaneously with the study of the penetrating radiation, determinations of the emanation content of the air should be made; also the ionization, the potential gradient, the barometric pressure, the temperature of the air, etc., at the same place. Up to the present, few correlated measurements of this kind have been made. Theories to account for the penetrating radiation or the ionization at one place are made from a few data as to the radium emanation content of the air or the average radium and thorium content of the rocks from various parts of the world, and the results are applied to certain specific instances. At present this is the best that can be done, but one must not forget the paucity of data of this kind and the very great errors that may be introduced in our present calculations.

(*i*) A radium and thorium survey should be made of the soils and rocks in the vicinity of the "atmospheric ionization" laboratory in order to correlate the radioactive content of the various winds with the radioactive materials of the surface of the ground over which they pass.

(*j*) In obtaining continuous records of the intensity of the penetrating radiation, the value of the potential gradient, etc., it would be important to use two or more apparatus of this kind at the same place in order to find how much of the variation is due to the characteristic properties of the apparatus.

Digitized by Google

RESULTS OF SOME LONG SERIES MAGNETIC OBSERVATORY OBSERVATIONS: COLABA (1846-1905), POLA (1847-1909), AND POTSDAM (1892-1900).

REVIEWED BY D. L. HAZARD.

Colaba Magnetic Record, 1846-1905.[1]

Magnetic work at Colaba was started in 1840, but it was not until 1846 that a regular system of observing was adopted. From that time to March, 1906, there is an unbroken series of observations. Photographic registration began in 1872. Prior to that time eye-readings were taken every hour except on Sundays and holidays.

Early in 1900 the city of Bombay decided to install electricity as motive power for its street cars. With the certainty of the line approaching within 1½ miles of the observatory, if not actually passing by it, it was apparent that the operation of the Colaba magnetograph would be seriously affected, and steps were therefore taken to establish a new observatory at a sufficient distance from Bombay to be free from the disturbing effect of the electric car lines. A suitable site was selected at Alibag, about 18 miles from Bombay, and after comparative observations had been secured at the two observatories for the years 1904 and 1905, the work at Colaba was discontinued in March, 1906.

The results of this remarkable series of observations are presented in the first volume, and their discussion follows in the second. The reader is at first overwhelmed by the mass of figures in tabular form which are presented for his study, but this feeling quickly gives place to one of admiration for the thorough manner in which the results have been discussed and the perseverance evinced in carrying such a tremendous piece of work to its conclusion. Every step of an investigation is given in detail and, wherever it is possible, the results are shown graphically as well as in tabular form. In the limited space here available it will be possible to give only a brief statement of the principal features of the work.

Absolute Observations.

A critical examination of the series of absolute observations of horizontal intensity extending from 1847 to 1867 led to the conclusion that they were only rough approximations and less reliable than the results from the Grubb horizontal force magnetometer. From 1868 to 1905 Kew magnetometer No. 23, made by Eliott Brothers, was used for determining D and H, and this 37-year series furnished data for

[1] Moos, N. A. F.: Magnetic Observations made at the Government Observatory, Bombay, for the Period 1846–1905, and their Discussion. Bombay, 1910. Parts 2; pp. 776; 111 plates.

Digitized by Google

a very instructive study of the life of this instrument. The constants determined at the beginning of the series were used to the end. Deflection observations were always made at two distances, and the moment of inertia was determined once a month up to 1880 and once in 3 months after that year. The results indicate a progressive change in both the moment of inertia and the first distribution coefficient, but as the instrument was found to be in close agreement with four other magnetometers with which it was compared in 1902-3, it was not deemed necessary or desirable to attempt to allow for the change. A study of the base-line values indicates that the average probable error of observation for a single determination of H was about $\pm 8\gamma$. A comparison of the decrease of the magnetic moment of the magnet with the increase of the first distribution coefficient indicated the possibility of a relation between them represented by the expression

$$P_0 \sqrt[4]{M} = \text{constant}$$

From 1846 to 1867 an old pattern Barrow dip circle was used, but the results can be considered only as rough approximations. From 1867 to 1905 the dip observations were made with Kew dip circle No. 49. It seems to have given satisfactory results except for the period from 1873 to 1887.

Variation Observations.

The magnetograph was installed in 1870. The adjustment of the D variometer was not changed from that time to 1905, but it was found impossible to secure uniformly dry and rarefied air inside the bell jar until July, 1876, and the results up to that time had to be corrected for the effect of moisture on the suspension fibers. After a comparison of the Grubb eye-reading declinometer with the variometer, it was possible to make use of the Grubb eye-readings as far back as 1861 with a fair degree of accuracy and back to 1846 as an approximation to the truth.

On account of the gradual loss of magnetism of the magnet of the H variometer there was a gradual drift of the zero (base-line value), which necessitated a readjustment from time to time. After these had been allowed for, a study of the drift indicated an annual variation superposed on the secular change. Eye-readings of the Grubb force magnetometer, which had been made hourly up to 1872, were made five times a day after that date. By a study of the variation of the zero after that date it was possible to make use of the series back to 1856. It was found that the thermometer did not correctly record the changes of temperature of the Grubb magnet, the former being more susceptible to change than the latter.

Digitized by Google

Vertical force variometer No. 1 was in operation from 1870 to 1905, and No. 2 from 1893 to 1905. The latter was more reliable. The ordinary scale value deflections at one distance were supplemented once a year by deflections at 5 distances in order to find the value of the distribution factor. The variation in the scale value of No. 2 was mainly a function of time. In the case of No. 1 the variation in scale value was ascribed partly to a variation in the curvature of the knife edges. A comparison of the two instruments for a period of 5 years indicated that there was some restriction of the motion of the magnet of No. 1 when Z was decreasing.

The change of paper and care of the magnetograph took place between 9 and 10 A. M. The tabulation of hourly ordinates was made from 10^h of one day to 9^h of the next, so that the diurnal variation curve would not be affected by any undetected disturbance of the instruments at the time of change. The results are presented in the form of differences from the mean of the month. The diurnal variation in temperature in the underground room was so small that allowance for it was not necessary.

Discussion of Results.

The general features of the *secular change* of H for the period covered by this series of observations are well represented by the formula

$$H = 37237 + 16.75t - 0.35t^2,$$

in which t is the time in years since 1871.5, the maximum occurring in 1895. The departures of H from this smooth curve amount in the extremes to 30γ and follow closely the variations in sun-spot activity, being negative in years of great activity and positive in quiet years. A period of 11.3 years seemed the most probable.

The curve deduced in 1896 to represent the secular change of D required no modification to fit it to the later observations. The maximum was reached about 1879. Here too there is some evidence of the influence of sun-spot activity.

The dip and vertical intensity were increasing throughout the period, the latter increasing from 12874γ in 1868 to 15083γ in 1905.

In the discussion of the variations of shorter period, principal dependence was placed on the photographic registration from 1872 to 1905. The series was divided into three 11-year periods, and the results were deduced from each period separately.

A variation with a period of six months was more pronounced than the yearly variation, but the amplitudes were small. In H there was also a well-defined variation having a period of 14 months.

The *diurnal variation* is discussed in great detail both for all days and for the five selected quiet days, and the coefficients of the Fourier series are derived for each month in each case. The results show little

Digitized by Google

difference except that the amplitude is somewhat less for the quiet days. In the case of H the maximum occurred at about 11^h and the time of minimum varied from 22^h in February to 18^h in June. For D there was a principal maximum at 8^h, a principal minimum at 12^h, and a secondary maximum at 16^h. The diurnal variations of Z and I were similar to that of D. In addition to graphical representation for the various elements, vector diagrams are given, showing the diurnal variation of force in the horizontal plane and also in the prime vertical plane.

The discussion of *magnetic disturbances* is confined almost exclusively to horizontal intensity, as their effect upon D and Z is usually very small in so low a magnetic latitude. The frequency and intensity of storms was greater at the equinoxes and less at the solstices. The most marked effect of a magnetic storm is a rapid decrease in H for a few hours, followed by a more gradual increase, two or three days usually being required to recover the loss. From this it follows that the mean of quiet days is almost invariably greater than the mean of all days. From a study of 113 abruptly beginning storms it was possible to construct type curves for that kind of disturbance. The curve for H shows a decrease of about 110γ in the first 11 hours, followed by a slow rise of only 16γ in the next 13 hours. In the case of D there was an average easterly motion of about $1'.2$ in the first 10 hours. For seven selected storms a comparison was made with the magnetograms from Kew, Pavlovsk, and Batavia, and it was found that while the beginnings were practically simultaneous, the divergence of the later portions indicated that secondary forces were called into play locally.

The *lunar-diurnal variation* was investigated and found to be different for different phases of the moon. For the whole lunation a period of 12 lunar hours was well defined.

Regarding the effect of *solar rotation* there was more indication of a 13.5-day period than of one of 27 or 27.5 days.

Comparison was made of the variations in the Earth's magnetism and various meteorological phenomena.

There is little evidence of any relation between magnetic disturbances and *earthquakes*. A comparison with the tabulated earthquake records indicates that, if anything, an earthquake is more likely to occur on a magnetically quiet day than on a disturbed one. There does appear to be some relation between *volcanic eruptions* and magnetic disturbances. There is a marked resemblance between the magnetic disturbances recorded at the times of the Krakatoa eruption in 1883, the eruption of Kelvet in 1901, and of Mont Pelee in 1902. While they begin abruptly with a small increase of H, they do not show the succeeding rapid decrease which is almost always present in storms beginning abruptly.

Digitized by Google

The magnetograph at Colaba recorded many peculiar disturbances of short duration, some of them occurring at the time of a seismic disturbance. The number of these disturbances increased from a few a year to 201 in 1902, and then fell off to only a few a year again. Two types were recognized, one where the motion of the magnet was so rapid as to produce simply a blur, and the other where the waves had a period of about 2 minutes. The first type is evidenty the result of a mechanical disturbance, but the second type must be explained in some other way.

Results of the Magnetic Observations at Pola, 1847-1909.[2]

While the magnetic declination was determined at Pola in 1806 by Beautemps-Beauprés and in 1819 and 1823 by Smith, these observations were necessarily lacking in accuracy, and it was not until 1847 that the first determination of the three magnetic elements was made by Kreil in connection with the first magnetic survey of Austria. Observations were made again at Pola in 1854 by Kreil and in 1868 by Schellander, but at different stations.

With the establishment of the Hydrographic Office in 1869 it was intended that frequent determinations of the magnetic elements should be made, but, except for a series of weekly observations of declination and horizontal intensity extending from October, 1874, to April, 1875, no systematic observations were made until 1881.

A new observatory building having an underground room for the variation instruments was completed in 1877, but not until the summer of 1880 was it possible to prevent the water from coming in and to keep the room dry enough to undertake the mounting of the instruments. Beginning with January, 1881, eye-readings of the declination variometer were made twice a day up to August, 1885, when the photographic registration began. Before that time an attempt was made to use collodium plates, but it proved a failure. At first the record was made in polar co-ordinates on a rotating disk. This method had so many serious disadvantages that in 1896 the recording apparatus was remodeled and a revolving drum substituted for the rotating disk.

Absolute observations of horizontal intensity began in March, 1882; variation observations in January, 1884; and photographic registration in 1886. In 1897 a new bifilar magnet was provided, having a much smaller temperature coefficient than the old one, and a damping device was added.

Absolute observations of dip were begun in 1881. Various dip circles were used at different times, but the results with them have been

[2] Kesslitz, W. von: Ergebnisse aus den erdmagnetischen Beobachtungen in Pola (1847-1909). Veröff. des Hydrog. Amtes der K. und K. Kriegsmarine in Pola. Gruppe V, Nr. 30, Pola, 1911. Pp. 74.

Digitized by Google

reduced to the standard of the Wild earth inductor, purchased in 1896. Beginning with 1877, efforts were made to get a vertical force variometer in satisfactory operation, but, although one was started in 1890, its record up to June, 1899, was practically worthless. Even after that date the results were not entirely satisfactory and a number of changes of variometer had to be made. In 1908 an attempt was made to diminish the disturbing effect of a nearby electric car line, which had become sufficient to broaden the vertical intensity curve to about 3 mm., except for the hours from 11 P. M. to 6 A. M. A compensating device was installed which at first worked satisfactorily, but after a year or two ceased to be of any value.

From this brief account of the operation of the Observatory it will be seen that the director had more than his share of difficulties to overcome, and no doubt the work of reducing the observations was correspondingly increased. He is to be congratulated on his success in deducing the homogeneous series of results presented in this publication.

The monthly and yearly mean values of the three magnetic elements are presented in tabular form up to the end of 1909.

The discussion of the diurnal variation was limited to the ten-year period of 1900-1909, as before that time the adjustment of the vertical force variometer was not satisfactory, and in 1910 the effect of the electric car line had become sufficient to render the results less reliable. The mean diurnal variation of the three magnetic elements, D, I, and H, of the total intensity, T, and of the three rectangular components, X, Y, and Z, are given for each month, and from these are derived the corresponding average values for the ten-year period. These average values are combined into three groups of four months each; namely, January, February, November, December; March, April, September, October; May, June, July, August. Finally a single series representing the average for the whole year is derived.

The harmonic analysis of the diurnal variation was carried out for D, X, Y, and Z, but only for the average monthly values for the ten-year period. The computation was carried to four terms, and interesting diagrams show graphically how the different terms enter into the makeup of the resultant curve. The average variations for the three groups and for the whole year are also represented graphically, showing the characteristic changes in phase and amplitude with change of season.

A study of the secular variation from 1847 to 1909 shows a steady decrease of west declination throughout the period, but the annual change, which had decreased to about 4′ in 1899, rose rapidly to more than 6′ in 1909, a phenomenon which has been noted at other places.

The publication also contains an investigation of the diurnal and annual variation of disturbances and a comparison of sun-spot and disturbance frequency.

Digitized by Google

Finally, vector diagrams are given showing the change in the direction of the Earth's magnetic intensity in the course of the day, for the three groups of four months.

Magnetic Observations at Potsdam, 1892-1900.[3]

In the annual publications of the Potsdam observatory for the years 1892 to 1900 the hourly values of *D*, *H*, and *Z* are given without accompanying explanatory data. The present volume contains the information regarding absolute observations, scale value determinations, base-line values, changes and adjustments of instruments, etc., needed for the proper interpretation of those results. The most important changes in that period were the purchase of a new standard magnetometer in 1893 and the erection of a new absolute observatory in 1897.

The original magnetic observatory is a stone building with underground rooms for the variation instruments and rooms on the ground floor for absolute observations. It was decided to locate the magnetic observatory on the grounds of the Astrophysical Observatory only after tests had shown that the buildings existing or contemplated at that time would have no appreciable effect on the magnetic instruments. While the magnetic observatory was under construction and soon after its completion additional buildings were constructed for the Astrophysical Observatory and the Geodetic Institute, containing such large masses of iron that the site of the magnetic observatory could no longer be considered free of disturbance. In order to be able to determine the effect on the absolute values of the magnetic elements produced by the erection of new buildings, a station was selected about 1 km. to the southeast, at which observations were made with great care from time to time and compared with the observatory results. A little later plans were prepared for the erection of a dome 22 meters inner diameter only about 160 meters from the magnetic observatory, involving the use of 255,000 kg. of iron. Under these conditions there could no longer be any doubt as to the necessity for providing a new place for absolute observations.

While the desirability of a site well removed from possible disturbance was recognized, questions of expediency finally led to the selection of a spot about 100 meters west of the old observatory and 260 meters from the proposed new dome. In the summer of 1896 the long contemplated magnetic survey of the observatory grounds and vicinity was begun. In 1896-7 measures of *D* and *H* were made at over fifty stations, and the spot selected for the new absolute observatory was found to be satisfactory.

[3] BRÜCKMANN, W.: Ergebnisse der magnetischen Beobachtungen in Potsdam, Ergänzungsband zu den Jahrgängen 1892–1900. Veröff. des K. Preuss. Met. Instituts. Nr. 232, Berlin, 1911. Pp. 100, with 3 plates.

Digitized by Google

During the erection of the great dome monthly observations were made to determine the effect in the old observatory of the masses of iron used in its construction, and after its completion tests were made of the effect of rotating the dome, which was found to be measurable on the variation instruments.

The new absolute observatory was begun in the Spring of 1897, and completed in May, 1898. It is 5 by 12 meters in size, with the longer axis approximately in the magnetic meridian. On the middle of the east side is an extension 4.2 m. wide and 2.4 m. deep, forming a vestibule and a room for astronomical observations. The main building is divided into 3 rooms with sliding doors. It is heated by 4 small gas stoves and lighted by electricity. The piers rest on solid masses of concrete about 1 meter thick and do not come into contact with the floor. Oscillations were made on each pier, to be sure that no local disturbance had been introduced in the construction, and a careful comparison was made of the absolute values in the new and old observatories.

The new Wanschaff magnetometer was received in November, 1892. A comparison with the former standard magnetometer led to the discovery that observations with the old instrument were subject to a constant error amounting in the case of the declination to over 4′, which was due apparently to magnetic impurities in the metal entering into its makeup. The two parts of the new magnetometer, the theodolite and the oscillation box, have separate bases and are mounted on separate piers. The horizontal circle of the theodolite is 27 cm. in diameter and is read by two micrometer microscopes directly to 0′.05. The magnets are tubular, with polished ends, and the relative length of the long and short magnets is such as to make the second distribution coefficient very nearly zero, so that the first distribution coefficient could be determined from deflections at two distances. From a comparison of the observed and smoothed out base-line values for 1908 it was concluded that for the Wanschaff magnetometer the probable error of a determination of H is only $\pm 1\gamma$, and of D only $\pm 0'.16$.

The Bamberg dip circle was continued as the standard instrument up to 1900, then giving place to a Wild-Eschenhagen earth inductor made by G. Schulze. The Weber earth inductor was found to be less reliable than the dip circle, and the latter was not altogether satisfactory.

During the years 1892-1900 no material changes were made in the variation buildings or instruments. The Wild-Edelmann set of eye-reading instruments continued to be used as a control over the photographically recording Mascart variometers. Eye-readings were taken three times a day and also at the time of the absolute observations. The scale values and temperature coefficients were determined in the usual way, but in the case of the Mascart Z variometer allowance was made

Digitized by Google

for the fact that the magnets of the D and Z variometers are quite dissimilar. Additional deflection observations were made with the Z magnet suspended by a silk fiber, so that its knife edges were vertical, the deflecting magnet being placed to the East and West. The corrective factor was found to be 1.0885 for deflections at a distance of 27 cm.

In addition to the regular work of the observatory, a number of special investigations were carried out. A declinometer was set up in a room in the office building and eye-readings were taken at 7^h, 14^h, and 21^h, for the purpose of getting early knowledge of the outbreak of a magnetic storm.

To investigate the question whether magnetic disturbances occur simultaneously at different places, simultaneous readings were made in 1895 in co-operation with the Wilhelmshaven Observatory, at selected hours and at 5-second intervals. The results of these observations led to the international simultaneous observations of 1896, in which 15 observatories joined.[4] They led also to the design and construction of the Eschenhagen type of magnetograph.

In the years 1899 and 1900 the observatory took part in the international observations made in co-operation with the Russo-Swedish expedition to Spitzbergen and the Danish to Iceland.

An investigation was made as to the character of the earthquake disturbances recorded by a magnetograph, and the conclusion was reached that they are purely mechanical.

In view of the threatened installation of an electric care line in Potsdam, a study was made of the effect of such a line in Spandau, and it was found that even at a distance of 15 km. there was an appreciable disturbance of very sensitive instruments.

The publication concludes with the tabulated absolute observations and base-line values and explanatory notes regarding changes of adjustment of the instruments, etc.

[4] ESCHENHAGEN, M.: Internationale magnetische Simultanbeobachtungen, 1896. Anhang zu Heft II der Ergebnisse für 1896.

Digitized by Google

THE PHYSICAL THEORY OF THE EARTH'S MAGNETIC AND ELECTRIC PHENOMENA.—NO. V.

(Continued)

By L. A. Bauer.

ON THE FORMATION OF THE EARTH'S MAGNETIC FIELD.

54. The opinion was expressed in §2 that it was doubtful whether "we should ever be able to solve the problem as to the origin of terrestrial magnetism until we had found out what causes the Earth's magnetism to vary." A general study of the time and spasmodic variations of the Earth's magnetic condition led to a relation which was put in the following tentative form (*l. c.*): "Alterations in the Earth's magnetic condition, whatever their nature or origin may be, appear to be distributed over the globe according to a law profoundly dependent upon that governing the distribution of the Earth's own primary magnetic forces." The exemplification and deduction of the consequences of this law are reserved for a future number.

It seems advantageous at this stage to take up a study of possible disturbing forces which, operating on a simple, uniform magnetization of the Earth about some diameter, such as that of the axis of rotation, could change it into the heterogeneous or complex magnetic system of forces actually observed. The method of the Gaussian analysis results in a cumbersome mathematical expression embracing a large number of terms. Though it has been in use for three-quarters of a century, and has counted among its devotees most eminent investigators, the precise physical interpretation of the harmonic terms beyond that of the first still awaits solution. Indeed, the general opinion has been that the departure from uniform magnetization is to be ascribed primarily to the heterogeneous structure and formation of the Earth's crust. A favorite theory referred the chief cause of irregularities in the distribution of the Earth's magnetic forces over its surface to the relative distribution of land and water.

And even with respect to the first simple harmonic, interpreted as a uniform magnetization about a diameter inclined at the present time between 11° and 12° to the rotation axis, it is often overlooked

Digitized by Google

that this represents merely an average state of affairs which may possibly have no real existence by itself, but, instead, may be simply the resultant of several operating systems. Discussion, therefore, as to the cause of the supposed uniform magnetization and as to the possible clue afforded by the near coincidence of magnetic and rotation axes, might possibly be preceded advantageously by an inquiry as to how the first three Gaussian coefficients defining the uniform magnetization are really made up. Otherwise might it not happen that we would be making difficulties for ourselves which do not exist?

Because of the above reasons, the method of decomposition and study of the Earth's magnetic field used in the series of papers PD. Nos. I to V (*cf.*, Resumé, §2) was introduced for the purpose of supplementing the results derived from the Gaussian analysis. It was thus found that there are certain general distorting systems acting over the entire Earth which exceed in importance the effects due to possible regional disturbances or to the distribution of land and water. For example, in the belt between 50° and 60° South, where water almost wholly predominates, and where, in consequence, fewer irregularities in the distribution of the Earth's magnetism might be expected, there, nevertheless, the Earth's general disturbing forces are such as to make the axis of uniform magnetization for 1885 incline 21° from the axis of rotation, whereas, for the corresponding zone in the Northern Hemisphere, in which land chiefly prevails, and where, therefore, local disturbances are more predominant, the same inclination is but 8½°.

The map of the residual magnetic field derived by eliminating from the total field that portion due to the average uniform magnetic system about an inclined diameter, as mentioned above, immediately revealed to the eye that the Gaussian harmonics, beyond the first, contained in them a system of disturbing or distorting forces which, as a first approximation, can be characterized as a double transverse or crosswise magnetization, the axes of which lie roughly near the planes of the circles of latitude, 40° N. and 50° S. The chief axes in the two planes are oppositely directed. For further details the reader must be referred to PD. No. I (*Terr. Mag.*, v. 4, p. 44).

Additional interest was aroused by the fact that the system of magnetization thus found had characteristics similar to those of the diurnal variation magnetic system disclosed by Schuster's researches (*l. c.*, pp. 46-49).

Digitized by Google

It is confidently believed that the foregoing facts will serve as crucial tests of any theory of the cause of the Earth's magnetism that may be advanced. Accordingly, it has been thought well to devote the present number to a more complete presentation of the phenomena.

General Facts of the Earth's Uniform Magnetic Field.

55. The potential of a sphere of radius, R, magnetized uniformly about a diameter, is as follows for a point P outside, whose position is fixed by the polar co-ordinates, u (the co-latitude counted positive and continuously from the north to south geographical pole), λ, (the longitude counted positive, east from Greenwich) and r (radius vector counted from the center of the sphere, positive outwards to the point P):

$$V_o = \frac{R^3}{r^2}(g_{10}\cos u + g_{11}\sin u\cos\lambda + h_{11}\sin u\sin\lambda). \qquad (16)$$

g_{10}, g_{11}, and h_{11} correspond, respectively, to the well-known Gaussian coefficients, g^0_1, g^1_1, h^1_1.

Let ρ be the intensity of magnetization per unit of volume; ξ_m, η_m, ζ_m its three rectangular components, the first along the axis of rotation (positive towards the North Pole), the other two in the equatorial plane (the positive directions being, respectively, towards the meridian of Greenwich and 90° east). Let the direction of the axis of magnetization be defined by the co-ordinates of its point of intersection with the surface in the Northern Hemisphere, *viz.*, $u_n = 90° - \phi_n$, λ_n; then if M is the magnetic moment about the magnetic axis and M_p and M_e, the rectangular components resolved along the axis of rotation and at right angles thereto (in the equatorial plane):

$$g_{10} = \frac{4\pi}{3}\xi_m = \frac{4\pi}{3}\rho\cos u_n \quad , \qquad (17)$$

$$g_{11} = \frac{4\pi}{3}\eta_m = \frac{4\pi}{3}\rho\sin u_n\cos\lambda_n \quad , \qquad (18)$$

$$h_{11} = \frac{4\pi}{3}\zeta_m = \frac{4\pi}{3}\rho\sin u_n\sin\lambda_n \quad , \qquad (19)$$

$$\rho = \sqrt{\xi^2_m + \eta^2_m + \zeta^2_m} = \frac{3}{4\pi}\sqrt{g^2_{10} + g^2_{11} + h^2_{11}} \quad , \qquad (20)$$

Digitized by Google

$$M_p = \frac{4\pi}{3}\,\xi_m \,.\, R^3 = c_p \,.\, R^3 \quad , \tag{21}$$

$$M_e = \frac{4\pi}{3}\,\sqrt{\eta^2_m + \zeta^2_m}\,.\,R^3 = \sqrt{g^2_{11} + h^2_{11}}\,.\,R^3 = c_e\,.\,R^3 \quad , \tag{22}$$

$$M = \sqrt{M^2_p + M^2_e} = \frac{4\pi}{3}\,\rho\,R^3 = \sqrt{g^2_{10} + g^2_{11} + h^2_{11}}\,.\,R^3 = c\,.\,R^3 , \tag{23}$$

$$\tan u_n = M_e \,/\, M_p = + \sqrt{g^2_{11} + h^2_{11}} \,/\, (-g_{10}) \quad , \tag{24}$$

$$\tan \lambda_n = (-h_{11}) \,/\, (-g_{11}) \quad . \tag{25}$$

Let the total magnetic intensity at P be resolved along three rectangular co-ordinates, X along a meridian positive towards the North, Y along a parallel of latitude, positive towards the East, and Z along the vertical positive towards the Nadir, then by partial differentiations of (14), bearing in mind the sense in which the variables increase, we have:

$$X_o = -\frac{R^3}{r^3}\,(g_{10}\sin u - g_{11}\cos u\cos\lambda - h_{11}\cos u\sin\lambda) \quad , \tag{26}$$

$$Y_o = \frac{R^3}{r^3}\,(g_{11}\sin\lambda - h_{11}\cos\lambda) \quad , \tag{27}$$

$$Z_o = -\frac{2R^3}{r^3}\,(g_{10}\cos u + g_{11}\sin u\cos\lambda + h_{11}\sin u\sin\lambda) \quad . \tag{28}$$

For a point P within the sphere,

$$V_w = r\,(g_{10}\cos u + g_{11}\sin u\cos\lambda + h_{11}\sin u\sin\lambda) \quad , \tag{29}$$

$$X_w = -\,(g_{10}\sin u - g_{11}\cos u\cos\lambda - h_{11}\cos u\sin\lambda) \quad , \tag{30}$$

$$Y_w = (g_{11}\sin\lambda - h_{11}\cos\lambda) \quad , \tag{31}$$

$$Z_w = (g_{10}\cos u + g_{11}\sin u\cos\lambda + h_{11}\sin u\sin\lambda) \quad . \tag{32}$$

If P be on the surface, hence $r = R$:

$$X_o = X_w\,;\; Y_o = Y_w \text{ and } Z_o = -\,2\,Z_w \quad . \tag{33}$$

Suppose now P to be on the surface and that the observed magnetic components, X, Y, Z, are due partly to a uniform magnetic system arising from agencies wholly inside the Earth whose potential is V_i (equal to the V_o above) and partly to a uniform magnetic system caused by agencies wholly outside the Earth's

Digitized by Google

surface having a potential V_e (equal to the V_w above). Let the coefficients $g'_{10}, g'_{11}, h'_{11}$ define V_i and $g''_{10}, g''_{11}, h''_{11}$, define V_e, then it will readily be seen from above that:

$$X = X_i + X_e = -(g'_{10} + g''_{10}) \sin u + (g'_{11} + g''_{11}) \cos u \cos \lambda + (h'_{11} + h''_{11}) \cos u \sin \lambda, \quad (34)$$

$$Y = Y_i + Y_e = (g'_{11} + g''_{11}) \sin \lambda - (h'_{11} + h''_{11}) \cos \lambda, \quad (35)$$

$$Z = Z_i + Z_e = -(2g'_{10} - g''_{10}) \cos u - (2g'_{11} - g''_{11}) \sin u \cos \lambda - (2h'_{11} - h''_{11}) \sin u \sin \lambda. \quad (36)$$

56. We are now ready to make application of the foregoing formulæ. Let us scale from the Neumayer magnetic charts for 1885 the values of the magnetic elements (D, declination, I, inclination, and H, the horizontal intensity) for points on the parallels 60° N, 50° N, 40° N, 30° N, 20° N, 10° N, Equator, 10° S, 20° S, 30° S, 40° S, 50° S, and 60° S, and 10° apart in longitude. For each of the parallels there are then 36 points at which D, I, and H have been scaled and for each of which in turn the rectangular components, $X = H \cos D$, $Y = H \sin D$, and $Z = H \tan I$, are obtained. Substituting these values in equations (34), (35), and (36), the coefficients (g, h) may be derived by the method of least squares, either separately for each parallel or at once for the entire zone 60° N to 60° S.

Table XII contains the results thus obtained on the supposition that the whole of the Earth's uniform magnetic system is due to magnetic and electric systems enclosed within the surface which, indeed, analysis has shown to be correct within a few per cent. Here, moreover, while a least square solution is made separately for each parallel, the components X, Y, Z are treated together so that there are 36×3 observation equations for any one parallel from which the three unknowns, g_{10}, g_{11}, and h_{11} are derived: from these, with the aid of equations (21) to (25), the quantities tabulated are obtained.

It must be borne in mind that the least square solutions are made on the supposition that the departures of the actually observed magnetic forces at points along a parallel from those due to a uniform magnetization partake of an accidental nature, so as to tend to cut themselves out in the summation for a parallel. This, likewise, analysis has proved to be true to a very high degree. Comparing, for example, the quantities in Table XII with those

Digitized by Google

of Adolf Schmidt's careful analysis, in which the unknowns are derived by retaining in the observational equations the successive harmonics to the sixth order, it is found that the differences are so trivial as not to vitiate the conclusions in the slightest degree.

TABLE XII.

The Earth's Magnetic Axis and Magnetic Moment for 1885 as determined separately for each parallel of latitude from a combined treatment of the three rectangular components, X, Y, Z.

Parallel	Axis		Moment /R^3			Ratio
	ϕ_n	λ_n E of Gr	C_p	C_e	C	C_p / C_e
°	°	°				
60 N	81.7	259.7	.308	.045	.311	6.89
50 N	81.3	267.9	.321	.049	.325	6.55
40 N	81.4	276.2	.328	.049	.332	6.63
30 N	81.3	278.9	.339	.052	.343	6.54
20 N	80.8	273.6	.341	.055	.345	6.16
10 N	80.6	272.4	.341	.057	.346	6.02
Equator	80.1	275.4	.337	.059	.342	5.71
10 S	78.6	283.2	.328	.066	.335	4.98
20 S	76.4	294.4	.321	.078	.330	4.13
30 S	73.9	305.7	.310	.090	.323	3.47
40 S	71.8	314.2	.302	.099	.318	3.04
50 S	69.7	320.3	.302	.112	.322	2.70
60 S	69.3	329.7	.302	.114	.323	2.65
60 N & S	77.4	311.8	.305	.068	.312	4.49
50 N & S	76.8	304.9	.312	.073	.321	4.25
40 N & S	77.3	301.7	.315	.071	.323	4.44
30 N & S	78.0	296.0	.325	.069	.332	4.71
20 N & S	78.8	285.7	.331	.066	.337	5.05
10 N & S	79.6	279.1	.334	.061	.340	5.46
Equator	80.1	275.4	.337	.059	.342	5.71

And this, too, in spite of the fact that his quantities depend upon scalings 5° apart in longitude, hence upon twice the number of points used in our calculations. How closely the quantities for the entire region 60° N to 60° S, as derived by these simple methods, will agree with those from the more elaborate analyses for 1885 is shown by the following comparison:

	Magnetic Axis ϕ_n °	λ_n °	Magnetic Moment (C. G. S. units)
Adolf Schmidt	78.54 N	291.48	.32378 R^3
Fritsche	78.48	291.85	.32186
Bauer (approximate values)	78.4	293.1	.3236

Digitized by Google

It is seen that the North Magnetic Pole of the uniform magnetic system as derived by the approximate method is only 18 statute miles distant from the mean of Schmidt's and Fritsche's positions. The approximate magnetic moment, $0.3236R^3$, differs but ¼ per cent from the mean of the more elaborately derived ones.

The figures in Table XII, except for revisions, are taken from Table VI, PD. II (*Terr. Mag.*, v. 6, 1901, p. 21), and the following conclusions, practically the same as those of 1901, may be drawn:

1. The inclination of the magnetic axis to the rotation axis is greater for parallels in the Southern Hemisphere than in the Northern Hemisphere. For the parallels 60 N. to the Equator[1] the inclination as deduced from the magnetic components combined is practically the same; viz., about 9°, whereas, for the Southern Hemisphere, the inclination increases with increase of southerly latitude at the average rate of about 0°.18 per degree latitude, so that for the parallel 60 S. the inclination amounts to 21°.

2. The meridional plane of the magnetic axis shifts its position almost uniformly to the eastward with descending latitude of parallel; *i. e.*, turns in the direction of the Earth's rotation. This fact might be expressed otherwise, thus: The meridional plane of the magnetic axis moves steadily from its mean position for the entire zone, in the Northern Hemisphere, in a direction contrary to that of the Earth's rotation, and in the Southern Hemisphere in the same direction as the Earth's rotation, as we go poleward. The amount of motion in longitude is on the average 0°.6 per degree latitude, the extreme values of λ_n being 260° E. for parallel 60 N., and 330° E. for parallel 60 S.

3. The value of the magnetic moment is a maximum for the equatorial belt, and decreases steadily in both hemispheres with departure from the belt.

These laws will be put in a mathematical form later.[2]

A mere casual inspection of the tabulated quantities will immediately show that the forces which operate over the Earth's surface so as to change the magnetic axis and magnetic moment of the uniform system in such a regular and systematic manner as is disclosed in the changes of ϕ_n, λ_n, c_p, c_e, and c, from parallel to parallel, are not such as may be ascribed to heterogeneous systems, but instead to those following perfectly definite laws which further study should reveal.

[1] This was erroneously printed in the original as 60° S.

[2] Dr. W. van Bemmelen, while investigating the secular motion of the magnetic axis, obtained the following direction angles of the axis for 1885 from declination data alone which fell in with the first two deductions above, viz:

For Zone	Φ_n	λ_n
60° N—30° S	80°	278° E
20 N—20 S	78	283
30 S—60 S	74	302

Digitized by Google

57. Table XIII puts the salient facts shown by the previous table in a still more suggestive form, and with its aid a number of questions of considerable importance to the theory and analysis of the Earth's magnetic phenomena can be answered definitely. We have now the direction angles of the magnetic axis and the values of the magnetic moment of its two rectangular components as derived from zones; the effect of the area of the region embraced can thus be seen to advantage.

Beginning with the column headed c_p, the quantity proportional to the component of the Earth's magnetic moment along the axis of rotation, it will be seen that the value increases systematically until somewhere near the equatorial region, after passing which it decreases and again systematically. The same fact is shown by the figures in column c, the quantity proportional to the total magnetic moment. At the same time it will be noticed that the respective quantities do not have quite the same values for corresponding zones in the Northern and in the Southern Hemisphere. However, the differences are once more systematic —the total magnetic moment, as well as its polar component, is invariably less, for the same zone, in the Southern than in the Northern Hemisphere. The equatorial component of the magnetic moment, as shown by the c_e column, on the other hand, is systematically greater in the Southern Hemisphere than in the Northern for the same zone. A formula containing a constant, a sine term, and a cosine term will represent the three sets of quantities quite closely.

What do the facts revealed by this table signify? Omitting for the present the phenomena asymmetrical about the equator, turn to the lower half of the table: the systematic increase of the qualtities, c_p and c, as well as the systematic decrease of the c_e quantities, with approach to the equatorial regions, are both made very evident. If we compute the average intensity of magnetization, ρ as derived from the observed magnetic elements from 60° North to 60° South, for which according to the table, the total magnetic moment, in C. G. S. units, is 0.324 R^3, we find with the aid of equation (23), $\rho = \frac{3}{4\pi} c = 0.0774$ whereas for the equatorial region, 10° N. to 10° S., the same quantity has risen to 0.0812, or an increase of five per cent. Assuming that the systematic decrease continues as a larger and larger portion of the Earth is embraced in the analysis, it is found as a first approximation, that the mag-

Digitized by Google

TABLE XIII.

The Earth's Magnetic Axis and Magnetic Moment for 1885 as derived from zones from a combined treatment of the three rectangular components, X, Y, Z.

Zone Embraced	Axis		Moment/R^3			Ratio
	ϕ_n	λ_n E of Gr	c_p	c_e	c	c_p / c_e
° °	°	°				
0 to 60 N.....	80.8	273.1	.326	.053	.330	6.15
0 to 50 N.....	80.8	274.2	.332	.054	.336	6.15
0 to 40 N.....	80.8	275.1	.336	.055	.340	6.14
0 to 30 N.....	80.7	274.9	.339	.056	.344	6.08
0 to 20 N.....	80.5	273.9	.340	.057	.345	5.97
0 to 10 N.....	80.3	274.0	.339	.058	.344	5.88
Equator	80.1	275.4	.337	.059	.342	5.71
0 to 10 S.....	79.4	279.5	.333	.062	.339	5.35
0 to 20 S.....	78.5	285.0	.328	.067	.335	4.91
0 to 30 S.....	77.6	290.8	.322	.071	.330	4.54
0 to 40 S.....	76.7	295.8	.316	.075	.325	4.24
0 to 50 S.....	75.9	300.2	.312	.078	.322	3.98
0 to 60 S.....	75.5	303.9	.309	.080	.320	3.86
60 N to 60 S	78.4	293.1	.317	.065	.324	4.86
50 N to 50 S	78.5	290.1	.321	.065	.328	4.92
40 N to 40 S	78.8	288.4	.325	.064	.331	5.05
30 N to 30 S	79.1	285.0	.330	.063	.336	5.20
20 N to 20 S	79.4	280.8	.333	.062	.339	5.36
10 N to 10 S	79.8	277.2	.335	.060	.341	5.56
Equator	80.1	275.4	.337	.059	.342	5.71

netic moment of the whole Earth, from 90° N to 90° S, may have dwindled to 0.309 R^3 or that the average intensity of magnetization may be but 0.0738, hence nearly 5 per cent less than the hitherto accepted value derived by the Gaussian method of analysis for the region 60° N to 60° S and nearly 10 per cent less than the value for the equatorial belt alone. While to be sure, the value of 0.309 R^3 is to be regarded as but an approximate one, since it depends somewhat upon extrapolation, other calculations would seem to show that it can not be far in the wrong. In any case the effect on the principal Gaussian coefficients of the neglected polar caps is more pronounced, as shown by the figures in the table, than has hitherto been believed.

Examining theoretically into the latter matter, it is indeed found that as far as the chief Gaussian coefficient is concerned, viz.—$g_{10} = c_p$, the neglected regions between the poles and the parallels 60° have nearly the same weight, namely, 23.1, as the much larger region 60 N to 60 S which has a combined weight of 25.9. It is thus forcibly impressed upon us how much more im-

Digitized by Google

portant it is to perfect the physical analysis than the Gaussian empirical one if we attempt to derive, from a limited region, correct values of the physical constants of the Earth's magnetism. As far as the second and third Gaussian coefficients are concerned, however, viz. g_{11}, h_{11} or $c_e = \sqrt{g^2_{11} + h^2_{11}}$ fairly close values may be derived from the region 60 N to 60 S; the weight of the latter is 52.1 against 12.9 for the two polar caps together.

Let us now turn to the facts shown by the table respecting the direction of the magnetic axis. We have:

ZONE	MAGNETIC AXIS	
	ϕ_m	λ_m
Equator to 60° N.	80°.8	273°.1 E.
Equator to 60° S.	75°.5	303°.9
60° N. to 60° S.	78°.4	293°.1

First, the pronounced asymmetry between the Northern and Southern Hemisphere should be noted, and in doing so it is well to recall that this asymmetry pertains to the portion characterized as a "uniform magnetization." It is out of two such asymmetrical systems that the resultant so called "uniform system," shown by the lowest figures, for 60° N to 60° S, is derived. It is also a matter of no little significance that the position of the North Magnetic Pole for the supposed uniform portion of the Earth's magnetism between the equator and 60° N, viz., 80°.8 N, 86°.9 W, and that of the South Magnetic Pole for the supposed uniform portion between the equator and 60° S, viz., 75°.5 S, 123°.9 E approach the positions of the actual Magnetic Poles; the latter are situated approximately 70° N, 97° W; 73° S, 155° E.

The effect of the areas embraced upon the values of the direction angles of the magnetic axis can again be seen from the tabulated figures. It is possible to represent the chief facts shown by the variation in ϕ_n and in λ_n by a formula composed of a constant, a sine term, and a cosine term. These formulae will be established later as also those for the magnetic moment values.

58. Having now become familiar with the chief facts exhibited in Tables XII and XIII, let us consider briefly the possible causes. It was found that, due to some cause or other, the Earth's intensity of magnetization instead of being uniform, systematically increased with approach towards the equator, irrespective of the hemisphere, whether the north or the south one. It is believed that this fact argues not for an actual magnetization of the Earth, but for the hypothesis that the Earth is an electromagnet, as

Digitized by Google

already tentatively advanced in the previous numbers. We may be wrong in the first hypothesis set up, but the belief that the Earth's magnetic phenomena are to be referred largely if not chiefly to some electric system, has been strengthened, it would appear, by the evidence now afforded. Assuming negative electric currents circulating inside the Earth from west to east along parallels of latitude whose strength varies as the sine of the co-latitude, then, if the Earth were not rotating, the magnetic elements measured on the surface would be the same as those derived from a uniform magnetization about the axis of rotation, the South Magnetic Pole being coincident with the North Pole. But as the Earth rotates, the currents may no longer follow strictly latitudinal paths, but may perhaps be turned by the Earth's deflecting force so as to flow toward the equatorial regions from both sides of the Equator. In this way, or by some other action of the Earth's centrifugal force, we may possibly account for the increased strength of the electric currents or increased apparent magnetization in the equatorial regions as found above. The same cause seemingly would also largely, if not wholly, account for the existence of the equatorial component of the Earth's apparent magnetization and especially the rapid variation in its magnitude from parallel to parallel. In brief, the question arises, whether the two rectangular components of the magnetic moment combined may not form practically the magnetic system symmetrical about the axis of rotation which would exist if the Earth did not rotate.

The asymmetry about the equator, as shown by the figures above, appears to be due to effects similar to those resulting from the flow of electric currents across the vertical component of the Earth's apparent magnetism. As the vertical component is directed oppositely in the two hemispheres, the effects are of opposite sign and hence the asymmetries; the effect would be additive in one hemisphere and subtractive in the opposite one. The differences in the tabulated quantities for corresponding regions in the two hemispheres give twice the effect. These matters will be more fully treated in the next number, where an attempt will be made to deduce the chief phenomena of the residual field, or of the "non-uniform" portion, from the same causes as are herein brought into evidence.

Sections 59 and 60 were written practically independently of the previous ones; it is therefore of added interest that they both point to conclusions similar to those in this section.

Digitized by Google

Average East-West Magnetic Component (Y_a).

59. Table XIV gives the average value of the east-west component, Y_a, as derived from scalings along a parallel of latitude using the Neumayer charts of 1885, the values being expressed in units of the fourth decimal C. G. S.; they are taken directly from PD. IV, Table I (*Terr. Mag.*, v. 9, 1904, p. 121). For the region embraced 60° N to 60° S, it will be seen that, in general, the plus sign occurs in the Northern Hemisphere and the minus sign in the Southern. Since the plus sign means an east Y component, the figures, if real, imply that the north end of the compass is deflected by some cause, in general, to the east or right in the Northern Hemisphere and to the west or to the left in the Southern. But these are the directions in which a body of mass m moving on or parallel to the Earth's surface with the relative velocity v is deflected from its path by the Earth's rotation, according as the body is moving in the North or in the South Hemisphere. This deflecting force for a frictionless Earth is $2mv\,\omega \cos u$, in which $\omega = 2\pi/T$ is the angular velocity.

Let us assume that a portion at least of the Earth's magnetism is due to electric currents, which for a magnetization symmetrical about the axis of rotation, similar to the Earth's principal field, must circulate along parallels of latitude from west to east, if the currents be negative ones. Suppose now that, owing to the Earth's rotation, these currents are subject to the deflecting effect spoken of above. Since the compass always sets itself at right angles to the current, the compass would have its north-seeking end turned by a certain amount to the East for northerly latitudes and to the west for southerly ones. Assume, then, that in some such manner a ΔY_1 arises which we may put proportional to the deflecting force due to the Earth's rotation, hence:

$$\Delta Y_1 = kv \cos u \ .$$

If v_e be the velocity concerned for a point on the equator, assume that the velocity for a point in co-latitude u is $v_e \sin u$, hence:

$$\Delta Y_1 = kv_e \sin u \cos u = y_1 \sin 2u \ .$$

Subjecting the observed data, Y_a, to this formula, we derive by least squares,

$$\Delta Y_1 = +0.00121 \sin 2u \ ,$$

which, while it reproduces the chief fact shown by the observed quantities, leaves, however, residuals showing a systematic run;

Digitized by Google

these may be represented largely by a simple sine term. Such a term may arise, if the disturbance effect, due to the Earth's rotation, produces even powers of the trigonometric functions in the expression for Y or causes an asymmetry by reason of which the magnetic axis no longer passes through the Earth's center.

TABLE XIV.

Comparison of observed and computed average values of the Magnetic Component Y for 1885.

Parallel	$Y_a \times 10^4$				
	1885 Obs'd	Bauer		Schmidt	
		Comp'd	O-C	Comp'd	O-C
60°N.....	+ 4	+ 7	— 3	+13	— 9
50 N.....	+12	+ 8	+ 4	+13	— 1
40 N.....	+15	+ 7	+ 8	+10	+ 5
30 N.....	+ 9	+ 5	+ 4	+ 6	+ 3
20 N.....	+ 0	+ 2	— 2	+ 0	0
10 N.....	—10	— 2	— 8	— 5	— 5
Equator..	—13	— 6	— 7	—10	— 3
10 S.....	— 8	—10	+ 2	—13	+ 5
20 S.....	— 8	—14	+ 6	—16	+ 8
30 S.....	—21	—16	— 5	—18	— 3
40 S.....	—28	—17	—11	—18	—10
50 S.....	—20	—16	— 4	—17	— 3
60 S.....	+14	—13	+27	—15	+29

Confining ourselves, at present, to the first degree effect of this disturbance:

$$\Delta Y_2 = y_2 \sin u \ ;$$

$$\Delta Y = \Delta Y_1 + \Delta Y_2 = Y_a = y_1 \sin 2u + y_2 \sin u \ . \quad (37)$$

Applying least squares we get, with the aid of the observed quantities, the formula in C. G. S. units:

$$Y_a = 0.00121 \sin 2u - 0.00062 \sin u \ . \quad (37a)$$

Digitized by Google

The computed values and resulting residuals are given in the table and some improvement over the previous formula has been effected; the run of the residuals is now not so systematic.

For the sake of comparison the table likewise contains Adolf Schmidt's computed quantities depending upon more unknowns and a more elaborate analysis. Looking over his residuals it will be seen that our limited formula, (37a), gives as satisfactory agreement as his. The mean square error for a single computed result is ± 0.00096 C. G. S. for his formula and ± 0.00094 C. G. S. for ours. In all other previous analyses, except Schmidt's, the Y_a values have been regarded as errors, as they should be if all of the Earth's magnetic forces could be referred to a potential.

Considerable doubt has, in fact, been put upon the correctness of the tabulated observed data, for if their existence is ascribed to vertical electric currents, which pass perpendicularly through the Earth's surface, an average current per sq. cm. is obtained which is about 10000 times that measured with atmospheric electric instruments.

In PD IV (*Terr. Mag.*, v. 9, pp. 127-128; *cf.*, also *Terr. Mag.*, v. 2, pp. 20-22) it was pointed out that the distribution of the supposed vertical currents was apparently connected with the general atmospheric circulation, *viz.*, that over the regions of low pressure the currents which could account for the magnetic effects would have to go upwards, if positive, and over the regions of high pressure, downwards. Let us test this matter again with the aid of formula (37a). In view of the possible uncertainty of the Y_a it is not deemed worth while to get still closer mathematical representation of the observed quantities than has been already reached with two unknowns.

By differentiating Y_a with respect to u we may get the rate of change per one degree of latitude for various latitudes, expressed in $\gamma = 0.00001$ C. G. S.:

$$dY_a = 4.22 \cos 2u - 1.08 \cos u. \qquad (37b)$$

If the quantities computed from this formula be plotted along with Ferrel's average annual barometric pressures for parallels of latitude, such a striking similarity is shown that, if we could regard the formula (37a) as representing physical facts, the conclusion would be inevitable that the magnetic phenomenon is subject to the same cause as that producing the salient features

Digitized by Google

of the distribution of pressure, *viz.*, the deflecting force due to the Earth's rotation. If this be true then we may possibly have opened to us a means for closing the gap at present existing between magnetic and electric measurements; instead of referring the magnetic effects exhibited by the Y_a values to vertical currents, we may find that they are due to the disturbance effects which enter because of the Earth's rotation. It may be possible, hence, that the magnetician and the electrician are measuring different things.

As fresh data will soon be available, it may be best to defer further inquiry into this part of our subject. However, it may be worth while to get some idea, at least, as to the possible deflection of the compass shown by the existing observed data for 1885, which may ultimately have to be referred to the effect of the Earth's rotation.

°	°	°	°
60 N	0.18 East	Equator	0.23 West
50 N	0.37 "	10°S	0.15 "
40 N	0.37 "	20 S	0.17 "
30 N	0.18 "	30 S	0.48 "
20 N	0.00	40 S	0.73 "
10 N	0.17 West	50 S	0.60 "
Equator	0.23 "	60 S	0.52 East

An interesting question which might be put at this juncture is: *Suppose the Earth uniformly magnetized about its axes of rotation. If the Earth did not rotate, the compass would everywhere point due north and south; will it continue doing so as the Earth rotates? If it does not, what may we conclude as to the nature or cause of the Earth's magnetization?*

Average North and Average Vertical Magnetic Components.

60. The following formulae have been tentatively deduced to represent the average values of the magnetic components along parallels of latitude, taking into account possible disturbance effects to the second degree, the cause of which is, at present, ascribed to the effects which may result from some action of the Earth's rotation and to the fact that the point of observation itself rotates with the Earth.

Their justification, at present, may rest upon the agreement of the computed results with the observational data;

$$X_a = x_1 \sin u + x_2 \sin^2 u + x_3 \cos u + x_4 \sin 2u \quad ,$$

$$Z_a = z_1 \cos u + z_2 \sin 2u + z_3 \sin u + z_4 \sin^2 u \quad .$$

Digitized by Google

Using the observed values given in Table XV, and applying least squares, the following formulæ are found:

$$X_a = (0.216 + 0.117 \sin u) \sin u - (0.095 - 0.149 \sin u) \cos u \,, \quad (38)$$

$$Z_a = 2\,(0.270 + 0.071 \sin u) \cos u + (0.095 - 0.109 \sin u) \sin u. \quad (39)$$

Table XV gives the comparison between observed quantities and those computed by the foregoing formulæ. A glance at the residuals O-C shows that, in general, a satisfactory agreement

TABLE XV.

Comparison between Observed and Computed Average Magnetic Components in C. G. S. units for 1885.

Parallel	X			Z			F		
	Obs'd	Co'd	O-C	Obs'd	Co'd	O-C	Obs'd	Co'd	O-C
60°N.....	.129	+.119	+.010	+.539	+.550	—.011	.554	.563	—.009
50 N.....	.185	+.188	—.003	+.500	+.500	.000	.533	.534	—.001
40 N.....	.239	+.246	—.007	+.429	+.426	+.003	.491	.492	—.001
30 N.....	.288	+.292	—.004	+.343	+.332	+.011	.448	.442	+.006
20 N.....	.323	+.322	+.001	+.229	+.224	+.005	.396	.392	+.004
10 N.....	.341	+.335	+.006	+.104	+.110	—.006	.357	.353	+.004
Equator...	.337	+.333	+.004	—.022	—.013	—.009	.338	.333	+.005
10 S.....	.316	+.317	—.001	—.138	—.127	—.011	.345	.342	+.003
20 S.....	.286	+.290	—.004	—.240	—.237	—.003	.373	.374	—.001
30 S.....	.252	+.258	—.006	—.324	—.331	+.007	.410	.420	—.010
40 S.....	.221	+.222	—.001	—.394	—.408	+.014	.452	.464	—.012
50 S.....	.191	+.186	+.005	—.464	—.467	+.003	.502	.503	—.001
60 S.....	.157	+.156	+.001	—.522	—.509	—.013	.545	.532	+.013

has been reached; the mean square error for X_a is ± 0.005 and for Z_a is ± 0.0085, quantities approaching the order of error of the present magnetic charts as revealed by the ocean observations of the magnetic survey vessel, the *Carnegie*. From the run of the present residuals it will also be seen that to represent them it would be necessary to have terms in our formulæ above the second

Digitized by Google

degree. However, it is not believed worth while to push the investigation further just now until more accurate data are available, which will be the case before very long. It is not really our object to aim so much at a complete representation of the present data as to discover terms which can be interpreted physically.

Equations (38) and (39) may be put in the following suggestive form:

$$X_a = 0.236 \sin (u - 23.°7) + 0.189 \sin (u + 51.°9) \sin u , \quad (38a)$$

$$Z_a = 0.548 \cos (u - 10.°1) + 0.179 \cos (u + 52.4) \sin u , \quad (39a)$$

In this form the origin of the various terms is made more significant. Taking the first terms, it will be seen that there is a direct connection with the magnetic axis; its average inclination to the axis of rotation is 23.°7 according to the X_a components and 10.°1 according to the Z_a components — but this is about the range shown in the values of ϕ_n in Table XII.

Next note the remarkable similarity in the second terms of the X_a and Z_a equations; these show a pronounced connection with the position of the poles of my residual field (PD I, *Terr. Mag.*, v. 4, p. 44), which were found to lie between parallels 40° and 50° north and then again between 40° and 50° south. This is the first time that, in the terms depending solely upon latitude, the presence of effects have been revealed, which hitherto have only been found with the aid of the coefficients of the longitude terms. It is thus seen once more that the system of forces symmetrical about the axis of rotation and the unsymmetrical one are very closely related to each other.

Preliminary Remarks on the Theory.

61. Sutherland[1] showed that a sphere rotating with an angular velocity ω and charged with electricity would have a magnetic potential at a point outside which did not partake of the sphere's rotation, of the form:

$$V = A \cos u / r^2 . \quad (40)$$

If E be the total charge and if it be uniformly distributed over the surface, $A = \frac{1}{3} \omega ER^2$; if E be uniformly distributed throughout the sphere then $A = \frac{1}{5} \omega ER^2$. It is thus seen that the potential

[1] *Terr. Mag.*, v. 5, p. 74; v. 8, p. 149; v. 9, p. 167.

Digitized by Google

of this magnetic field is precisely similar to the one arising from a uniform magnetization about the axis of rotation or for a sphere with an infinitely short magnet of finite moment at its center.

The question has recently been raised by Schuster[1] as to the modifications which must be made if the point of observation be on and rigidly fixed to the rotating electrically charged sphere instead of outside of it. If no change entered then the magnetic components for a point on the surface, and supposing the charge uniformly distributed throughout, would be:

$$X_1 = -\frac{E\omega}{5} \sin u/R;\ Z = -\frac{2E\omega}{5} \cos u/R\ . \qquad (41)$$

Now, according to Schuster, the effect due to the fact that the point is rigidly fixed to the sphere and hence moves with it, would be to give an X_2 component $= E\,\omega \sin u/R$, hence:

$$X = X_1 + X_2 = \frac{4E\omega}{5} \sin u/R, \qquad (42)$$

$$Z = -\frac{2E\omega}{5} \cos u/R. \qquad (43)$$

Accordingly, if E be negative, then for a sphere rotating in the same direction as the Earth, in the Northern Hemisphere X would be negative and Z positive, or the north end of the magnetic needle would point south in the hemisphere where that end is below the horizon. If E be positive, then in the Northern Hemisphere the end which points north would point above the horizon. Since both deductions are contrary to the facts on the Earth, Schuster concludes that the chief part of the Earth's magnetism can not be due to the magnetic field arising from the rotation of a charged sphere. It might be pointed out, however, that the hypothesis would still be able to account for the small portion of the Earth's magnetism hitherto referred to an external potential. As shown, if E be positive, X is positive, and Z negative in the Northern Hemisphere, which agrees with the facts ascribed to an external potential according to the method of the Gaussian analysis (*cf.* PD I, Table I, *Terr. Mag.*, v. 15, p. 117). It is thus seen how, if we accept Schuster's term, a possible rotation effect may profoundly modify our views and methods of analysis.

[1] A critical examination of the possible causes of terrestrial magnetism. *Proc. Phys. Soc. London*, v. 24, pt. III, pp. 127–133, paragraphs 8 and 12.

Digitized by Google

MAGNETIC DECLINATIONS AND CHART CORRECTIONS IN THE ATLANTIC OCEAN ACCORDING TO THE OBSERVATIONS ON BOARD THE *CARNEGIE*, JUNE, 1910, TO MARCH, 1911.[1]

By L. A. Bauer and W. J. Peters.

Table I.

Gardiner's Bay, New York, to Vieques, Porto Rico, June to July, 1910.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1910	° ′	° ′	°	°	°	°	°	°	°
June 30	41 03N	71 23W	—12.1	—11.6	—11.3	—11.2	—0.5	—0.8	—0.9
30	40 35N	69 42W	—12.8	—12.3	—12.3	—12.0	—0.5	—0.5	—0.8
July 3	37 54N	62 30W	—15.3	—14.6	—14.9	—14.8	—0.7	—0.5	—0.4
6	38 08N	52 02W	—21.6	—19.7	—20.3	—19.5	—1.9	—1.3	—2.1
7	38 00N	50 50W	—21.8	—20.0	—20.6	—19.8	—1.8	—1.3	—2.0
7	37 55N	50 01W	—22.1	—20.2	—20.9	—20.1	—1.9	—1.2	—2.0
9	37 22N	48 34W	—22.1	—20.3	—21.0	—20.3	—1.8	—1.1	—1.8
9	36 36N	47 55W	—21.7	—19.9	—20.8	—19.8	—1.8	—0.9	—1.9
10	36 14N	47 39W	—21.9	—19.8	—20.8	—19.7	—2.1	—1.1	—2.2
10	35 52N	46 58W	—22.1	—19.6	—20.6	—19.7	—2.5	—1.5	—2.4
12	34 25N	46 26W	—20.7	—19.0	—19.7	—18.9	—1.7	—1.0	—1.8
13	33 39N	45 57W	—20.3	—18.7	—19.2	—18.6	—1.6	—1.1	—1.7
13	33 38N	45 56W	—20.6	—18.7	—19.1	—18.5	—1.9	—1.5	—2.1
14	33 22N	45 52W	—20.5	—18.6	—19.1	—18.5	—1.9	—1.4	—2.0
14	32 56N	45 55W	—20.0	—18.3	—18.7	—18.2	—1.7	—1.3	—1.8
14	32 44N	45 56W	—20.0	—18.2	—18.6	—18.1	—1.8	—1.4	—1.9
15	31 46N	45 57W	—19.5	—17.6	—17.9	—17.5	—1.9	—1.6	—2.0
15	30 24N	46 02W	—18.8	—16.9	—16.8	—16.9	—1.9	—1.9	—2.0
16	29 49N	46 07W	—18.5	—16.5	—16.4	—16.5	—2.0	—2.1	—2.0
16	29 37N	46 07W	—18.0	—16.4	—16.3	—16.4	—1.6	—1.7	—1.6
16	29 26N	46 07W	—18.5	—16.3	—16.2	—16.4	—2.2	—2.3	—2.1
17	27 41N	46 44W	—17.3	—15.5	—15.0	—15.4	—1.8	—2.3	—1.9
18	26 27N	47 30W	—16.0	—14.8	—13.9	—14.9	—1.2	—2.1	—1.1
18	25 00N	48 23W	—15.4	—13.8	—12.9	—13.7	—1.6	—2.5	—1.7
18	24 48N	48 32W	—15.0	—13.6	—12.7	—13.5	—1.4	—2.3	—1.5
19	23 55N	49 55W	—13.9	—12.8	—11.8	—12.5	—1.1	—2.1	—1.4
19	22 58N	51 20W	—12.4	—11.7	—10.9	—11.4	—0.7	—1.5	—1.0
20	21 31N	53 58W	—10.0	— 9.4	— 8.7	— 9.2	—0.6	—1.3	—0.8
20	21 25N	54 08W	—10.2	— 9.3	— 8.7	— 9.2	—0.9	—1.5	—1.0
21	20 10N	56 40W	— 7.6	— 7.1	— 6.7	— 7.2	—0.5	—0.9	—0.4
21	20 05N	56 50W	— 7.9	— 7.0	— 6.6	— 7.1	—0.9	—1.3	—0.8
22	19 28N	58 15W	— 6.8	— 6.1	— 6.0	— 5.8	—0.7	—1.1	—0.8
22	19 09N	59 23W	— 5.6	— 5.3	— 5.1	— 5.3	—0.3	—0.5	—0.3
23	18 55N	61 09W	— 5.0	— 4.1	— 4.1	— 4.0	—0.9	—0.9	—1.0
23	18 45N	62 36W	— 4.3	— 3.4	— 3.6	— 3.0	—0.9	—0.7	—1.3
23	18 42N	62 53W	— 4.2	— 3.1	— 3.5	— 2.9	—1.1	—0.7	—1.3
24	18 08N	64 24W	— 3.2	— 2.3	— 2.4	— 2.0	—0.9	—0.9	—1.2

[1]For the observations in the North Atlantic on the first cruise of the *Carnegie*, 1909–1910. see *Terr. Mag.*, v. 15, pp. 57–82 and 143–144. For special tests made at Gardiner's Bay see *Terr. Mag.*, v. 15, pp. 129–142.

Digitized by Google

TABLE II.

Vieques to Para, Brazil, August to September, 1910.

(Minus indicates west declination, plus east declination.)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1910	° ′	° ′	°	°	°	°	°	°	°
Aug. 18	19 34N	66 20W	— 2.5	— 2.1	— 2.1	— 2.0	—0.4	—0.4	—0.5
18	19 50N	66 26W	— 2.5	— 2.2	— 2.2	— 2.0	—0.3	—0.3	—0.5
19	22 42N	67 04W	— 3.5	— 3.2	— 3.1	— 3.1	—0.3	—0.4	—0.4
19	22 56N	67 02W	— 3.7	— 3.4	— 3.2	— 3.2	—0.3	—0.5	—0.5
20	24 13N	66 44W	— 4.1	— 4.1	— 4.0	— 4.0	0.0	—0.1	—0.1
20	25 22N	66 15W	— 4.9	— 4.9	— 4.7	— 4.8	0.0	—0.2	—0.1
20	25 36N	66 13W	— 5.0	— 5.1	— 4.8	— 4.9	+0.1	—0.2	—0.1
21	28 12N	65 05W	— 7.2	— 7.0	— 6.6	— 6.6	—0.2	—0.6	—0.6
21	28 23N	64 56W	— 7.1	— 7.1	— 6.8	— 6.7	0.0	—0.3	—0.4
22	30 45N	63 00W	— 9.7	— 9.4	— 9.2	— 9.0	—0.3	—0.5	—0.7
22	30 58N	62 48W	—10.2	— 9.7	— 9.6	— 9.4	—0.5	—0.6	—0.8
23	32 58N	61 30W	—12.6	—11.5	—11.5	—11.4	—1.1	—1.1	—1.2
23	33 03N	61 24W	—12.2	—11.6	—11.5	—11.5	—0.6	—0.7	—0.7
24	33 12N	61 21W	—12.7	—11.8	—11.6	—11.7	—0.9	—1.1	—1.0
25	33 08N	58 44W	—15.5	—13.2	—13.0	—12.8	—2.3	—2.5	—2.7
26	33 10N	57 03W	—15.7	—13.9	—14.0	—13.7	—1.8	—1.7	—2.0
26	32 40N	55 42W	—16.4	—14.3	—14.1	—14.1	—2.1	—2.3	—2.3
29	32 20N	53 50W	—16.7	—15.0	—14.8	—14.8	—1.7	—1.9	—1.9
30	32 08N	53 12W	—16.9	—15.1	—15.0	—15.0	—1.8	—1.9	—1.9
30	31 39N	52 27W	—16.8	—15.2	—14.9	—15.0	—1.6	—1.9	—1.8
Sep. 2	28 04N	46 46W	—17.2	—15.7	—15.1	—15.7	—1.5	—2.1	—1.5
3	26 32N	45 57W	—16.9	—15.4	—14.6	—15.5	—1.5	—2.3	—1.4
3	25 55N	46 05W	—16.8	—15.2	—14.2	—15.2	—1.6	—2.6	—1.6
3	25 45N	46 02W	—17.0	—15.2	—14.1	—15.2	—1.8	—2.9	—1.8
4	24 44N	45 37W	—16.7	—15.0	—13.9	—15.1	—1.7	—2.8	—1.6
4	23 40N	45 20W	—16.6	—14.8	—13.6	—15.0	—1.8	—3.0	—1.6
4	23 29N	45 11W	—16.2	—14.8	—13.5	—15.0	—1.4	—2.7	—1.2
5	22 36N	44 33W	—16.1	—14.8	—13.4	—14.9	—1.3	—2.7	—1.2
5	21 37N	44 08W	—16.4	—14.8	—13.2	—14.9	—1.6	—3.2	—1.5
5	21 25N	44 00W	—16.0	—14.8	—13.2	—14.9	—1.2	—2.8	—1.1
6	20 21N	43 20W	—15.8	—14.9	—13.2	—15.0	—0.9	—2.6	—0.8
6	19 06N	42 37W	—15.9	—14.9	—13.1	—15.0	—1.0	—2.8	—0.9
8	15 06N	40 14W	—15.8	—15.3	—13.4	—15.3	—0.5	—2.4	—0.5
9	14 29N	39 45W	—15.9	—15.4	—13.5	—15.4	—0.5	—2.4	—0.5
9	14 24N	39 43W	—15.7	—15.4	—13.5	—15.4	—0.3	—2.2	—0.3
10	13 01N	39 37W	—15.6	—15.3	—13.4	—15.3	—0.3	—2.2	—0.3
11	10 24N	39 10W	—15.2	—15.3	—13.4	—15.2	+0.1	—1.8	0.0
11	10 15N	39 03W	—15.5	—15.2	—13.4	—15.2	—0.3	—2.1	—0.3
13	7 53N	37 15W	—16.0	—15.9	—14.8	—15.8	—0.1	—1.2	—0.2
13	7 44N	37 10W	—16.2	—15.9	—14.7	—15.8	—0.3	—1.5	—0.4
14	6 46N	36 52W	—16.1	—16.1	—14.8	—15.9	0.0	—1.3	—0.2
15	6 25N	36 50W	—16.1	—16.1	—14.9	—15.9	0.0	—1.2	—0.2
17	5 52N	35 48W	—16.5	—16.5	—15.3	—16.3	0.0	—1.2	—0.2
19	4 13N	37 18W	—15.9	—15.7	—14.7	—15.5	—0.2	—1.2	—0.4
19	3 16N	37 59W	—15.8	—15.3	—14.0	—15.1	—0.5	—1.8	—0.7
19	3 10N	38 08W	—15.1	—15.3	—14.0	—15.0	+0.2	—1.1	—0.1
20	2 28N	39 14W	—14.3	—14.5	—13.1	—14.3	+0.2	—1.2	0.0
20	1 53N	40 20W	—13.8	—13.7	—12.2	—13.6	—0.1	—1.6	—0.2
20	1 47N	40 37W	—13.5	—13.5	—12.1	—13.4	0.0	—1.4	—0.1
21	1 06N	42 18W	—12.3	—12.4	—10.9	—12.2	+0.1	—1.4	—0.1
21	0 30N	43 38W	—11.6	—11.3	— 9.8	—11.0	—0.3	—1.8	—0.6
21	0 21N	43 53W	—11.3	—10.9	— 9.5	—10.7	—0.4	—1.8	—0.6
22	0 01N	45 16W	—10.2	— 9.9	— 8.5	— 9.8	—0.3	—1.7	—0.4
22	0 09 S	46 12W	— 9.7	— 9.2	— 7.9	— 9.0	—0.5	—1.8	—0.7
23	0 27 S	47 18W	— 9.1	— 8.3	— 6.9	— 8.2	—0.8	—2.2	—0.9

Digitized by Google

TABLE III

Para to Rio de Janeiro, Brasil, October to December, 1910.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1910	° ′	° ′	°	°	°	°	°	°	°
Oct. 17	0 14N	48 01W	— 8.3	— 8.0	— 6.5	— 7.7	—0.3	—1.8	—0.6
17	1 18N	48 06W	— 8.5	— 8.1	— 6.4	— 7.8	—0.4	—2.1	—0.7
17	1 30N	48 09W	— 8.3	— 8.1	— 6.4	— 7.8	—0.2	—1.9	—0.5
18	3 19N	48 27W	— 8.5	— 8.2	— 6.1	— 7.9	—0.3	—2.4	—0.6
18	4 32N	49 03W	— 8.3	— 8.1	— 6.0	— 7.7	—0.2	—2.3	—0.6
18	4 45N	49 06W	— 8.2	— 8.1	— 6.0	— 7.7	—0.1	—2.2	—0.5
19	5 44N	48 54W	— 8.4	— 8.3	— 6.1	— 8.0	—0.1	—2.3	—0.4
19	6 22N	48 41W	— 8.6	— 8.5	— 6.4	— 8.2	—0.1	—2.2	—0.4
21	6 02N	47 48W	— 9.3	— 9.2	— 7.0	— 8.9	—0.1	—2.3	—0.4
21	6 10N	47 11W	— 9.6	— 9.6	— 7.3	— 9.4	0.0	—2.3	—0.2
21	6 16N	47 03W	— 9.7	— 9.7	— 7.4	— 9.4	0.0	—2.3	—0.3
22	7 35N	45 49W	—10.8	—10.8	— 8.5	—10.5	0.0	—2.3	—0.3
23	9 20N	45 51W	—11.2	—11.2	— 8.8	—10.9	0.0	—2.4	—0.3
23	10 36N	45 47W	—11.4	—11.5	— 9.0	—11.2	+0.1	—2.4	—0.2
23	10 50N	45 47W	—11.5	—11.6	— 9.0	—11.3	+0.1	—2.5	—0.2
24	11 38N	45 43W	—11.9	—11.8	— 9.2	—11.5	—0.1	—2.7	—0.4
25	12 36N	45 51W	—12.0	—11.9	— 9.5	—11.6	—0.1	—2.5	—0.4
26	14 41N	44 44W	—13.2	—12.9	—10.5	—12.5	—0.3	—2.7	—0.7
26	15 42N	44 13W	—13.4	—13.7	—11.3	—13.2	+0.3	—2.1	—0.2
26	15 55N	44 10W	—13.7	—13.7	—11.4	—13.4	0.0	—2.3	—0.3
27	18 26N	43 33W	—14.7	—14.6	—12.5	—14.2	—0.1	—2.2	—0.5
27	18 35N	43 33W	—14.9	—14.6	—12.5	—14.2	—0.3	—2.4	—0.7
28	17 09N	42 51W	—15.4	—14.6	—12.5	—14.4	—0.8	—2.9	—1.0
28	16 58N	42 48W	—15.2	—14.6	—12.4	—14.4	—0.6	—2.8	—0.8
30	13 42N	41 18W	—15.1	—14.7	—12.6	—14.7	—0.4	—2.5	—0.4
30	12 50N	41 06W	—15.2	—14.8	—12.4	—14.6	—0.4	—2.8	—0.6
Nov. 1	11 58N	40 10W	—14.9	—15.1	—13.0	—15.0	+0.2	—1.9	+0.1
1	12 08N	39 22W	—15.2	—15.5	—13.5	—15.3	+0.3	—1.7	+0.1
1	12 13N	39 15W	—15.6	—15.5	—13.7	—15.3	—0.1	—1.9	—0.3
2	11 56N	38 32W	—15.9	—15.8	—14.0	—15.7	—0.1	—1.9	—0.2
2	11 50N	38 26W	—16.0	—15.8	—14.0	—15.7	—0.2	—2.0	—0.3
3	11 36N	37 49W	—16.2	—16.0	—14.5	—15.8	—0.2	—1.7	—0.4
4	12 16N	37 22W	—16.7	—16.2	—14.9	—16.1	—0.5	—1.8	—0.6
4	12 08N	37 18W	—16.6	—16.3	—14.9	—16.1	—0.3	—1.7	—0.5
6	11 31N	36 20W	—16.8	—16.6	—15.2	—16.4	—0.2	—1.6	—0.4
9	10 51N	35 23W	—16.8	—16.9	—15.7	—16.9	+0.1	—1.1	+0.1
9	11 03N	35 19W	—17.1	—16.9	—15.7	—16.8	—0.2	—1.4	—0.3
10	9 56N	34 43W	—17.9	—17.1	—16.0	—17.0	—0.8	—1.9	—0.9
10	9 46N	34 41W	—17.5	—17.1	—16.0	—17.0	—0.4	—1.5	—0.5
12	6 44N	33 34W	—17.6	—17.5	—16.6	—17.2	—0.1	—1.0	—0.4
17	1 03N	33 08W	—17.9	—17.8	—17.2	—17.5	—0.1	—0.7	—0.4
17	0 53N	33 15W	—17.7	—17.8	—17.0	—17.5	+0.1	—0.7	—0.2
18	1 39 S	34 06W	—18.0	—17.4	—16.6	—17.2	—0.6	—1.4	—0.8
18	1 53 S	34 12W	—17.4	—17.4	—16.6	—17.1	0.0	—0.8	—0.3
19	3 20 S	34 24W	—17.2	—17.2	—16.5	—17.1	0.0	—0.7	—0.1
19	3 53 S	34 25W	—16.4	—17.2	—16.5	—17.1	+0.8	+0.1	+0.7

Digitized by Google

TABLE III—Continued.

Para to Rio de Janeiro, Brazil, October to December, 1910.

(Minus indicates west declination, plus east declination.)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1910	° ′	° ′	°	°	°	°	°	°	°
Nov. 19	3 42 S	34 14W	—16.8	—17.3	—16.6	—17.1	+0.5	—0.2	+0.3
20	4 42 S	34 24W	—17.0	—17.3	—16.6	—17.1	+0.3	—0.4	+0.1
20	5 59 S	34 52W	—17.0	—16.9	—16.3	—16.7	—0.1	—0.7	—0.3
23	10 24 S	34 45W	—16.8	—16.8	—16.6	—16.7	0.0	—0.2	—0.1
23	10 41 S	34 50W	—16.4	—16.8	—16.4	—16.6	+0.4	0.0	+0.2
26	17 59 S	36 58W	—15.0	—14.8	—15.0	—15.0	—0.2	0.0	0.0
26	18 06 S	37 00W	—14.9	—14.7	—15.0	—15.0	—0.2	+0.1	+0.1
27	18 58 S	37 31W	—14.0	—14.2	—14.6	—14.5	+0.2	+0.6	+0.5
27	19 40 S	37 52W	—14.2	—13.8	—14.1	—14.1	—0.4	—0.1	—0.1
27	19 50 S	37 57W	—14.4	—13.7	—14.1	—14.1	—0.7	—0.3	—0.3
28	21 34 S	39 11W	—13.8*	—12.6	—13.1	—12.8	—1.2*	—0.7*	—1.0*
Dec. 1	22 52 S	40 06W	—12.2	—11.7	—12.2	—11.8	—0.5	0.0	—0.4
1	23 09 S	41 04W	—11.3	—11.1	—11.3	—11.0	—0.2	0.0	—0.3

* Conditions very unfavorable for accurate work.

TABLE IV.

Rio de Janeiro to Buenos Aires, Argentina, December, 1910, to January, 1911.

(Minus indicates west declination, plus east declination.)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1910	° ′	° ′	°	°	°	°	°	°	°
Dec. 30	24 00 S	42 51W	— 9.6	— 9.6	— 9.9	— 9.7	0.0	+0.3	+0.1
31	24 15 S	43 10W	— 9.5	— 9.3	— 9.5	— 9.5	—0.2	0.0	0.0
1911									
Jan. 1	24 30 S	42 55W	— 9.3	— 8.7	— 8.9	— 8.8	—0.6	—0.4	—0.5
1	25 25 S	42 42W	— 9.2	— 9.3	— 9.6	— 9.5	+0.1	+0.4	+0.3
1	25 39 S	42 38W	— 9.8	— 9.4	— 9.6	— 9.6	—0.4	—0.2	—0.2
2	26 48 S	41 37W	— 9.8	—10.0	—10.2	—10.2	+0.2	+0.4	+0.4
2	26 50 S	41 32W	—10.4	—10.1	—10.2	—10.2	—0.3	—0.2	—0.2
3	26 59 S	42 38W	— 9.6	— 9.1	— 9.4	— 9.2	—0.5	—0.2	—0.4
4	28 52 S	44 20W	— 7.6	— 7.4	— 7.7	— 7.3	—0.2	+0.1	—0.3
5	30 33 S	44 52W	— 5.9	— 6.5	— 6.6	— 6.5	+0.6	+0.7	+0.6
5	30 44 S	45 04W	— 6.5	— 6.2	— 6.5	— 6.3	—0.3	0.0	—0.2
7	30 15 S	47 26W	— 4.6	— 4.6	— 4.9	— 4.2	0.0	+0.3	—0.4
7	30 14 S	47 28W	— 4.3	— 4.6	— 4.9	— 4.2	+0.3	+0.6	—0.1
8	30 37 S	47 59W	— 4.5	— 4.0	— 4.0	— 3.6	—0.5	—0.5	—0.9
10	31 36 S	47 53W	— 3.9	— 3.8	— 3.9	— 2.5	—0.1	0.0	—1.4
10	31 45 S	47 58W	— 4.0	— 3.8	— 3.9	— 2.5	—0.2	—0.1	—1.5
16	35 09 S	56 47W	+ 4.7	+ 4.0	+ 4.5	+ 5.4	+0.7	+0.2	—0.7
17	34 51 S	57 25W	+ 5.1	+ 4.7	+ 5.0	+ 5.9	+0.4	+0.1	—0.8

Digitized by Google

TABLE V

Buenos Aires to Cape Town, Africa, February to March, 1911.

(Minus indicates west declination, plus east declination.)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1911	° ′	° ′	°	°	°	°	°	°	°
Feb. 15	34 51 S	57 25W	+ 5.0	+ 4.8	+ 5.0	+ 5.8	+0.2	0.0	—0.8
21	35 42 S	55 49W	+ 4.3	+ 4.0	+ 4.0	+ 4.8	+0.3	+0.3	—0.5
22	36 17 S	54 32W	+ 3.2	+ 3.1	+ 3.1	+ 3.7	+0.1	+0.1	—0.5
22	36 24 S	53 45W	+ 2.8	+ 2.5	+ 2.5	+ 3.2	+0.3	+0.3	—0.4
23	35 59 S	52 03W	+ 0.7	+ 1.0	+ 0.9	+ 1.3	—0.3	—0.2	—0.6
23	35 52 S	51 54W	+ 1.0	+ 1.0	+ 0.7	+ 1.2	0.0	+0.3	—0.2
24	36 17 S	51 36W	+ 0.5	+ 0.7	+ 0.5	+ 0.8	—0.2	0.0	—0.3
25	36 37 S	50 06W	— 0.4	— 0.6	— 0.5	— 0.2	+0.2	+0.1	—0.2
25	36 44 S	49 52W	— 0.6	— 0.6	— 0.8	— 0.2	0.0	+0.2	—0.4
26	37 14 S	48 32W	— 1.5	— 1.6	— 1.6	— 1.3	+0.1	+0.1	—0.2
27	38 42 S	44 00W	— 5.3	— 5.1	— 4.6	— 4.7	—0.2	—0.7	—0.6
28	39 14 S	40 04W	— 8.2	— 7.7	— 7.4	— 7.5	—0.5	—0.8	—0.7
Mar. 1	39 29 S	37 46W	— 9.7	— 9.2	— 9.1	— 9.0	—0.5	—0.6	—0.7
2	39 40 S	34 26W	—11.6	—11.4	—11.4	—11.2	—0.2	—0.2	—0.4
3	39 58 S	31 24W	—14.0	—13.2	—13.4	—13.1	—0.8	—0.6	—0.9
3	40 00 S	31 07W	—14.0	—13.5	—13.6	—13.2	—0.5	—0.4	—0.8
5	40 37 S	25 11W	—17.5	—17.0	—17.4	—16.6	—0.5	—0.1	—0.9
10	41 12 S	7 31W	—26.0	—25.9	—25.8	—26.0	—0.1	—0.2	0.0
11	41 01 S	6 26W	—26.2	—26.6	—26.3	—26.4	+0.4	+0.1	+0.2
11	40 45 S	5 02W	—26.4	—27.2	—27.0	—27.0	+0.8	+0.6	+0.6
11	40 43 S	4 50W	—26.5	—27.4	—27.1	—27.2	+0.9	+0.6	+0.7
12	39 48 S	1 03W	—28.5	—28.9	—28.7	—28.9	+0.4	+0.2	+0.4
13	39 25 S	1 00 E	—28.5	—29.3	—29.3	—29.3	+0.8	+0.8	+0.8
13	39 23 S	1 02 E	—28.7	—29.3	—29.3	—29.3	+0.6	+0.6	+0.6
14	39 26 S	1 28 E	—28.8	—29.4	—29.4	—29.5	+0.6	+0.6	+0.7
14	39 16 S	2 49 E	—28.7	—29.7	—29.7	—29.8	+1.0	+1.0	+1.1
15	39 03 S	4 51 E	—29.4	—30.0	—30.1	—30.1	+0.6	+0.7	+0.7
15	39 01 S	5 04 E	—29.5	—30.0	—30.1	—30.1	+0.5	+0.6	+0.6
16	38 19 S	7 50 E	—29.8	—30.2	—30.2	—30.4	+0.4	+0.4	+0.6
16	38 15 S	8 06 E	—29.6	—30.2	—30.3	—30.4	+0.6	+0.7	+0.8
17	37 50 S	9 49 E	—29.5	—30.1	—30.3	—30.5	+0.6	+0.8	+1.0
18	36 45 S	12 25 E	—29.2	—29.9	—30.1	—30.3	+0.7	+0.9	+1.1
19	35 03 S	16 36 E	—28.5	—28.4	—29.6	—29.1	—0.1	+1.1	+0.6
20	34 00 S	18 14 E	—27.7	—27.4	—29.0	—28.4	—0.3	+1.3	+0.7

Digitized by Google

LETTERS TO EDITOR

CIRCULAR CONCERNING REPRODUCTION AND EXCHANGE OF CURVES OF GREAT DISTURBANCE.

[Translation.]

Director Abels at Ekaterinbourg having called attention to the fact that the usefulness of the reproduction and exchange of the magnetic curves on days of great disturbances or other interesting days, recommended at Innsbruck (1905, Bericht p. 31) and at Berlin (1910, Bericht p. 68), would become greater if the same days were chosen by all the observatories participating, the Executive Bureau has requested us to indicate kindly on the quarterly publications of "magnetic character" a certain number of days which seem especially suited to a reproduction.

In order to comply with this request, and after having consulted Prof. Schmidt, we have the intention of indicating on the quarterly tables, beginning on Jan. 1, 1912, two series of days suitable for the reproduction: a first list to the number of 5 to 10 per year, including the days of greatest disturbance, the reproduction of which seems indispensable, and a second list, including days of considerable disturbance which will admit of a reproduction without omissions in a large number of observatories. The total number of days proposed will not exceed 15 per year.

The first list will be indicated by ‡, the second by * at the base of the page under the date of the day proposed. In order to facilitate the choice and to profit as much as possible by the information which the aspect of the magnetic curves furnishes, we request the directors who have the intention of having the curves reproduced to indicate kindly on their quarterly list the days and hours which seem most favorable for the reproduction.

Moreover, it goes without saying that the choice of the beginning and end of the part reproduced will be always left to the discretion of the directors themselves. The asterisks will indicate only the dates of the moment of the maximum disturbance (Greenwich time), and we merely express the wish that the part reproduced shall generally extend over 24 hours at least, but shall not exceed 30 hours. A disturbance of more than 30 hours' duration should be indicated, if necessary, by two dates, and reproduced on two sheets.

De Bilt, April, 1912. E. VAN. EVERDINGEN.

Digitized by Google

DIE ERDMAGNETISCHEN ELEMENTE IN TSINGTAU, CHINA.

Die nachstehenden erdmagnetischen Elemente sind die ersten Resultate der Beobachtungen mit dem kürzlich hier eingetroffenen Stationstheodolit von Tesdorpf. Obschon es sich nur um einige Tagesmittel handelt, an die eventuell noch kleine Korrektionen anzubringen sind, möchte ich die Daten doch umgehend veröffentlichen, da sie vielleicht schon Verwendung finden können. Für Anfang März 1912 ist der Tageswert in:

Deklination *D*	Inklination *I*	Intensität Horizontal (H)	Intensität Vertikal (Z)
3° 59′.0 W	52° 05′.7 N	0.30886	0.39668

Im Oktober 1908 hat Herr Edmunds (Carnegie Institution of Washington) magnetische Beobachtungen in Tsingtau ausgeführt und dabei gefunden:

D	*I*	*H*	*Z*
3° 48′.1 W	52° 21′.5 N	0.30766	0.39890

Das hiesige Instrument ist auf Potsdam bezogen, das von Herrn Edmunds jedenfalls auf Washington, die gegenseitigen Beziehungen sind mir jedoch zur Zeit nicht bekannt. Da die letzteren Beobachtungen zudem auf einem jetzt nicht mehr vorhandenen, jedenfalls aber nicht störungsfreien Backsteinpfeiler ausgeführt worden sind, lässt sich die jährliche Aenderung der Elemente nur unsicher aus den oben gegebenen Daten ableiten.

B. MEYERMANN.

Tsingtau, 1912, März 8.

REQUEST FOR DATA.

The undersigned respectfully requests that papers and not yet pubished observations of earth-magnetic and air-electric phenomena, purple light, halos, coronae, luminous night-clouds, aurora borealis and australis and zodiacal-light for June 30 to July 1, 1908, September 25, 1909, and May 19, 1910, be sent to him.

D. STENQUIST,

May 30, 1912. *Frejgatan 69, Stockholm, Sweden.*

Digitized by Google

NOTES

5. Personalia. We regret to be obliged to record the deaths of *Prof. A. Lawrence Rotch,* the eminent meteorologist and founder of the justly famed Blue Hill Observatory, at Boston, on April 7th last, at the age of 51 years; *Prof. Dr. J. B. Messerschmitt,* director of the magnetic and of the seismological observatory at Munich, Germany, on April 12th, also at the age of 51 years; *Dr. S. Róna* was appointed director of the Meteorological and Magnetic Observatory.

6. New Magnetic Survey of Japan. Preparations are being made, under the superintendence of *Prof. Tanakadate,* to send out four parties for making a new magnetic survey of Japan, to be completed within two years. The same general scheme of work will be followed according to which the first survey of about 18 years ago was so successfully accomplished under Prof. Tanakadate's direction.

7. British Admiralty Magnetic Chart. The preparation of the British Admiralty chart of lines of equal magnetic declination has been recently transferred from the Hydrographic Department of the Admiralty to the Magnetic and Meteorological Department of the Royal Observatory, Greenwich.

8. Magnetic Storms, Jan. 1 to Mar. 31, 1912. The Superintendent of the Coast and Geodetic Survey reports that there were no magnetic storms during this period.

ABSTRACTS AND REVIEWS

HOFFMANN, KARL. *Experimentelle Prüfung der durch verschiedene Messungsanordnungen in einem homogenen elektrischen Felde hervorgerufenen Störungen der Niveauflächen.*[1]

In order properly to interpret the results directly obtained in the study of atmospheric electricity it is necessary to know in what way the Earth's field as influenced by the topography of the neighborhood, by the presence of buildings, and by the observer and his instruments, differs from the field that would exist were the surface of the Earth flat and smooth, and devoid of observer and instruments. The absence of such information greatly reduces the value of many observations that have been made in the past.

In his experimental study of the distortion of the equipotential surface that may be produced by these various factors the author makes use of the fact that in two geometrically similar systems that are similarly charged the equipotential surfaces are similar. Hence, if A and B are two points in one system, and a and b are two corresponding points in the second system, the ratio of the drop of potential at A to the drop of potential at B equals the ratio of the drop of potential at a to the drop of potential at b.

[1] *Dissertation,* Technische Hochschule, München, C. Wolf & Sohn, 1911, 67 pp., 12 tables.

Digitized by Google

Consequently if in a sufficiently extended uniform electric field we introduce miniature models of the disturbing features with which we are concerned and measure the drop of potential at the point corresponding to that at which the measurement of atmospheric potential gradient is made and also the drop at a point far removed from the disturbing cause, the ratio of the second by the first will be the factor by which the observed drop of potential in the atmosphere must be multiplied in order to obtain the drop that would have been observed had there been none of the disturbing factors. This is the method adopted by the author.

Having shown by direct observations, in a case amenable to calculation, that the equipotential surface as mapped by means of a water-dropper having a long capillary outlet (so that the bulk of the instrument was entirely removed from the region under study), agreed with those calculated, he built miniature models of mountains, houses, masts, etc., and similarly mapped the equipotential surfaces around them when placed in a previously uniform field. Thus he obtained the reduction factors for numerous cases.

A similar study of a field around a flame collector and around a polonium collector showed that the fields above them are very distorted. If a wind is blown across the polonium plate the equipotential surface above it flattens out and the plate takes a potential a few per cent higher than that corresponding to the undisturbed potential at its height.

The author also investigated the fields around an Elster and Geitel apparatus, kites, airships, etc. The observations with the Elster and Geitel apparatus clearly show the necessity for surrounding the discharging body with a conductor, preferably of gauze.

Included in the work is a series of plates showing the experimentally determined equipotential surface, and thus giving ocular evidence of the magnitude of the distortion.

N. E. DORSEY.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.

STEINER, DESIDER. *The Effect of the Frequency of an Alternating Magnetizing Field upon the Hysteresis Loss of the Ferromagnetic Alloys of Aluminium, Copper, and Manganese.*[1]

Heusler's alloys of aluminium, copper, and manganese have been shown by previous investigators to possess magnetic properties dependent upon their thermal conditions and chemical composition. By proper annealing[2] after casting, a stable magnetic state is reached such that each alloy, for every temperature below the critical point, shows a definite susceptibility which, in contrast with that of iron, steel, iron alloys, nickel, and cobalt, generally increases as the temperature is reduced to that of liquid air.[3] In some specimens a susceptibility equal to one-third that of the best iron was obtained. The static hysteresis loss[4] can be reduced by thermal treatment to zero for many specimens.

[1] *Dissertation*, Tech. Hochschule, Darmstadt, J. A. Barth, Leipzig, 1911.

[2] HEUSLER and RICHARZ, *Zeits. f. anorgan. Chem.* 61, p. 267, 1909. A. D. ROSS, *Journ. Inst. Metals*, Vol. IV, No. 2, 1910, p. 77.

[3] *Ibid*, p. 76, table III.

[4] ASTEROTH, *Dissertation*, Marburg, 1907.

Digitized by Google

Heusler[5] found maximum susceptibility when the alloy contained about 13 per cent aluminium, or, according to later experiments, 6 per cent manganese. One of the most remarkable results of the investigation of these alloys was obtained by Ross[6] from specimens which suffered a diminution in susceptibility, hysteresis loss, and coercive force due to quenching at about 400° C., but which with lapse of time regained almost all of their original susceptibility, while the hysteresis loss and the coercive force remained unaltered from their reduced values.

Similar experiments[7] upon the magnetic alloys of copper, manganese, and tin show that their magnetic properties depend upon heating and chemical composition, the susceptibility increasing with decrease in temperature. Critical temperatures vary from 225° to 275° C.; the Heusler alloys show a greater range than this. The tin alloys were found to fall into two distinct groups characterized by their chemical composition; a suggested explanation of this phenomenon is the fact that manganese and tin form two magnetic compounds, Mn_4Sn and Mn_2Sn, which may be constituents of solid solutions in the alloy, thus giving rise to the magnetic properties. Likewise, the theory has been advanced[8] that the Heusler alloys owe their magnetic properties to the formation of solid solutions containing the intermetallic compound Cu_2Al; as this is the only magnetic compound of copper and aluminium[9], the theory seems to explain the fact that the Heusler alloys do not fall into two or more magnetic groups as do the tin alloys.

In the present experiments the hysteresis and eddy current losses of iron and of the Heusler alloys were studied for various frequencies of the magnetizing currents between 2,000 and 5,000 cycles per second. The first apparatus employed, that of K. E. F. Schmidt[10], consists of a Tesla machine in the primary circuit and a secondary circuit containing inductance, capacity and the magnetizing coil. By proper adjustment, the secondary current may be made to oscillate in resonance with the primary current of known frequency; the specimen of metal is then inserted into the magnetizing coil, and resonance is again secured; observing the two values of the secondary current corresponding to these two resonance points, and the amount of energy necessary to magnetize the coil when empty, the total energy loss due to the specimen may be calculated. Subtracting the computed eddy current loss from this total, the hysteresis loss is obtained for the given current frequency.

The use of high frequencies, causing surface concentration of the lines of magnetic induction in the specimens magnetized, rendered Steinmetz's[11] simple formula for the eddy current loss inapplicable; instead, this loss was computed for each frequency on the basis of Maxwell's[12] law of distribution. The expression for the loss derived from this law involves the permeability of the specimen, which was assumed to be $\mu = 500$, because this value of μ leads to values of the total energy loss which agree with those found by the second method.

[5] Vortrag in der Sitzung der Marburger Gesellschaft vom 16 Juni 1908.
[6] *Ibid*, p. 78.
[7] A. D. ROSS AND R. C. GRAY, *Proc. Roy. Soc.* Edinburgh XXXI, part 1, No. 5, p. 85.
[8] A. D. ROSS, *Jour. Inst. Metals*, v. IV, No. 2, 1910.
[9] A. D. ROSS AND R. C. GRAY, *Ibid*, p. 99.
[10] *Ann. d. Phys.*, 14, p. 50, 1904.
[11] *Electrotechn. Zeitschr.* 16, p. 668, 1895.
[12] J. J. THOMSON, Electrician, p. 599, 1892.

Digitized by Google

The second apparatus consisted essentially of a Braun's tube in which the stream of cathode rays impinged upon a calcium tungstate screen; the trace of the spot of light was photographed, giving the hysteresis curve with sufficient definition to justify quantitative measurements of the area, which determine the value of the sum of the hysteresis and eddy current losses.

By both methods four specimens of the alloys (of different, but known, chemical composition) were examined. Since the strength of the magnetizing field was the same in each case, comparison of the results is justified, with the following conclusions:

1. The Heusler alloys behave like iron for high frequencies; but for the same strength of the magnetizing field, the maximum magnetic induction, and hence also the hysteresis loss, is less for the alloys than for iron.

2. As the frequency of the magnetizing current is increased up to a critical value, the corresponding hysteresis loss for the alloys and for iron increases at first rapidly, then more slowly; for frequencies greater than the critical value, the hysteresis loss decreases with increase in frequency. The critical frequency is different for different specimens.

3. Contrary to previous supposition, the values of the eddy current loss, as found by calculation, decrease with increase in frequency, more markedly in iron than in the alloys. The explanation of this decrease is the greater surface concentration of the lines of magnetic induction with higher frequencies.

4. Both methods show an increase in the sum of the hysteresis and eddy current losses as the frequency approaches 4,300 cycles per second, beyond which the total loss decreases.

5. The author infers from the photographic curves an increase in the total loss with decrease in the percentage of manganese.

6. After the specimens had been heated in boiling Xylol at 140° C. for eight hours, and then cooled slowly, the same results were obtained as before.

7. The alloys are in general very hard and brittle, so it is questionable whether they can be used practically in spite of their valuable magnetic properties.

ROGERS H. GALT.

Department of Terrestrial Magnetism,
Carnegie Institution of Washington.

HJORT, V. *The Comparison of Two Simultaneously Registered Curves of the Variation of the Vertical Intensity.*[1]

The disturbance of the vertical intensity of the Earth's magnetic field on September 25 and 26, 1909, was recorded photographically at the observatory of Rude Skov by two types of Lloyd's balance, one of Mascart's, the other of Edelmann's design. The present paper is a comparison of the curves obtained from the two instruments. The curves themselves, which show sharp maxima and minima, paired off with occasional points of no variation, are not reproduced in the article; but a table of measurements made from them is given to illustrate the author's conclusions.

The magnet of each instrument was placed in the magnetic meridional plane; hence the expression for the variation of the vertical intensity involved the corresponding change in horizontal intensity, which was determined by means

[1] An Extract from L'Annuaire Magnétique Années, 1907, pub. by L'Inst. Mét. de Danemark.

Digitized by Google

of the bifilar declinometer. In principle the two Lloyd balances are identical, though of different mechanical construction. The magnet of the Edelman instrument rests upon two points, that of the Mascart instrument upon an agate knife edge; the former, which was fitted with a damping device, gave twice the sensibility of the latter, which was undamped. Accidental errors may result from failure of the bar to return exactly to its original position, which is the greatest fault of the Lloyd balance.

Since the two instruments give curves in close agreement from point to point, the author concludes that the two forms of balance are equally well adapted to determining changes in the vertical intensity. R. H. GALT.

LIST OF RECENT PUBLICATIONS

Terrestrial Magnetism.

ANGENHEISTER. Die Island-Expedition im Frühjahr 1910. Erster Teil, Die erdmagnetischen Beobachtungen. Reprint Göttingen, Nach. Ges. Wiss. 1911, (34 with charts).

BAUER, L. A. The Magnetic Declination in Prince George's County. Maryland Geological Survey. Prince George's County. Baltimore, 1911 (215-218).

BAUER, L. A. Report on the Lines of Equal Magnetic Declination in Maryland for 1910. Maryland Geological Survey. Baltimore, Md., 1911, v. 9, (331-338).

BOMBAY AND ALIBAG OBSERVATORIES. Report of the Director, Bombay and Alibag Observatories for the year ending December 31, 1911. Bombay, 1912 (16) 33 cm. P.

BIDLINGMAIER, F. Übersicht über die Tätigkeit des Erdmagnetismus im II. Halbjahr, 1911. Blatt 4. Berlin, Veröff. K. Obs., Wilhelmshaven, 1912 (4 and 1 pl.) 32 1-2 cm. P.

BIRKELAND KR. Orages Magnétiques et Aurores Polaires. Bruxelles, C. R. Congrès, International de Radiologie et d'Electricité, Sciences Physiques, Tome I, 1911 (1-26).

BIRKELAND, RICHARD. Mouvement d'une particle électrisée dans un champ magnétique. Arch. Sci. Phys. Genève, v. 33, No. 1, Jan. 15, 1912, (32-51); No. 2, Feb. 15, 1912, (151-175).

CANADA NAVAL SERVICE. Report of the Department of the Naval Service for the Fiscal Year ending March 31, 1911. (Includes a report on magnetic declination in Hudson's Bay.) Ottawa, 1911, (56 with 15 pl. and 1 map.) 25 cm.

CHREE, C. Some Phenomena of Magnetic Disturbances at Kew. London, Phil. Trans. R. Soc. Series A, v. 210.

DANEMARK INSTITUT MÉTÉOROLOGIQUE. Annuaire Magnétique. (Années 1907-1908). Copenhagen, 1911.

Digitized by Google

Terrestrial Magnetism

and

Atmospheric Electricity

VOLUME XVII SEPTEMBER, 1912 NUMBER 3

A NEW TYPE OF COMPASS DECLINOMETER.[1]

By R. L. Faris.

The magnetic field work of the United States Coast and Geodetic Survey in the United States and outlying territory under their jurisdiction is performed by regularly equipped magnetic parties in charge of magnetic observers, and as incidental work by survey parties engaged in geodetic, topographic, hydrographic, or other surveying duties. The regular magnetic parties are supplied with a complete magnetic outfit consisting of a theodolite magnetometer, dip circle, pocket chronometer, small accessories, and a nonmagnetic observing tent. General survey parties, especially when working in remote regions, such as Alaska or the Philippines, are usually furnished with a compass declinometer or other instrument for determining the magnetic declination in connection with their triangulation or other work, where the azimuths of lines are already known. In this way it has been possible to ascertain the magnetic declination at many places at nominal cost and in advance of the general magnetic survey.

Experience with other types of compass declinometer and with dip circle compass attachments has shown that acceptable results for magnetic declination are readily obtainable with the pivot suspension needle. The older type of compass declinometer with prism for reading the needle's position and invertible needle for elimination of magnetic axis error, was liable to have a more or less variable index correction depending partly upon the position of the reading prism. Inversion of the needle for elimination of the effect of the lack of coincidence of geometric and magnetic axis

[1] Published by permission of the Superintendent of the United States Coast and Geodetic Survey.

Digitized by Google

now seems unnecessary with an appropriately designed needle. In such a needle it is believed that its magnetic axis will remain practically unchanged between successive standardizations, or the change may be assumed as uniform unless due to accident.

In supplying the need for additional compass declinometers it was desired to eliminate or minimize the objectionable features of the older type of instrument. The present type of instrument was constructed in the instrument shop of the Coast and Geodetic Survey from designs prepared by Mr. E. G. Fischer, Chief of the Instrument Division.

Figure 1 shows the general appearance of the instrument mounted on its tripod ready for use. It is essentially a compass needle enclosed in a rectangular needle box with peep sights, which is rigidly mounted on a graduated horizontal circle. The needle lifter and some of the other details are novel and have been worked out with great care. The base rests on three leveling screws and has double centres of brass. The horizontal circle, which is 152.4 mm. in diameter, is read by two verniers to single minutes of arc. The needle is mounted in a rectangular box the inside dimensions of which are 164 mm. long, 31.5 mm. wide, and 17 mm. deep. This box is secured to the cover plate of the horizontal circle. At each end of the box is a graduated arc (arc graduated to 10 minutes), about 10° in extent on each side of the zero in the middle. Vertical peep sights, 40 mm. high, are attached to the ends of the box so that the zeros of the graduations, the needle pivot and the peep slits are in the same vertical plane. The top edge of the needle is straight, its ends and the apex of the agate cup suspension being in a straight line. The needle lifter is of special design and direct acting, and so arranged that the instrument can not be packed in its packing case without first lifting the needle off its suspension pivot. When the needle is raised off the pivot it is held firmly in place between the lifter and a flat brass spring in the top of the needle box, as shown in Figure 2. The needle and horizontal circle readings are made with the aid of three reading microscopes as seen in Figure 1. The instrument is leveled by means of an adjustable circular level mounted on the cover plate of the horizontal circle.

The needle is 148.5 mm. long and is made of "magnet steel;" the dimensions of its cross-section are 0.6 mm. by 3.5 mm. The width of the peep-sight slits is 0.5 mm., and that of the holes is 5.0 mm.

Digitized by Google

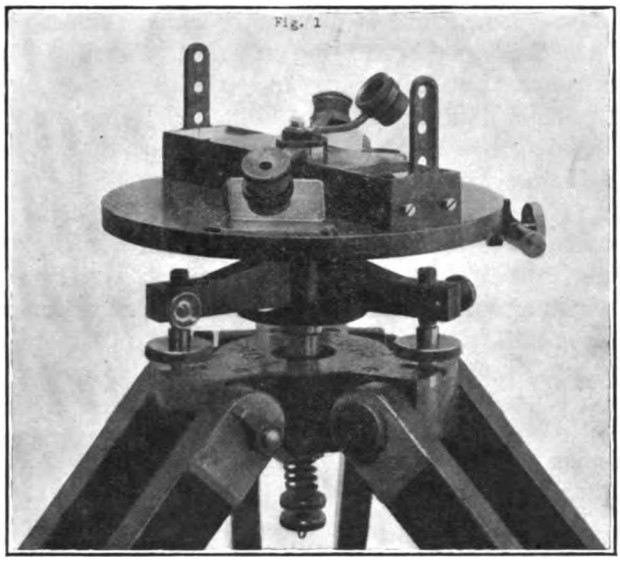


FIG. 1. *Declinometer ready for use.*

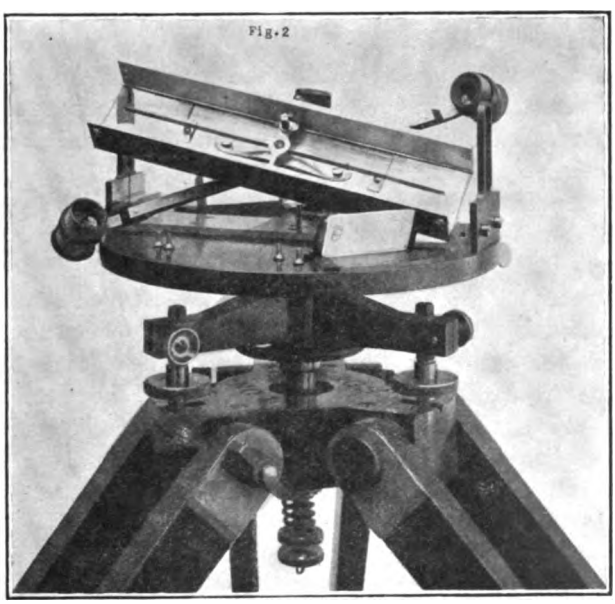


FIG. 2. *View of needle box and needle lifter.*

Digitized by Google

The whole instrument is compactly built, the height of the needle above the tripod head being only about 80 mm. The weight of the instrument and its packing case is 5.0 kilograms, and of the tripod about 3.5 kilograms.

Before being tested in the field the instrument was compared with the Cheltenham magnetic observatory standard declinometer in order to ascertain its index error, if any. In the course of these experiments it was found that the slits in the peep-sights, which were originally only 0.25 mm. in width, were too narrow and did not permit sufficient illumination for pointing on distant reference marks. The slits were subsequently widened to 0.5 mm., which has been found satisfactory.

In the comparisons at Cheltenham, observations with the instrument were made with the north end of the needle over six different parts of the horizontal circle, these being distributed at about equal intervals (about every 60 degrees). This was done in order to have an additional test, under actual working conditions, for the existence of magnetic properties in the instrument. The materials from which the instrument was constructed were all tested, with a specially designed and mounted needle, before being used, and the results of the comparisons at Cheltenham confirmed the results of these material tests.

Table I, on page 113, contains the results of the comparison observations at Cheltenham, before and after the peep-sight slits were widened, and after the instrument had been used at magnetometer stations in the field.

From Table I it seems that the widening of the peep-sight slits had the effect of decreasing the index correction, the correction now being about +2.′0 for west declination. The weighted mean correction (+2.′6) obtained from the twelve field stations by comparison with a field magnetometer, is somewhat larger (0.′6) than the correction obtained from the observatory comparisons, but the difference is well within the probable error of observation. The results of the observatory comparisons in various positions of the needle with reference to the horizontal circle shows that the material of which the instrument is constructed is free from magnetic impurities.

When using this instrument in the field at the stations given in Table I it was found that the needle required rather too frequent rebalancing to overcome the changes of magnetic dip at the different stations occupied. To remedy this inconvenience the point

Digitized by Google

of suspension (agate cup) has since been raised, likewise the ends of the needle have been slightly bent upwards to bring them in line with the suspension point.

Figure 3 is a general view of the mounting of the needle and reading microscope used in testing the materials entering into the

TABLE I.—*Results of Comparison Observations with Compass Declinometer No. 2.*

Station	Date	Magnetometer	Comp. Dec. No. 2	Correction	W'ght	Mean (w'ght'd)	Observer
	1911	° ′	° ′				
Cheltenham, Md.	Apr. 27	5 45.9W	5 43.2W	+2.7			J .E. Burbank
" "	"	46.9	42.9	+4.0			
" "	"	48.6	46.2	+2.4			
" "	"	48.4	44.5	+3.9			
" "	"	48.3	43.9	+4.4			
" "	"	48.0	44.0	+4.0		+3.6[a]	
" "	July 31	40.4	38.2	+2.2			J. E. Burbank
" "	"	40.1	37.6	+2.5			
" "	"	42.0	39.9	+2.1			
" "	"	47.5	45.7	+1.8			
" "	"	50.7	48.8	+1.9			
" "	"	50.4	48.4	+2.0		+2.1[b]	
	1912						
Greenville, S. C.	Mar. 2	0 16.2W	0 13.5W	+2.7			O. H. Gaarden
Laurens, S. C.	15	0 10.8W	0 06.8W	+4.0			
Abbeville, S. C.	18	0 03.2W	0 00.4W	+2.8			
Lexington, S. C.	30	0 00.8 E	0 02.4 E	+1.6			
Chester, S. C.	Apr. 5	2 34.7W	2 26.8W	+7.9			
Chesterfield, S. C.	9	1 02.9W	1 00.9W	+2.0			
Bishopville, S. C.	13	0 27.3W	0 22.9W	+4.4			
Florence, S. C.	18	0 58.7W	0 54.0W	+4.7			
" "	22	1 05.5W	1 01.3W	+4.2			
" "	23	1 05.5W	1 00.0W	+5.5			
Dillon, S. C.	25	1 36.6W	1 34.8W	+1.8	2		
St. Matthews, S. C.	May 2	0 18.4 E	0 17.7 E	—0.7	4	+2.6	
Cheltenham, Md.	June 4	5 50.5W	5 49.7W	+0.8	2		S. G. Townshend. Jr.
" "	"	52.0	50.6	+1.4	2		
" "	"	52.4	52.5	—0.1	2		
" "	"	55.3	53.6	+1.7	2		
" "	"	55.2	51.7	+3.5	2		
" "	"	55.2	51.8	+3.4	2	+1.8	

(*a*) Before slits in peep-sights were widened.
(*b*) After slits in peep-sights were widened.

construction of Compass Declinometer No. 2. This testing apparatus consists of a long magnetic needle mounted in a wooden casing upon a pivot. The needle is made of "magnet steel," and is 201.4 mm. in length with a cross-section of 0.3 mm. by 3.2 mm. The wooden casing is mounted on a metal (brass) base provided with three foot screws. On top of the case is mounted a reading

Digitized by Google

microscope containing a graduated glass scale to facilitate the noting of the motion of the needle when tests are being made. The inside dimensions of the wooden case are 206 mm. in length with a cross-section of 8.0 mm. by 3.5 mm. About 10 mm. at each end of the needle casing is made of very thin material to enable

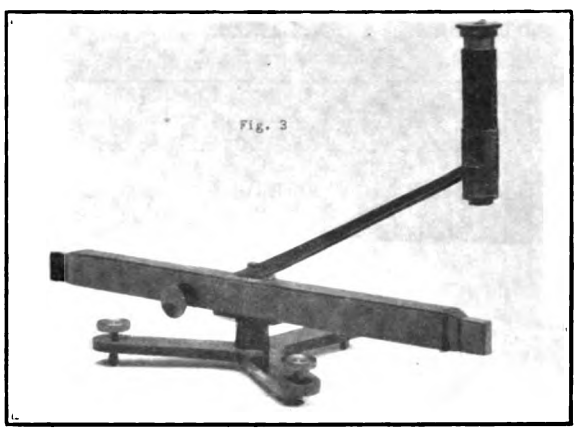


FIG. 3. *Apparatus for testing for magnetic impurities.*

the test piece to be brought as close as possible to the needle. The test piece can be brought within about 1.5 mm. of the needle by bringing the needle ends to rest slightly to one side of their central position in the needle case. A lifter is provided for raising the needle off its pivot when not in use.

United States Coast and Geodetic Survey,
Washington, D. C.

Digitized by Google

THE PHYSICAL THEORY OF THE EARTH'S MAGNETIC AND ELECTRIC PHENOMENA.—No. VI.

(Continued)

By L. A. Bauer.

ON THE ORIGIN OF THE EARTH'S MAGNETIC FIELD.[1]

62. The results obtained in the previous number, if not considered as definitely proving that a part at least of the Earth's magnetism is in some manner dependent upon the Earth's rotation, gave strong indications that such, indeed, is the case. Perhaps the result of greatest importance was that the forces which cause the Earth's magnetization to depart from that of a uniformly magnetized sphere follow, as a first approximation, certain definite and comparatively simple laws. When, for example, the Earth's magnetic moment was obtained separately for each parallel of latitude from 60 N. to 60 S., on the assumption that the measured magnetic forces are those resulting from a uniformly magnetized sphere, a systematic increase in the value was found in both hemispheres with approach towards the equator. This fact, as will be seen, furnishes a crucial test of the sufficiency of the theories already propounded to account for the origin of the Earth's magnetism.

Let us restrict ourselves again, as in the previous number, to the portion of the Earth's magnetic field symmetrical about the axis of rotation. If X be the field magnetic component, directed along the meridian, positive towards the North, and Z be the vertical component, positive downwards, then to whatever the field may be due, it is possible to express these two components as follows, u being the co-latitude of the point of observation supposed to be on the surface:

$$X = f_x(u) \cdot \sin u , \tag{44}$$

$$Z = Z_0 + 2 f_z(u) \cdot \cos u . \tag{45}$$

[1] A summary of the chief results of this article was presented at the meeting of the Astronomical and Astrophysical Society of America, Pittsburgh, August 28, 1912.

Digitized by Google

The two functions of u, $f_x(u)$ and $f_z(u)$, contain the chief secrets of the origin of the field; they might therefore, be appropriately called the "characteristic functions." Since it is possible, as far as external effects are concerned, that precisely the same function results for various origins of an internal magnetic field, our only hope of disclosing the precise cause, or at least narrowing down the number of possible causes, is in the time and geographical variations of the characteristic functions. Could we bring our magnet into the laboratory it might be possible to get the desired information by judiciously conducted experiments.

The Z_o term in (45) is the value of Z when cos u = o, *i. e.*, for points on the equator, as here $u = 90°$; the term, as will be seen later, arises when the field is asymmetrical about the equator.

If the magnetic field be not only symmetrical with respect to the axis of rotation, but also uniform, the two functions are constant over the whole Earth. If this magnetic field arises solely from internal agencies, we have:

$$f_x(u) = f_z(u) = c_p = \frac{4}{3}\pi\rho . \qquad (46)$$

$c_p = M/R^3$, M being the magnetic moment and R the Earth's mean radius; ρ is the intensity of magnetization per unit of volume, or its equivalent, according as the field is due to magnetic, or electric agencies, or to both. Precisely the same expressions result, however, if our observations are confined solely to the surface and we are prevented from penetrating into the interior of our magnet, whether the field be due to an elemental magnet at the Earth's center, or to a uniformly magnetized sphere, or to an appropriate system of electric currents, or to a rotating electrically charged sphere, or to a combination of these causes. Hence, in this instance, no help would be gained from the geographical variations of the characteristic functions, since they both would have the same value everywhere; the only possible hope of solving the problem of the actual cause of the field in this case might possibly be in the time variations and the extremely sudden and comparatively large changes which occur at times during magnetic storms.

Fortunately it was found in the previous number that the characteristic functions are not constant but vary in a systematic and simple manner from parallel to parallel, the value of the functions increasing towards the equator in both the Northern

Digitized by Google

and Southern Hemispheres. Previously, however, we did not derive $f_x(u)$ and $f_z(u)$ separately. This has now been done in Table XVII where the tabulated quantities have been derived in the same manner as explained for Table XII, §55 and §56, except that this time the X and Z components have been treated separately. There is thus avoided any assumption as to a combined potential or as to whether the magnetic systems are all internal or all external. This table again shows that the variations in the respective quantities from parallel to parallel follow certain definite laws.[2]

TABLE XVII.

The Earth's Magnetic Axis and Magnetic Moment for 1885 as determined separately for each parallel of latitude from a separate treatment of the rectangular components, X, Y, Z, supposing a uniform magnetization.

Parallel	Axis					Moment /R^3							Ratio	
	ϕ_n		λ_n (E of Gr.)			c_p		c_e			c		c_p / c_e	
	X	Z	X	Y	Z	X	Z	X	Y	Z	X	Z	X	Z
°	°	°	°	°	°									
60 N.....	76.3	82.5	287.4	254.6	232.1	.259	.311	.063	.045	.041	.266	.314	4.10	7.58
50 N.....	76.0	81.8	298.9	266.0	252.8	.288	.326	.072	.048	.047	.297	.329	4.02	6.90
40 N.....	75.6	82.2	297.3	272.5	271.4	.313	.334	.080	.049	.045	.322	.337	3.89	7.35
30 N.....	75.4	81.9	293.3	275.8	277.8	.333	.343	.086	.052	.049	.344	.346	3.85	6.99
20 N.....	73.5	80.8	296.2	276.7	271.3	.344	.335	.108	.054	.054	.359	.340	3.38	6.17
10 N.....	62.5	79.4	322.0	277.5	270.0	.346	.300	.179	.057	.056	.389	.305	1.92	5.35
Equator....	....	80.6	.....	280.6	274.1	.337	.352	....	.061	.058	.337	.....	.....	....
10 S.....	43.6	80.4	173.5	287.8	284.0	.320	.396	.337	.067	.067	.464	.402	0.95	5.88
20 S.....	58.1	76.6	185.3	296.4	293.0	.305	.352	.189	.074	.084	.358	.361	1.61	4.20
30 S.....	68.6	72.7	207.0	305.0	311.2	.291	.324	.114	.083	.101	.313	.340	2.54	3.21
40 S.....	73.5	69.2	241.5	314.0	321.3	.289	.306	.085	.093	.116	.300	.327	3.38	2.64
50 S.....	73.8	65.4	274.4	322.0	328.7	.298	.302	.086	.101	.139	.310	.332	3.45	2.19
60 S.....	72.5	63.1	296.1	329.0	345.8	.315	.302	.099	.105	.152	.329	.338	3.17	1.98

Let us first confine our attention to that portion of the uniform magnetization which is symmetrical about both the axis of rotation and the equator. Then, under the supposition, the unknowns determined by least squares reduce to the $(-g_{10})$, or the quantity proportional to the magnetic moment parallel to the axis of rotation, *viz.*, to c_p. The required quantities are obtained by combining the results for the corresponding parallels N and S. The provisionally computed quantities based on the empirical formulæ below are also given.

[2] Some of the quantities for the equator were derived with the aid of the scaled magnetic elements for parallels 5° N and 5° S. No attempt at weighting has, at present, seemed warranted in the least square solutions.

Digitized by Google

Parallel	Magnetic Moment Parallel to Axis of Rotation / R^3					
	Observed Values from Component		Computed Values from Component		Observed—Computed	
	X	Z	X	Z	X	Z
60 N and S....	.287	.306	.280	.302	+.007	+.004
50 " " "....	.293	.315	.294	.315	—.001	.000
40 " " "....	.301	.320	.307	.326	—.006	—.006
30 " " "....	.312	.334	.317	.336	—.005	—.002
20 " " "....	.324	.343	.325	.342	—.001	+.001
10 " " "....	.333	.348	.330	.347	+.003	+.001
Equator.....	.337	.352	.331	.348	+.006	+ 004

A systematic increase with approach to the equator is disclosed by both columns of figures, which may be *tentatively* formulated thus:

$$M_p / R^3 = 0.2281 + 0.103 \sin u \text{ for } X \text{ components},$$

$$M_p / R^3 = 0.2549 + 0.093 \sin u \text{ for } Z \text{ components}.$$

These expressions, as is seen from the above table, reproduce the observed quantities, on the average, within 1⅓ per cent, the maximum difference between observed and computed quantity being 2⅓ per cent. [See the superior expressions (74*a*) and (75*a*), and accompanying remarks on page 134.]

Comparing the above expressions, it will be seen that the corresponding coefficients differ by about 10 per cent from each other; these differences are not to be ascribed chiefly to errors of observation or irregularities in the respective components X and Z, but may be due partly, at least, to the different way in which the various systems involved combine to produce the horizontal component, X, from that by which the vertical component is produced. Consulting equations (34) and (36) it follows, if we confine ourselves to the internal magnetic systems, that

$$M_{p,i} / R^3 = 0.2460 + 0.096 \sin u .$$

This expression should be regarded, however, as only a first approximation, a more correct one being established later.

The intensity of magnetization per unit of volume, as deduced from the preceding preliminary formula is:

$$\rho = M_{p,i} / \frac{4\pi}{3} R^3 = 0.0588 + 0.0229 \sin u .$$

Digitized by Google

TABLE XVIII.

Average Values of the Magnetic Components and of the Characteristic Functions for 1842-45, 1880 and 1885.

Parallel	X_a			Z_a			$f_x(u)$			$f_z(u)$		
	1840–5	1880	1885	1840–5	1880	1885	1840–5	1880	1885	1840–5	1880	1885
°												
60 N	+.1284	+.1282	+.1293	+.5484	+.5457	+.5394	+.2568	+.2564	+.2586	+.3236	+.3273	+.3244
50 N	+.1840	+.1826	+.1854	+.5039	+.5000	+.5003	+.2862	+.2841	+.2884	+.3367	+.3402	+.3413
40 N	+.2361	+.2394	+.2395	+.4399	+.4322	+.4294	+.3082	+.3125	+.3127	+.3514	+.3527	+.3514
30 N	+.2833	+.2887	+.2883	+.3452	+.3429	+.3426	+.3271	+.3334	+.3329	+.3572	+.3640	+.3650
20 N	+.3208	+.3236	+.3231	+.2346	+.2309	+.2292	+.3414	+.3444	+.3438	+.3602	+.3684	+.3678
10 N	+.3381	+.3408	+.3409	+.1121	+.1055	+.1042	+.3433	+.3461	+.3462	+.3571	+.3652	+.3646
Equator	+.3372	+.3349	+.3370	—.0119	—.0212	—.0224	+.3372	+.3349	+.3370	.....	.....	.....
10 S	+.3252	+.3146	+.3156	—.1260	—.1408	—.1377	+.3302	+.3195	+.3205	+.3283	+.3445	+.3318
20 S	+.2990	+.2858	+.2862	—.2291	—.2401	—.2402	+.3182	+.3041	+.3046	+.3175	+.3199	+.3184
30 S	+.2677	+.2518	+.2519	—.3198	—.3289	—.3244	+.3091	+.2908	+.2909	+.3080	+.3076	+.3020
40 S	+.2341	+.2204	+.2211	—.4087	—.4018	—.3937	+.3056	+.2877	+.2886	+.3086	+.2960	+.2887
50 S	+.2002	+.1909	+.1915	—.4809	—.4651	—.4635	+.3114	+.2970	+.2979	+.3061	+.2898	+.2880
60 S	+.1613	+.1578	+.1573	—.5445	—.5246	—.5221	+.3226	+.3156	+.3146	+.3075	+.2906	+.2885
60 N & S	+.1448	+.1430	+.1433	+.5464	+.5352	+.5308	+.2897	+.2860	+.2866	+.3156	+.3090	+.3064
50 N & S	+.1921	+.1868	+.1884	+.4924	+.4826	+.4819	+.2988	+.2906	+.2932	+.3214	+.3150	+.3146
40 N & S	+.2351	+.2299	+.2303	+.4243	+.4170	+.4116	+.3069	+.3001	+.3006	+.3300	+.3244	+.3200
30 N & S	+.2755	+.2702	+.2701	+.3325	+.3359	+.3335	+.3181	+.3121	+.3119	+.3326	+.3358	+.3335
20 N & S	+.3099	+.3047	+.3046	+.2318	+.2355	+.2347	+.3298	+.3242	+.3242	+.3388	+.3442	+.3431
10 N & S	+.3316	+.3277	+.3282	+.1190	+.1232	+.1210	+.3368	+.3328	+.3334	+.3427	+.3548	+.3482
Equator	+.3372	+.3349	+.3370	—.0119	—.0212	—.0224	+.3372	+.3349	+.3370	.....	.....	.....

Digitized by Google

Using this tentative expression only for the region embraced by the observed quantities, equator to parallel 60°, it is found that ρ is 0.070 for parallel 60° and 0.082 for the equator, showing an increase of about 17 per cent.

63. Returning to equations (44) and (45), we have:

$$f_x(u) = X \operatorname{cosec} u, \tag{47}$$

$$f_z(u) = \frac{1}{2}(Z - Z_0) \sec u. \tag{48}$$

The X and Z in above equations may be taken with sufficient approximation to be the same as the average values X_a and Z_a of the components due to the entire Earth's field at equidistant points along a parallel of latitude, *e. g.*, as has been done here, at 10° longitude intervals. Table XVIII contains the values of X_a and Z_a in C. G. S. units for three epochs, 1840-45, 1880 and 1885, as derived from the scalings of the respective magnetic charts; then with the aid of equations (47) and (48) the values of the characteristic functions, $f_x(u)$ and $f_z(u)$, are obtained. It will be seen that the functional values increase systematically towards the equator for each component and for each epoch. The remarkable parallelism in the run of the two functions will be seen best from Fig. 1, page 135.

No theory of the Earth's magnetism has thus far been propounded which will reproduce the variation in the characteristic functions as found above. In fact the attempts heretofore have been directed to account merely for a "uniform" magnetization of the Earth, for which $f_x(u) = f_z(u) =$ constant; it was believed that the geographical variations in the constant of magnetization or of electrification, whichever it be, were of secondary importance and mainly to be referred to purely local or regional causes and that, therefore, they were more or less of an accidental nature as far as their distribution over the Earth was concerned. It was not fully appreciated that there are distorting or disturbing systems, embracing the whole Earth, which far outweigh the effects from the so-called "local disturbances" or irregularities in the distribution of the Earth's magnetism. The table exhibits well the variations in the functional values; the equatorial values are about 13 to 17 per cent higher than the mean values for the parallels 60 N and S.

Digitized by Google

Following the line of argument of this series of papers, we shall reverse the usual order of procedure and begin with an attempt to ascertain the cause of the geographical variations, or disturbances, which act so as to make the Earth's actual magnetic field depart from that of a simple type. It is our hope that by this avenue we may ultimately be led to clues as to the origin of the primary or fundamental field; at least the experiment may be worth trying when it is remembered that the course heretofore followed has not led to any decisive or universally accepted results.[3]

Turning to the Gaussian form of analysis our characteristic functions may be written as follows, if we stop with the fourth order harmonic and consider only the zonal harmonics since we are at present dealing solely with effects symmetrical about the axis of rotation:

$$f_x(u) = -\frac{R^3}{r^3}\Big[g_{10} + 2\frac{R}{r}g_{20}\cos u + 3\frac{R^2}{r^2}g_{30}(\cos^2 u - \frac{1}{5}) + 4\frac{R^3}{r^3}g_{40}(\cos^2 u - \frac{3}{7})\cos u\Big], \quad (49)$$

$$f_z(u) = -\frac{R^3}{r^3}\Big[g_{10} + \frac{3}{2}\frac{R}{r}g_{20}\cos u + \frac{4}{2}\frac{R^2}{r^2}g_{30}(\cos^2 u - \frac{3}{5}) + \frac{5}{2}\frac{R^3}{r^3}g_{40}(\cos^2 u - \frac{6}{7})\cos u\Big], \quad (50)$$

$$Z_0 = \frac{R^4}{r^4}(g_{20} - \frac{3}{7}\frac{R^2}{r^2}g_{40}). \quad (51)$$

g_{10}, g_{20}, g_{30}, g_{40} are the well-known Gaussian coefficients $g^{1,0}$, $g^{2,0}$, $g^{3,0}$, and $g^{4,0}$; they have been derived heretofore by the method of least squares, supposing them to be strictly independent of each other, which, of course, may not necessarily be the case; in fact it will be seen that this question has an important bearing on the theoretical value of the coefficients.

If we assume now that the field is also symmetrical about the equator, then the terms containing odd powers of cos u must drop

[3] The reader who desires to familiarize himself with the difficulties attaching to the chief hypotheses thus far advanced to account for the Earth's magnetism will do well to read the following suggestive papers:

SCHUSTER, A. A critical examination of the possible causes of terrestrial magnetism, *Proc. Phys. Soc.* London, v. 24, pt. III, p. 127, and SWANN, W. F. G., The Earth's Magnetic Field, *Phil. Mag.*, July, 1912, v. 24, p. 80.

Digitized by Google

out of our functions. Furthermore, Z_0 must be zero, otherwise Z will not be symmetrical about the equator. Hence:

$$f_x(u) = -\frac{R^3}{r^3}\left[g_{10} + 3\frac{R^2}{r^2}g_{30}(\cos^2 u - \frac{1}{5})\right], \qquad (49a)$$

$$f_z(u) = -\frac{R^3}{r^3}\left[g_{10} + 2\frac{R^2}{r^2}g_{30}(\cos^2 u - \frac{3}{5})\right]. \qquad (50a)$$

For points on the Earth's surface, $r = R$ = Earth's mean radius, so that:

$$f_x(u) = -\left[g_{10} + 3g_{30}(\cos^2 u - \frac{1}{5})\right], \qquad (49b)$$

$$f_z(u) = -\left[g_{10} + 2g_{30}(\cos^2 u - \frac{3}{5})\right]. \qquad (50b)$$

In order that the functional values may conform with the facts shown in Table XVIII, the bracketed quantity in each case must be negative and reach a numerical maximum in the equatorial region. Since $(\cos^2 u - \frac{1}{5})$, as also $(\cos^2 u - \frac{3}{5})$, continually decreases as u approaches 90°, it follows that for each function g_{30} must be of opposite sign to g_{10} in order to correspond with the observational facts of the Earth's magnetism. This is actually the case as is seen from the following values of the two coefficients found by various analysts and tabulated in PD III (*Terr. Mag.*, v. 8, 1903, p. 105):

Coefficient	Harmonics Embraced.							
	To Fourth Order			To Sixth Order				
	Adams 1840–'45	Adams 1880	Neu.–Pet. 1885	Adams 1840–'45	Fritsche 1840–'45	Adams 1880	Fritsche 1885	Schmidt 1885
g_{10}	—.3217	—.3161	—.3157	—.3219	—.3227	—.3168	—.3164	—.3173
g_{30}	+.0245	+.0286	+.0244	+.0242	+.0215	+.0268	+.0256	+.0234

There can not, therefore, be any question now as to the validity of the fact discovered and its value as a criterion for testing theories of the Earth's magnetism. This fact briefly stated is: *As a first approximation, and as far as points on the surface of the Earth are concerned, the portion of the Earth's magnetic field symmetrical*

Digitized by Google

about the axis of rotation is similar to the magnetic field of a uniformly magnetized sphere which has superposed on it another field whose intensity in both hemispheres increases systematically from parallel to parallel with approach towards the equator; which of the two fields is the fundamental one remains to be found out.

It should be pointed out that from the Gaussian analysis we are unable to say whether or not g_{10} and g_{30} are intrinsically of opposite sign; a theoretical formula might, for example, connect the two terms of the bracketed quantities in (49*b*) and (50*b*) by a minus sign instead of a plus, as Gauss would have it. Furthermore we are unable to say thus far whether or not the two coefficients are really independent of each other; it will be found that these questions are of no small importance.

Present Theories of the Earth's Magnetism.

64. As already stated, nearly all theories advanced thus far to account for the Earth's magnetism have been confined to the discovery of a cause for a field equivalent to that of a uniformly magnetized sphere. For this case (49*b*) and (50*b*) become:

$$\boldsymbol{f}_x(u) = \boldsymbol{f}_z(u) = -g_{10} = \text{constant} = c_p = M_p/R^3 \ . \qquad (50c)$$

These theories do not attempt to explain the systematic increase in the values of our characteristic functions with approach toward the equator disclosed in Table XVIII. The equations above show that a theory which will embody this fact must be one yielding a potential which includes, at least, the third harmonic, $P_3(u)$, whereas the theories just mentioned stop with the first, $P_1(u)$.

Recently, however, a valuable paper by Dr. W. F. G. Swann[4] has appeared, in which a critical examination is made of recent theories and several tentative theories of his own are worked out. Of the five cases considered in the appendix to his paper, three fall in the category of simple cases, namely, such as produce a uniform magnetic field, symmetrical about both the axis of rotation and the equator. He works out, however, two additional cases of considerable interest, yielding higher harmonics in addition to the first; in each case, the potential expression leads to an infinite series. Under case (3) Dr. Swann discusses the magnetic field due to an electrically charged sphere of radius

[4] See footnote 3.

Digitized by Google

R, rotating with an angular velocity ω, the current density being merely a function of ω. He obtains as the potential at any point defined by our polar co-ordinates r, u, and not connected with the sphere:

$$V = F(\omega)\left[0.194\frac{R^4}{r^2}P_1(u) + 0.123\frac{R^6}{r^4}P_3(u) + \cdots\right];$$

substituting the values of P_1 and P_3:

$$V = F(\omega)\left[0.194\frac{R^4}{r^3}\cos u + 0.3075\frac{R^6}{r^4}(\cos^3 u - \frac{3}{5})\cos u + \cdots\right].$$

This expression as it stands can only be used in deriving the vertical component, Z, and this will be sufficient to test the theory. In order to derive the X component for a point rigidly connected with the rotating charged sphere, an additional term would enter into V as shown by Schuster (*cf.* §61) as also in Swann's paper. We have:

$$Z = -\left(-\frac{\partial V}{\partial r}\right) = -2F(\omega)\left[0.194\frac{R^4}{r^5} + 0.615\frac{R^6}{r^5}(\cos^2 u - \frac{3}{5}) + \cdots\right]\cos u;$$

hence our characteristic function becomes for $r = R$:

$$f_z(u) = -RF(\omega)\left[0.194 + 0.615(\cos^2 u - \frac{3}{5}) + \cdots\right].$$

As u approaches 90° it is seen that the bracketed quantity, as far as given, diminishes instead of increasing as it should to conform with the facts of the Earth's magnetism.

Dr. Swann next considers the case of the magnetic field due to the rotation of a charged sphere in which each atom is arranged with its doublet axis perpendicular to the axis of rotation. If n is the number of doublets per cc., h the doublet separation, e the charge, let $k = 2\pi^2 n e h$; the test of the theory is again made simply with the aid of the Z characteristic function. For $r = R$ we get, after various reductions, kindly verified by Dr. Swann:

$$f_z(u) = -2kR\left[1.556 - 1.384\sin^2 u \cdots\right].$$

It will be seen that the bracketed quantity decreases instead of increasing with approach to the equator. Dr. Swann wishes it em-

Digitized by Google

phasized that his aim was not to work out a complete theory of the Earth's magnetic field "but simply to examine (mainly in regard to the order of the magnitude) the kind of magnetic effects which might be produced by the rotation of a body of the dimensions of the Earth when the field was supposed to arise as the result of a doublet separation."

It remains to point out, furthermore, that the cases thus far do not provide for any asymmetry about the equator.

Before passing to the next section, consider briefly the effect of superposing upon a uniform magnetic field symmetrical about the axis of rotation a secondary one which is due to an electric current of strength i lying in the equatorial plane and passing around the Earth's center from east to west below the surface in a circle of radius a. The potential V and the component Z due to this superposed field, are for $r>a$, in conformity with our notation:

$$V = -2\pi i\left[\frac{1}{2}\frac{a^2}{r^2}P_1(u) - \frac{3}{8}\frac{a^4}{r^4}P_3(u) + \frac{5}{16}\frac{a^6}{r^6}P_5(u) - \cdots\right],$$

$$Z = 2\pi i\left[\frac{a^2}{r^3} - \frac{15}{4}\frac{a^4}{r^5}(\cos^2 u - \frac{3}{5}) + \cdots\right]\cos u\ .$$

If the supposed current is at a sufficient distance below the surface, we may write for points on the surface:

$$f_z(u) = 2\pi i\frac{a^2}{R^3}\left[1 - \frac{15}{4}\frac{a^2}{R^2}(\cos^2 u - \frac{3}{5})\right]\ .$$

The tendency of the bracketed quantity would accordingly be to increase towards the equator. Now if the uniform portion of the Earth's magnetism symmetrical about the axis of rotation is to be referred primarily to a system of electric currents, circulating along parallels of latitude from east to west, then these currents must vary from parallel to parallel according to the sine of the co-latitude. If the currents are due to the rotation of an electrically charged Earth, then the surface density, or the volume density, must be constant. If σ be the surface density, then the current for any zone bounded by the parallels of co-latitude, u and $u + du$, due to rotation, is $R^2\omega\sigma\sin^2 u\,du$. To obtain increased strength towards the equator over that which will give a uniform magnetic field, it is accordingly necessary to find a cause which will make σ depart from uniformity and increase, in each hemisphere, towards the equator. We shall then get effects ap-

Digitized by Google

proximating to those arising from the above hypothetical case of an additional current in the equatorial regions.

If we assume $\sigma = \sigma_0 + \sigma_1 \sin u$, the potential of the magnetic field arising from rotation of the charge involves an integral which can not be evaluated in finite terms; we are also confronted with the difficulty of finding an adequate cause for such a distribution of the charge. If, however, we take $\sigma = \sigma_0 + \sigma_1 \sin^2 u$ then, as will be seen, the potential reduces in a remarkable and beautiful manner to a very simple, finite expression which closes with the third harmonic P_3; the physical explanation for this distribution of the charge also appears simplified.

Magnetic Field of a Rotating Charged Sphere.

65. CASE I. *A sphere of radius R revolves with angular velocity ω round a central axis and is charged with electricity of uniform surface density, σ_0 ; to determine its magnetic field.* The rotational motion of the charge on the zone bounded by the parallels of co-latitude u and $u + du$, gives rise to a current $R\omega \sin u \,.\, \sigma_0 R du$, whose magnetic potential is equal to the product of its strength by the solid angle subtended at the point by the periphery of the zone. According to the hypothesis, the strength of the current is $R^2 \omega \sigma_0 \sin u \, du$. Taking at first the point on the axis of rotation at a distance r from the center, the solid angle is:

$$2\pi \left[1 - (r - R \cos u)\,(r^2 + R^2 - 2rR \cos u)^{-\frac{1}{2}} \right] .$$

Hence the magnetic potential V' of the whole sphere at this point is:

$$V' = 2\pi R^2 \omega \sigma_0 \int_0^\pi \left[1 - \frac{r - R \cos u}{(r^2 + R^2 - 2rR \cos u)^{\frac{1}{2}}} \right] \sin u \, du$$

$$= 2\pi R^2 \omega \sigma_0 \frac{2R^2}{3r^2} \quad ;$$

finally by spherical harmonic analysis the potential at any point, r, u, not attached to the revolving sphere is:

$$V = \frac{4}{3} \pi R^2 \omega \sigma_0 \frac{R^2}{r^2} \cos u \quad . \tag{52}$$

This case has already been discussed by others, more recently by Schuster and Swann[5]; a corrective term must be applied if the point is on the revolving sphere. The following two cases are new.

Digitized by Google

CASE II. *A sphere of radius R rotating about a central axis is electrically charged with a surface density varying from parallel to parallel so that if the density be σ_1 along the equator it is $\sigma_1 \sin^2 u$ for any other parallel; to find its magnetic field.*

V' denoting the same as before we have:

$$V' = 2\pi R^2 \omega \sigma_1 \int_0^{\pi} \left[1 - \frac{r - R\cos u}{(r^2 + R^2 - 2rR\cos u)^{\frac{1}{2}}} \right] \sin^3 u \, du \quad ;$$

the integral though involving some labor reduces to a finite expression consisting of but two terms:

$$V' = \frac{8}{5} \pi R^2 \omega \sigma_1 \left(\frac{2}{3} \frac{R^2}{r^2} - \frac{1}{7} \frac{R^4}{r^4} \right) .$$

With the aid of spherical harmonics, the potential, V, at any point not rigidly attached to the rotating charged sphere is:

$$V = \frac{4}{3} \pi R^2 \omega \sigma_1 \left[\frac{4}{5} \frac{R^2}{r^2} \cos u - \frac{3}{7} \frac{R^4}{r^4} \left(\cos^2 u - \frac{3}{5} \right) \cos u \right] . \quad (53)$$

It will be noticed that this case provides a uniform magnetic field (first term) and superposed upon it a field (second term) whose intensity varies from parallel to parallel. Applying our criterion, using again only the vertical component so as to avoid the complications previously mentioned:

$$Z = -\left(-\frac{\partial V}{\partial r}\right) = -\frac{8}{3} \pi R^2 \omega \sigma_1 \left[\frac{4}{5} \frac{R^2}{r^3} - \frac{6}{7} \frac{R^4}{r^5} \left(\cos^2 u - \frac{3}{5} \right) \right] \cos u ;$$

hence for $r = R$, the characteristic function becomes:

$$f_z(u) = -\frac{8}{3} \pi R \omega \sigma_1 \left[\frac{4}{5} - \frac{6}{7} \left(\cos^2 u - \frac{3}{5} \right) \right] .$$

At the poles, the value of the bracketed quantity is 0.46; it then steadily increases until reaching the value of 1.31 at the equator; thus we have a cause that operates in the right direction, but the variations produced in the value of the characteristic function are much more rapid than is disclosed by the observational data in Table XVIII. A combination, however, of cases I and II will obviate the difficulty encountered with case II alone.

[1] See, for example, references in footnote 3, also ROWLAND, *Physical Papers*, 179–183, and SUTHERLAND, *Terr. Mag.* v. 5, 73–74.

Digitized by Google

CASE III. *A rotating charged sphere as before, the surface density, however, varying directly as the cosine of the co-latitude so that* $\sigma = \sigma_2 \cos u$; *to determine the magnetic field.*

We have:

$$V' = 2\pi R^2 \omega \sigma_2 \int_0^\pi \left[1 - \frac{r - R\cos u}{(r^2 + R^2 - 2rR\cos u)^{\frac{1}{2}}} \right] \cos u \sin u \, du$$

$$= \frac{8}{15} \pi R^2 \omega \sigma_2 \frac{R^3}{r^3} \quad ;$$

hence by spherical harmonics the magnetic potential at a point r, u, not rigidly connected with the sphere is:

$$V = \frac{8}{15} \pi R^2 \omega \sigma_2 \frac{R^3}{r^3} P_2(u) = \frac{4}{5} \pi R^2 \omega \sigma_2 \frac{R^3}{r^3} (\cos^2 u - \frac{1}{3}) \quad . \qquad (54)$$

From this is obtained:

$$Z = -\frac{12}{5} \pi R^2 \omega \sigma_2 \frac{R^3}{r^4} (\cos^2 u - \frac{1}{3}) \quad ;$$

for $r = R$, the characteristic function is:

$$f_z(u) = -\frac{12}{5} \pi R \omega \sigma_2 \cos u,$$

hence, though symmetrical about the axis of rotation, the function is asymmetrical about the equator. The value of Z does not reduce to zero at the equator and we have:

$$Z_o = +\frac{4}{5} \pi R \omega \sigma_2 \, .$$

Case I gave us the first harmonic, No. II, the first and third, and No. III, the second. Accordingly, if we assume the following distribution of the surface density:

$$\sigma = \sigma_0 + \sigma_1 \sin^2 u + \sigma_2 \cos u,$$

the potential of the magnetic field resulting from the rotation of the charged sphere at any point outside not partaking of the rotation is found by combining (52), (53), and (54):

$$V = \frac{4}{3} \pi R^2 \omega \left[(\sigma_0 + \frac{4}{5} \sigma_1) \frac{R^2}{r^2} P_1(u) + \frac{2}{5} \sigma_2 \frac{R^3}{r^3} P_2(u) \right.$$

$$\left. - \frac{6}{35} \sigma_1 \frac{R^4}{r^4} P_3(u) \right] \quad . \qquad (55)$$

Digitized by Google

If it is desired, closed expressions involving still higher harmonics may be obtained; thus a $\sigma_3 \sin^4 u$ term gives us the first, third, and fifth harmonic; a term involving an odd power of sin u leads to an infinite series. Terms containing even powers of cos u, *e. g.*, $\cos^2 u$, since this is equal to $1 - \sin^2 u$, increase σ_0 by σ_1 and lead to the same harmonics, but with reversed sign, as the terms containing even powers of $\sin^2 u$. Odd powers of cos u will give closed expressions containing the even harmonics, the second, fourth, etc., according to the power. It is thus seen how it is possible to build up any desired harmonic expression. We also see that if the sphere is electrically charged, or magnetized, in a manner symmetrical about both the axis of rotation and the equator, then only the odd harmonics will appear in the potential. If the distribution of the charge, or the magnetization, while symmetrical about the axis of rotation is not so about the equator, then the even harmonics appear. This information will be of use later on; a clearer insight than we possessed before as to the part played by the respective harmonics in the representation of a magnetic field has now been obtained.

For the present we shall stop with the above expression for which some physical explanation may possibly be found.

The potential expressions above, as distinctly stated, apply only to a point not attached to the rotating charged sphere; when the point is on the sphere an additional term enters, as has already been pointed out by Schuster and Swann. This difficulty we must seek to remove.

66. Suppose that instead of a surface charge we have now a body charge, of density ρ distributed throughout the sphere according to the law:

$$\rho = \rho_0 + \rho_1 \sin^2 u, \qquad (56)$$

ρ_0 and ρ_1 being constants.

We already know from above that this will give us a magnetic field symmetrical about both the axis of rotation and the equator. It will be observed that ρ varies only with the co-latitude and is accordingly constant along any one radius of the sphere. Substituting in equations (55) and (56), $\rho_0\, dR$ for σ_0 and $\rho_1\, dR$ for σ_1 and integrating for R from zero to R, there results for the magnetic potential at a point not connected with the rotating sphere:

$$V = \frac{4}{15}\pi\omega R^3\left[(\rho_0 + \frac{4}{5}\rho_1)\frac{R^2}{r^2} P_1(u) - \frac{6}{49}\rho_1 \frac{R^4}{r^4} P_3(u)\right] \ldots \qquad (57)$$

Digitized by Google

Next, suppose that throughout a sphere of radius R there are electric charges of the same sign, the volume density following the law given (56), and throughout a sphere of radius $R - dR$ there are opposite charges following the same law of density; furthermore, assume that the two total charges, though opposite in sign, are equal in amount.

If $\rho' = \rho'_0 + \rho'_1 \sin^2 u$ be the density of the body charge distributed throughout the sphere of radius $R - dR$, we shall have according to the hypothesis:

$-\rho'_0 (R - dR)^3 = \rho_0 R^3$, hence $\rho'_0 = -\rho_0 (1 + 3\frac{dR}{R})$, if we neglect higher powers of dR/R; similarly $\rho'_1 = -\rho_1 (1 + 3\frac{dR}{R})$.

Accordingly, after various reductions:

$$V' = -\frac{4}{15}\pi\omega R^3 \Big[(\rho_0 + \frac{4}{5}\rho_1)(\frac{R^2 - 2RdR}{r^2}) P_1(u) - \frac{6}{49}\rho_1 (\frac{R^4 - 4R^3 dR}{r^4}) P_3(u) \Big];$$

calling the former potential (57), V_1, there results as the combined potential for the two opposite and equal charges:

$$V = V_1 + V' = \frac{8}{15}\pi\omega R^2 dR \Big[(\rho_0 + \frac{4}{5}\rho_1)\frac{R^2}{r^2} P_1(u) - \frac{12}{49}\rho_1 \frac{R^4}{r^4} P_3(u) \Big] ;$$

substituting now the values of the zonal harmonics, $P_1(u) = \cos u$ and $P_3(u) = \frac{5}{2}(\cos^3 u - \frac{3}{5}\cos u)$, and putting $\frac{8}{15}\pi\omega R\, dR = K$, we get finally:

$$V = KR \Big[(\rho_0 + \frac{4}{5}\rho_1)\frac{R^2}{r^2}\cos u - \frac{30}{49}\rho_1 \frac{R^4}{r^4}(\cos^3 u - \frac{3}{5}\cos u) \Big]. \quad (58)$$

The potential in this form is complete and there is no longer any necessity for making the restriction that the point shall be outside the sphere and not partake of its rotation. The term which enters when the point is rigidly connected with the sphere, *i. e.*, when $r = R$, is of opposite sign for the opposite and equal charges, hence drops out in the final expression. The total charge in the sphere (sum of plus and minus charges) being zero, the electric field which by rotation of the containing body gives rise to the magnetic field of which (58) is the potential, would not be brought

Digitized by Google

to light by atmospheric electric measurements but only by magnetic observations.[6]

We may now obtain by partial differentiation the magnetic components X and Z, taking into account the sense in which the variables increase:

$$X = -\frac{1}{r}\left(-\frac{\partial V}{\partial u}\right)$$

$$= -KR\left[\left(\rho_0 + \frac{4}{5}\rho_1\right)\frac{R^2}{r^3} - \frac{90}{49}\rho_1\frac{R^4}{r^5}\left(\cos^2 u - \frac{1}{5}\right)\right]\sin u; \qquad (59)$$

$$Z = -\left(-\frac{\partial V}{\partial r}\right)$$

$$= -2KR\left[\left(\rho_0 + \frac{4}{5}\rho_1\right)\frac{R^2}{r^3} - \frac{60}{49}\rho_1\frac{R^4}{r^5}\left(\cos^2 u - \frac{3}{5}\right)\right]\cos u. \qquad (60)$$

Putting $r = R$:

$$X = -K\left[\left(\rho_0 + \frac{4}{5}\rho_1\right) - \frac{90}{49}\rho_1\left(\cos^2 u - \frac{1}{5}\right)\right]\sin u = f_x(u)\sin u; \qquad (59a)$$

$$Z = -2K\left[\left(\rho_0 + \frac{4}{5}\rho_1\right) - \frac{60}{49}\rho_1\left(\cos^2 u - \frac{3}{5}\right)\right]\cos u = 2f_z(u)\cos u. \qquad (60a)$$

67. Consider next briefly the case of a body charge producing a magnetic field symmetrical about the axis of rotation of the sphere but not about its equator. Only the simplest case will be discussed now, *viz.*, that in which the volume density is:

$$\rho = \rho_2 \cos u. \qquad (61)$$

Substituting $\rho_2 dR$ for σ_2 in (54) and integrating for R from zero to R, it is found that

$$V = \frac{4}{45}\pi\omega R^3\rho_2\frac{R^3}{r^3}P_2(u). \qquad (62)$$

Suppose again as in the previous section that an equal and opposite charge is distributed throughout a sphere of radius $R - dR$, the volume density being $\rho' = \rho'_2 \cos u$. According to the hypothesis,

$$-\rho'_2(R - dR)^3 = \rho_2 R^3, \text{ hence } \rho'_2 = -\rho_2\left(1 + 3\frac{dR}{R}\right) \text{ and}$$

$$V' = -\frac{4}{45}\pi\omega R^3\rho_2\left(\frac{R^3 - 3R^2dR}{r^3}\right)P_2(u);$$

[6] *Cf.* SUTHERLAND, *Terr. Mag.*, v. 8, 1903, pp. 49-52.

Digitized by Google

calling (62) V_1, the combined potential for the two opposite and equal charges becomes:

$$V = V_1 + V' = \frac{4}{15}\pi\omega R^2 dR\rho_2 \frac{R^3}{r^3} P_2(u)$$
$$= \frac{3}{4} K R \rho_2 \frac{R^3}{r^3}(\cos^2 u - \frac{1}{3}) . \quad (63)$$

Here again we may omit the restriction which applied to equations (57) and (62), *viz.*, that the point of observation shall not be rigidly connected with the rotating charged sphere. Accordingly after differentiation of V to get X and Z, we may put $r = R$, hence:

$$X = -\frac{3}{2} K \rho_2 \cos u \sin u = f_x(u) \sin u ; \quad (64)$$

$$Z = -\frac{9}{4} K \rho_2 \cos^2 u + \frac{3}{4} K \rho_2 = \frac{3}{2} f_z(u) \cos u + Z_0 ; \quad (65)$$

$$f_x(u) = f_z(u) = -\frac{3}{2} K \rho_2 \cos u ; \quad (66)$$

$$Z_0 = \frac{3}{4} K \rho_2 . \quad (67)$$

It will be seen that the characteristic functions, though symmetrical about the axis of rotation are not so about the equator since cos u changes sign in crossing the equator, hence the asymmetry about the equator of the magnetic field considered in this section. The value of the vertical component Z is not zero for the equator, but $\frac{3}{4} K \rho_2$.

Earth's Magnetic Field Symmetrical About the Axis of Rotation and About the Equator.

68. The formulæ to be numerically tested, for the case considered in this section, are (59*a*) and (60*a*). Adapting them to least square computation, X, Z being the quantities given in Table XVIII, we have for each parallel, after putting $-K\rho_0 = x_0$ and $-\frac{90}{49} K \rho_1 = x_1$ and collecting the various terms:

$$X_a = x_0 \sin u + x_1 (\frac{143}{225} - \cos^2 u) \sin u ; \quad (68)$$

$$Z_a = 2x_0 \cos u + 2x_1 (\frac{188}{225} - \frac{2}{3} \cos^2 u) \cos u . \quad (69)$$

Digitized by Google

The values of the unknowns as resulting from the least square computations using the data for 1885, are as follows:

From X_a : $x_0 = -K\rho_0 = +0.2882$; $x_1 = -\frac{90}{49}K\rho_1 = +0.0711$,
hence $-K\rho_1 = +0.0348$;

From Z_a : $x_0 = -K\rho_0 = +0.2652$; $x_1 = -\frac{90}{49}K\rho_1 = +0.1088$,
hence $-K\rho_1 = +0.0592$.

The above values of x_0 and x_1 yield computed values of X_a and Z_a agreeing, on the average, with the observed ones, for the portion of the Earth's magnetism considered in this section, to within one per cent. From a combined treatment of the X_a and Z_a components, the resulting values are:

$$x_0 = -K\rho_0 = +0.2845;\ x_1 = -\frac{90}{49}K\rho_1 = +0.0724,$$
$$\text{hence } -K\rho_1 = +0.0394 \quad . \qquad (70)$$

Substituting these last values in the expression for V (58), we get:

$$V = -R\left[(0.2845 + 0.0315)\frac{R^2}{r^2}\cos u - 0.0241\frac{R^4}{r^4}\left(\cos^3 u - \frac{3}{5}\cos u\right)\right]$$

$$= -R\left[0.3160\frac{R^2}{r^2}\cos u - 0.0241\frac{R^4}{r^4}\left(\cos^3 u - \frac{3}{5}\cos u\right)\right] . \qquad (71)$$

The equivalent expression resulting from the Gaussian analysis, stopping with the fourth harmonic, is:

$$V = R\left[g_{10}\frac{R^2}{r^2}\cos u + g_{30}\frac{R^4}{r^4}\left(\cos^3 u - \frac{3}{5}\cos u\right)\right] ; \qquad (72)$$

substituting the values of g_{10} and g_{30} for 1885 as obtained from the analysis by Neumayer and Petersen, which closed with the fourth harmonic (see page 122), we get:

$$V = R\left[-0.3157\frac{R^2}{r^2}\cos u + .0242\frac{R^4}{r^4}\left(\cos^3 u - \frac{3}{5}\cos u\right)\right]. \qquad (72a)$$

From the Gaussian empirical formula, it would not be possible to say whether, or not, the two significant coefficients g_{10} and g_{30} are in any way related. We have ascertained that whatever produces the second term may also enter into the first. Thus comparing the unknowns in the two expressions (58) and (72), it is found that:

$$K\left(\rho_0 + \frac{4}{5}\rho_1\right) = g_{10} \text{ and } -\frac{30}{49}K\rho_1 = g_{30}. \qquad (73)$$

Digitized by Google

By (59a), (60a), (68) and (69) we have:

$$f_x(u) = x_0 + x_1\left(\frac{143}{225} - \cos^2 u\right), \tag{74}$$

$$f_z(u) = x_0 + x_1\left(\frac{188}{225} - \frac{2}{3}\cos^2 u\right). \tag{75}$$

Using the mean observed values of the characteristic functions given in Table XVIII., resulting from the available magnetic charts, 1840-1885, and applying least squares, it is found that:

$$f_x(u) = 0.2910 + 0.0686\left(\frac{143}{225} - \cos^2 u\right), \tag{74a}$$

$$f_z(u) = 0.2792 + 0.0844\left(\frac{188}{225} - \frac{2}{3}\cos^2 u\right). \tag{75a}$$

The values computed with the aid of these formulæ, as well as the observed ones and the differences, observed-computed value, will be found in Table XIX. It will be seen from the table, as also from Fig. 1, that our theoretical formulæ reproduce satisfactorily the law of increase in the characteristic functions towards the equator as shown by the observed values; the computed values agree with the observed ones, in general, to within less than one per cent, thus showing an improvement on the empirical and tentative formulæ of §62.

TABLE XIX.

Observed Values of the Characteristic Functions as derived from the three sets of available magnetic charts, 1840-45, 1880 and 1885, and the computed values by (74a) and (75a); all expressed in C. G. S. units.

Parallel	$f_x(u)$			$f_z(u)$		
	Obs'd	Comp'd	O-C	Obs'd	Comp'd	O-C
60 N and S.	.2874	.2832	+.0042	.3103	.3076	+.0027
50 N and S.	.2942	.2944	—.0002	.3170	.3167	+.0003
40 N and S.	.3025	.3063	—.0038	.3248	.3265	—.0017
30 N and S.	.3140	.3175	—.0035	.3340	.3357	—.0017
20 N and S.	.3261	.3266	—.0005	.3420	.3432	—.0012
10 N and S.	.3343	.3326	+.0017	.3486	.3481	+.0005
Equator.	.3364	.3346	+.0018	.3510*	.3498	+.0012

*Obtained graphically.

Digitized by Google

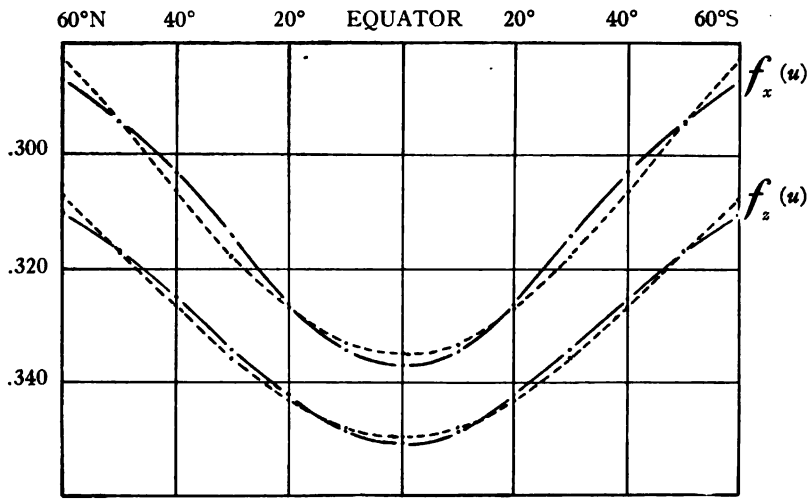


FIG. 1. Observed (full curve) and computed (broken curve) values of the characteristic functions for the mean epoch 1840-1885, and for the portion of the Earth's magnetism symmetrical about both the axis of rotation and the equator. (If the Earth's magnetic field were strictly uniform, the curves would become straight lines parallel to the axis of abscissas; instead, as will be seen, the departure from uniformity of the Earth's field, even for the portion considered, amounts, between parallels 60° and the equator, to 17 per cent for the upper curves, and 13 per cent for the lower ones.)

The residuals, while small, show a systematic run, indicating that our expression (58) is not yet complete, and that an additional harmonic, probably the fifth, besides those of the first and third order, will enter into the final formula. It may also be pointed out that if (58) were the complete potential of the Earth's magnetic field, then, theoretically, the value of x_0 should be the same, whether derived from the X or from the Z components; as is seen from (74a) and (75a), the values differ from each other by about four per cent. A similar statement applies to the two x_1 values; these differ from each other by about 21 per cent. These same difficulties, however, are encountered with the Gaussian analysis in its original form; thus, in Dr. Adolf Schmidt's elaborate analysis the establishment of a single expression was abandoned, and, instead, a separate one was established for each component, just as was done above. It should be recalled that in the establishment of (58) only *internal* agencies have thus far been considered; to obtain a more complete representation of the Earth's magnetic field, it will be necessary to take into account the possibility of

Digitized by Google

effects arising from systems of forces originating above the Earth's surface, as also such as may cut the surface.

In Fig. 1 the characteristic function curves have been drawn as though there were strict symmetry of the Earth's magnetic field about the equator, which, however, is not quite the case; the treatment of the asymmetries about the equator must be reserved for a future section.

69. The next question of interest is, which is the primary magnetic field, the one resulting from the rotation of the ρ_0 electrical charges, the ρ_0 field, or the one due to the ρ_1 charges, the ρ_1 field. It was found from the observational data for 1885 that $K\rho_0 = -0.2845$ and $K\rho_1 = -0.0394$: since K is a numerical constant $= \frac{8}{15}\pi\omega R dR$, it follows that ρ_0 and ρ_1 are both negative. Hence the charge distributed throughout the outer sphere of radius R, the density of which at any point is $\rho = \rho_0 + \rho_1 \sin^2 u$ is negative, whereas the assumed equal and opposite charge distributed throughout the inner sphere of radius $R—dR$, of density $\rho' = \rho_0' + \rho_1' \sin^2 u$, is positive. We accordingly must seek for some agency which causes a separation of the electric charges in each atom composing the Earth, the positive charge being driven inward or toward the Earth's center and the negative charge outward or towards the surface.

It is of added interest that not only do the general facts of the Earth's magnetism require that the outer rotating charge be negative, but that the facts disclosed by atmospheric electric instruments likewise require a surface negative electric charge the source of which has not yet been satisfactorily explained. It is possible that the latter may represent a residual quantity, *viz.*, instead of an exact balance of the two opposite charges as assumed in deriving our magnetic potential expression, may there not be a slight preponderance of one charge, *e. g.*, the negative one, over the other?

The residual quantity required to satisfy atmospheric electric measurements regarding the Earth's charge could not be detected magnetically. Assume, for example, that the Earth has a surface electric charge of uniform density $\sigma_0 = -2.65 \times 10^{-4}$ electrostatic units or about -8.83×10^{-15} electromagnetic units, as shown by electric measurements in middle latitudes. Then the magnetic field which ensues during the Earth's rotation, according to (42),

Digitized by Google

(43), and (52), will produce the following magnetic components, expressed in C. G. S. units:

$$X = -3.43 \times 10^{-10} \sin u; \quad Z = 3.43 \times 10^{-10} \cos u; \tag{76}$$

hence, even the maximum possible values, when sin u or cos $u = 1$, are at least 3,000 times smaller than the limit of magnetic measurements. It will also be noticed from the signs of X and Z, that this field would make the end of the needle which dips in the Northern Hemisphere point south. (*cf.* §61.)

To return to our question, whether the ρ_0 or the ρ_1 field is the primary; this will depend, in the first instance, upon an adequate physical cause. The ρ_0 field corresponds strictly to a uniformly magnetized Earth parallel to its axis of rotation; as already stated there is at present no generally accepted theory to account for this magnetic field. In common with Sutherland, we assumed that it arose from the rotation of an electric charge of uniform density ρ_0 distributed throughout a sphere of radius R equal to the Earth's mean radius, and of an equal and opposite charge distributed uniformly throughout a sphere of radius $R - dR$. From (70) we have $K\rho_0 = \frac{8}{15}\pi\omega R\,dR\,\rho_0 = -0.2845$; repeating Sutherland's calculations,[7] it is found that dR is about 7×10^{-9}, or on the order of magnitude for the diameter of ordinary molecules. Hence, to quote Sutherland[7], "if the negative and the positive electricity in the Earth are spread over concentric spheres whose radii differ by only the diameter of a single molecule, they can account for the Earth's primary magnetic field. This separation of the two electricities would produce no external electric effect." So far good, but when we inquire as to what has caused the separation of the atomic charges and what maintains their separate existence, then difficulties enter.

Pass next to the ρ_1 field. We enter now wholly new territory. This field we supposed to be caused by the rotation of an electric charge, of varying density, *viz.*, $\rho = \rho_1 \sin^2 u$, distributed throughout a sphere equal to the Earth's mean radius and an equal and opposite charge following the same law of density but distributed throughout an inner sphere of radius $R - dR$. From (70) we have $K\rho_1 = \frac{8}{15}\pi\omega R dR\rho_1 = -0.0394$. Making the same calculation as before, dR is found to be about 1×10^{-9}. Hence to account for this field

[7] *Terr. Mag.*, v. 8, 1903, p. 51.

Digitized by Google

the radii of the concentric spheres containing the equal and opposite charges only have to differ by about one-tenth of the diameter of an ordinary molecule.

Resolve the Earth's centrifugal force, $mr\omega^2$, where m is the mass of the particle acted upon, r the radius of the circle of latitude, and ω the angular velocity, into two components, one along the line connecting the point with the Earth's center and the other along a meridian, tangential to the surface. If the point where the mass m is situated is in co-latitude u, the component of the centrifugal force acting away from the Earth's center is $mR\omega^2 \sin^2 u$, R being the Earth's mean radius; this component acts in opposition to the Earth's gravitational force and we shall term it the radial component of the Earth's centrifugal force. The effect of the other component, $mR\omega^2 \sin u \cos u$, acting tangentially to the Earth's surface in the direction towards the equator, will be considered later.

Suppose that due to the radial component of the Earth's centrifugal force, a separation of the bound electric charges in each atom of the Earth takes place, the positive charge being driven in one direction and the negative in the opposite, through an infinitesimal distance. We may doubtless assume, as a first approximation, that the free atomic charge will be distributed throughout the Earth according to the law of variation in the separating agency. Hence if the density of the body charge may be set proportional to $mR\omega^2 \sin^2 u$, and if ρ_1 is the value for $u = 90°$, *i. e.*, for the equator, then we have for any other latitude $\rho = \rho_1 \sin^2 u$; but this is the same law as was assumed in deriving what we have termed the "ρ_1 field." The magnetic potential due to this distribution alone, according to (58) and (70), was found to be for 1885:

$$V_1 = KR\rho_1 \left[\frac{4}{5} \frac{R^2}{r^2} \cos u - \frac{30}{49} \frac{R^4}{r^4} (\cos^3 u - \frac{3}{5} \cos u) \right]$$

$$= -R \left[0.0315 \frac{R^2}{r^2} \cos u - 0.0241 \frac{R^4}{r^4} (\cos^3 u - \frac{3}{5} \cos u) \right]. \quad (77)$$

The first term gives a uniform magnetic field whose axis coincides with the axis of rotation, the second produces a magnetic field which while symmetrical both about the axis of rotation and the equator is not uniform, but acts so as to cause an apparent increase in the intensity of the Earth's uniform magnetic field in going from the poles to the equator. If then this field, the origin of which we may possibly have to refer to some action of the Earth's cen-

Digitized by Google

trifugal force, gives rise to a term equivalent to a uniform magnetization, the question arises whether we may not be able to account thus for the whole of the Earth's uniform magnetic field. To examine this, put $\rho_0 = 0$ in equations (59a) and (60a) and obtain $K\rho_1$ by the method of least squares, using the 1885 data; the value —0.3335 is found. Hence

$$V_1 = -R\left[0.2668\,\frac{R^2}{r^2}\cos u - 0.2041\,\frac{R^4}{r^4}\left(\cos^3 u - \frac{3}{5}\cos u\right)\right]. \quad (78)$$

We have here in the first term a uniform magnetic field of the order required; unfortunately, however, the second term gives rise to a much more rapid increase in our characteristic functions than disclosed by observation. Accordingly, we can not do away entirely with the ρ_0 distribution, or some cause must be found which will reduce the effect of the second term of (78). The law of distribution of the free electric charges assumed above, is to be regarded, as distinctly stated, merely as a first approximation. Strictly the density will also be a function of the radius and the dR assumed as constant in deriving (58) will itself vary, being a maximum in the equatorial plane and reducing to zero along the axis of rotation; these matters will be investigated more carefully later.

It should be pointed out that for the ρ_1 field, as for the ρ_0, the observed facts require that the negative charge is on the outside and the positive on the inside.

The ρ_0 field may of course also be interpreted as resulting from an actual uniform magnetization of the Earth about the axis of rotation. If ρ_0 be the intensity of magnetization per unit of volume, then for 1885 according to (70):

$$V_o = \frac{4}{3}\pi\,\rho_0\,R^3\,\frac{R^2}{r^2}\cos u = -0.2845\,\frac{R^3}{r^2}\cos u \quad . \quad (79)$$

Comparing this with the corresponding term in (77) it is seen that it is only about nine times larger.

Assume for the sake of argument that the ρ_1 field is the primary one. Then we have the Earth rotating in a magnetic field which we may perhaps regard as not partaking of the rotation both by reason of its formation and its symmetry about the axis of rotation.[8] If the Earth's magnetic permeability is uniform it will, accordingly, become magnetized uniformly parallel to the axis of rotation; if the magnetic permeability is not uniform then the magnetization will be irregular. The simplest hypothesis will be

[8] *Cf.* SUTHERLAND, *Terr. Mag.* v. 5, 1900, p. 77. Also, KENNARD, E. H., Unipolar induction, *Phil. Mag.* (6), 23, pp. 937-941, 1912.

Digitized by Google

a uniform magnetization parallel to a diameter inclined to the axis of rotation and this we may resolve into two rectangular components: one component parallel to the axis of rotation and the other parallel to a diameter in the equatorial plane. According to the previous paragraph the potential of the first portion would be about nine times the corresponding potential of the inducing field. If the ρ_0 field has come from the ρ_1 in the manner supposed, the characteristic coefficients ρ_0 and ρ_1 may be expected to be of the same sign, as was actually found.

If the assumption in the previous paragraph is correct, *viz.*, that the magnetically permeable and electrically conducting Earth is rotating in a magnetic field which does not rotate with the Earth, then the effects from induced electromotive forces, in addition to those from induced magnetization, will require to be taken into account; consideration of these, however, will be deferred until a later number.

71. *Chief Conclusions.* Confining attention at present to the portion of the Earth's magnetic field, which is symmetrical about both the axis of rotation and the equator, then the following chief results were found:

(1). If the intensity of magnetization be computed separately for each parallel over the region best covered by magnetic observations, 60° N. to 60° S., on the basis that the average values of the magnetic components along a parallel correspond to those of a uniform magnetization parallel to the axis of rotation, then a systematic increase in the intensity of magnetization is observed in both hemispheres in going from parallel to parallel towards the equator. Thus, for example, for the mean of parallels 60 N. and S., this constant in C. G. S. units was found to be 0.070, whereas for the equator it is 0.082, hence an increase of 17 per cent. This fact, when mathematically expressed, was found to furnish a valuable criterion for testing the sufficiency of the theories thus far advanced to account for the origin of the Earth's magnetism; not a single one met the requirements.

(2). The portion of the Earth's magnetic field considered is similar to that of a uniformly magnetized sphere which has superposed on it another field whose intensity in both hemispheres increases systematically from parallel to parallel, with approach towards the equator. A closed, physical expression (58) was found, consisting of the zonal harmonics of the first and third order, which gave a representation of the observed facts to within one per cent.

Digitized by Google

MAGNETIC DECLINATIONS AND CHART CORRECTIONS OBTAINED BY THE *CARNEGIE* FROM BATAVIA TO MANILA AND THENCE TO SUVA, FIJI, NOVEMBER, 1911, TO JUNE 5, 1912.[1]

By L. A. Bauer and W. J. Peters.

Table I.

Batavia to Manila, November, 1911, to February, 1912.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1911	° ′	° ′	°	°	°	°	°	°	°
Nov. 22	5 59 S	106 46 E	+ 0.5	+ 0.6	+ 0.6	+ 0.5	—0.1	—0.1	0.0
23	6 07 S	105 49 E	+ 0.6	+ 0.3	+ 0.4	+ 0.3	+0.3	+0.2	+0.3
26	9 33 S	106 33 E	+ 0.4	— 0.3	+ 0.1	— 0.3	+0.7	+0.3	+0.7
27	11 13 S	106 01 E	+ 0.2	— 1.0	— 0.2	— 1.0	+1.2	+0.4	+1.2
28	11 44 S	105 44 E	— 0.5	— 1.1	— 0.3	— 1.1	+0.6	—0.2	+0.6
28	12 07 S	105 34 E	— 0.5	— 1.2	— 0.5	— 1.3	+0.7	0.0	+0.8
30	16 22 S	101 45 E	— 3.4	— 3.0	— 2.0	— 3.1	—0.4	—1.4	—0.3
Dec. 1	17 49 S	100 46 E	— 4.0	— 3.8	— 2.5	— 3.9	—0.2	—1.5	—0.1
1	19 03 S	99 13 E	— 5.2	— 4.6	— 3.0	— 4.7	—0.6	—2.2	—0.5
2	21 20 S	96 32 E	— 7.8	— 6.1	— 4.2	— 6.4	—1.7	—3.6	—1.4
3	22 38 S	95 08 E	— 9.1	— 7.0	— 5.0	— 7.4	—2.1	—4.1	—1.7
4	25 16 S	92 50 E	—12.1	— 9.0	— 7.6	—10.0	—3.1	—4.5	—2.1
5	28 58 S	89 56 E	—16.0	—12.8	—11.5	—14.0	—3.2	—4.5	—2.0
6	29 22 S	89 35 E	—16.7	—13.5	—12.0	—14.2	—3.2	—4.7	—2.5
6	29 42 S	89 29 E	—16.9	—13.8	—12.3	—14.5	—3.1	—4.6	—2.4
7	31 22 S	89 31 E	—18.3	—15.4	—13.7	—15.4	—2.9	—4.6	—2.9
8	32 29 S	89 38 E	—20.0	—16.4	—14.8	—16.5	—3.6	—5.2	—3.5
9	34 40 S	91 34 E	—21.0	—18.0	—16.4	—18.0	—3.0	—4.6	—3.0
9	36 03 S	93 17 E	—21.9	—19.0	—17.2	—19.0	—2.9	—4.7	—2.9
11	36 14 S	97 18 E	—20.0	—17.1	—15.8	—17.1	—2.9	—4.2	—2.9
11	35 56 S	97 44 E	—19.0	—16.6	—15.0	—16.6	—2.4	—4.0	—2.4
12	35 38 S	98 34 E	—17.7	—15.8	—14.7	—15.8	—1.9	—3.0	—1.9
12	34 19 S	100 16 E	—16.1	—13.7	—12.8	—13.9	—2.4	—3.3	—2.2
13	33 11 S	101 22 E	—14.6	—12.4	—11.4	—12.4	—2.2	—3.2	—2.2
13	32 06 S	102 13 E	—12.1	—11.0	—10.1	—11.0	—1.1	—2.0	—1.1
14	30 12 S	103 34 E	—10.4	— 8.5	— 8.0	— 8.8	—1.9	—2.4	—1.6
15	29 09 S	103 58 E	—10.0	— 7.5	— 7.0	— 7.8	—2.5	—3.0	—2.2
15	27 58 S	104 42 E	— 8.0	— 6.6	— 5.6	— 6.9	—1.4	—2.4	—1.1
16	26 45 S	105 34 E	— 6.8	— 5.8	— 4.7	— 6.0	—1.0	—2.1	—0.8
17	22 58 S	106 27 E	— 4.2	— 4.2	— 2.9	— 4.3	0.0	—1.3	+0.1
17	21 16 S	107 01 E	— 3.1	— 3.4	— 2.3	— 3.6	+0.3	—0.8	+0.5
18	19 39 S	107 44 E	— 2.2	— 2.6	— 1.8	— 2.8	+0.4	—0.4	+0.6
19	17 30 S	108 39 E	— 0.8	— 1.9	— 1.0	— 2.0	+1.1	+0.2	+1.2
19	16 58 S	108 54 E	— 0.5	— 1.7	— 0.7	— 1.7	+1.2	+0.2	+1.2
20	15 26 S	109 40 E	— 0.4	— 1.2	— 0.3	— 1.2	+0.8	—0.1	+0.8

[1] For previous tables see *Terr. Mag.* v. 15, pp. 57–82, 129–142, 143–144; v. 16, pp. 133–136; v. 17, pp. 31–32, 97–101.

Digitized by Google

TABLE I.—Continued.

Batavia to Manila, November, 1911, to February, 1912.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1911	° ′	° ′	°	°	°	°	°	°	°
Dec. 21	14 48 S	110 11 E	—0.3	—0.9	0.0	—0.9	+0.6	—0.3	+0.6
21	14 03 S	110 41 E	+0.4	—0.6	0.0	—0.6	+1.0	+0.4	+1.0
22	12 44 S	111 51 E	+0.7	0.0	+0.4	+0.1	+0.7	+0.3	+0.6
23	11 59 S	112 26 E	+1.0	+0.3	+0.6	+0.3	+0.7	+0.4	+0.7
23	11 04 S	113 07 E	+1.0	+0.5	+0.9	+0.6	+0.5	+0.1	+0.4
24	10 24 S	113 44 E	+1.6	+0.8	+1.0	+0.7	+0.8	+0.6	+0.9
25	8 36 S	116 40 E	+1.8	+1.3	+1.7	+1.2	+0.5	+0.1	+0.6
26	7 48 S	117 44 E	+2.2	+1.4	+1.9	+1.4	+0.8	+0.3	+0.8
27	7 23 S	118 50 E	+2.2	+1.6	+2.1	+1.5	+0.6	+0.1	+0.7
28	5 43 S	119 58 E	+2.6	+1.8	+2.1	+1.8	+0.8	+0.5	+0.8
29	5 36 S	120 14 E	+1.0	+1.8	+2.1	+1.8	—0.8	—1.1	—0.8
30	5 43 S	120 59 E	+2.5	+1.9	+2.2	+1.9	+0.6	+0.3	+0.6
1912									
Jan. 1	5 15 S	123 23 E	+2.9	+2.1	+2.4	+2.1	+0.8	+0.5	+0.8
2	3 53 S	123 38 E	+2.8	+2.1	+2.5	+2.1	+0.7	+0.3	+0.7
3	3 09 S	124 03 E	+2.7	+2.2	+2.5	+2.2	+0.5	+0.2	+0.5
3	3 01 S	124 28 E	+2.8	+2.2	+2.5	+2.2	+0.6	+0.3	+0.6
5	2 34 S	124 28 E	+2.9	+2.2	+2.5	+2.3	+0.7	+0.4	+0.6
6	2 52 S	125 44 E	+2.9	+2.3	+2.5	+2.3	+0.6	+0.4	+0.6
6	3 31 S	125 59 E	+2.9	+2.4	+2.5	+2.3	+0.5	+0.4	+0.6
11	1 18 S	128 26 E	+3.0	+2.4	+2.5	+2.4	+0.6	+0.5	+0.6
11	1 07 S	128 32 E	+2.8	+2.5	+2.4	+2.4	+0.3	+0.4	+0.4
12	0 36 S	128 48 E	+3.0	+2.4	+2.3	+2.4	+0.6	+0.7	+0.6
13	0 08 S	128 25 E	+3.0	+2.4	+2.3	+2.3	+0.6	+0.7	+0.7
16	0 24N	129 27 E	+2.5	+2.4	+2.2	+2.4	+0.1	+0.3	+0.1
16	0 28N	129 33 E	+2.5	+2.5	+2.2	+2.4	0.0	+0.3	+0.1
18	0 49N	130 53 E	+2.7	+2.5	+2.3	+2.4	+0.2	+0.4	+0.3
18	0 38N	131 26 E	+2.7	+2.5	+2.4	+2.5	+0.2	+0.3	+0.2
19	0 26N	132 23 E	+3.1	+2.7	+2.5	+2.7	+0.4	+0.6	+0.4
20	1 17N	135 54 E	+3.1	+3.0	+2.8	+3.0	+0.1	+0.3	+0.1
21	2 10N	136 46 E	+3.3	+3.0	+2.8	+3.0	+0.3	+0.5	+0.3
22	3 08N	138 31 E	+3.2	+3.1	+2.9	+3.1	+0.1	+0.3	+0.1
23	4 10N	137 40 E	+3.1	+3.0	+2.6	+2.9	+0.1	+0.5	+0.2
23	5 02N	137 12 E	+2.9	+2.7	+2.4	+2.8	+0.2	+0.5	+0.1
24	6 32N	136 36 E	+2.5	+2.4	+2.1	+2.4	+0.1	+0.4	+0.1
24	7 38N	136 00 E	+2.0	+2.0	+1.8	+2.1	0.0	+0.2	—0.1
25	8 45N	135 28 E	+2.0	+1.9	+1.7	+1.9	+0.1	+0.3	+0.1
25	9 30N	134 28 E	+1.9	+1.5	+1.4	+1.6	+0.4	+0.5	+0.3
26	11 38N	132 18 E	+1.4	+0.9	+1.0	+1.0	+0.5	+0.4	+0.4
27	12 45N	131 03 E	+0.9	+0.6	+0.7	+0.8	+0.3	+0.2	+0.1
27	13 42N	130 00 E	+1.0	+0.3	+0.4	+0.6	+0.7	+0.6	+0.4
28	14 59N	124 44 E	+0.5	+0.1	+0.1	+0.4	+0.4	+0.4	+0.1
29	16 42N	126 52 E	+0.2	—0.1	—0.2	+0.1	+0.3	+0.4	+0.1
31	18 42N	120 36 E	+0.1	—0.1	—0.1	0.0	+0.2	+0.2	+0.1
31	17 22N	119 50 E	+0.3	+0.3	+0.2	+0.3	0.0	+0.1	0.0
Feb. 1	15 46N	119 36 E	+0.5	+0.6	+0.6	+0.6	—0.1	—0.1	—0.1
1	14 38N	120 10 E	+1.0	+0.7	+0.6	+0.7	+0.3	+0.4	+0.3
2	14 18N	120 28 E	+0.9	+0.7	+0.6	+0.7	+0.2	+0.3	+0.2

Digitized by Google

TABLE II.

Manila, Philippine Islands, to Suva, Fiji Islands, March 24 to June 5, 1912.

(*Minus indicates west declination, plus east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections.		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1912	° ′	° ′	°	°	°	°	°	°	°
Mar. 24	14 23N	120 24 E	+1.0	+0.8	+0.7	+0.8	+0.2	+0.3	+0.2
25	15 54N	119 58 E	+0.8	+0.5	+0.4	+0.5	+0.3	+0.4	+0.3
26	16 19N	119 20 E	+0.6	+0.4	+0.4	+0.5	+0.2	+0.2	+0.1
27	17 10N	119 34 E	+0.4	+0.2	+0.3	+0.3	+0.2	+0.1	+0.1
28	17 03N	119 12 E	+0.3	+0.3	+0.3	+0.3	0.0	0.0	0.0
29	18 40N	118 49 E	+0.1	+0.1	0.0	+0.1	0.0	+0.1	0.0
31	18 40N	120 15 E	+0.3	0.0	—0.1	0.0	+0.3	+0.4	+0.3
April 3	21 37N	119 40 E	—0.5	—0.6	—0.5	—0.5	+0.1	0.0	0.0
3	21 03N	120 18 E	—0.5	—0.5	—0.5	—0.5	0.0	0.0	0.0
5	20 08N	120 54 E	—0.4	—0.4	—0.4	—0.3	0.0	0.0	—0.1
7	19 12N	122 26 E	—0.4	—0.4	—0.3	—0.3	0.0	—0.1	—0.1
8	20 17N	122 20 E	—0.4	—0.6	—0.6	—0.5	+0.2	+0.2	+0.1
8	21 14N	122 25 E	—0.8	—0.8	—0.8	—0.8	0.0	0.0	0.0
9	22 20N	122 28 E	—0.9	—1.0	—1.0	—0.9	+0.1	+0.1	0.0
14	24 53N	128 00 E	—1.9	—2.2	—2.2	—2.0	+0.3	+0.3	+0.1
15	26 09N	129 52 E	—2.3	—2.7	—2.5	—2.4	+0.4	+0.2	+0.1
16	26 47N	132 19 E	—2.6	—2.8	—2.7	—2.5	+0.2	+0.1	—0.1
19	30 40N	137 07 E	—3.5	—3.5	—3.5	—3.3	0.0	0.0	—0.2
20	30 40N	138 43 E	—3.5	—3.2	—3.3	—3.1	—0.3	—0.2	—0.4
20	30 35N	139 08 E	—3.2	—3.2	—3.2	—3.0	0.0	0.0	—0.2
22	30 47N	143 26 E	—2.9	—2.4	—2.5	—2.5	—0.5	—0.4	—0.4
23	30 58N	145 30 E	—2.5	—2.0	—2.0	—2.1	—0.5	—0.5	—0.4
23	31 01N	147 14 E	—2.4	—1.5	—1.4	—1.7	—0.9	—1.0	—0.7
25	31 16N	150 35 E	—1.2	—0.4	—0.6	—0.7	—0.8	—0.6	—0.5
25	31 09N	152 32 E	—0.6	+0.4	0.0	0.0	—1.0	—0.6	—0.6
26	30 38N	154 38 E	+0.4	+1.1	+1.0	+0.9	—0.7	—0.6	—0.5
26	30 27N	156 51 E	+1.0	+1.9	+1.9	+1.9	—0.9	—0.9	—0.9
27	30 26N	158 44 E	+1.8	+2.5	+2.6	+2.5	—0.7	—0.8	—0.7
27	30 34N	160 52 E	+2.4	+3.2	+3.3	+3.4	—0.8	—0.9	—1.0
28	30 33N	162 50 E	+3.5	+3.9	+4.2	+4.1	—0.4	—0.7	—0.6
30	29 51N	167 25 E	+5.4	+5.8	+5.9	+5.9	—0.4	—0.5	—0.5
30	28 37N	168 36 E	+6.0	+6.5	+6.4	+6.4	—0.5	—0.4	—0.4
May 1	27 15N	169 54 E	+6.8	+7.1	+7.0	+7.0	—0.3	—0.2	—0.2
1	25 46N	171 01 E	+7.1	+7.8	+7.6	+7.6	—0.7	—0.5	—0.5
2	24 24N	171 37 E	+7.6	+8.2	+8.0	+7.9	—0.6	—0.4	—0.3
2	22 51N	171 56 E	+7.8	+8.4	+8.2	+8.1	—0.6	—0.4	—0.3
3	19 49N	172 57 E	+8.3	+8.9	+8.6	+8.5	—0.6	—0.3	—0.2
4	18 34N	173 16 E	+8.4	+9.0	+8.7	+8.7	—0.6	—0.3	—0.3
5	16 04N	173 48 E	+8.7	+9.0	+8.9	+8.9	—0.3	—0.2	—0.2
6	13 24N	174 58 E	+8.7	+9.0	+9.1	+9.0	—0.3	—0.4	—0.3
6	12 12N	175 45 E	+8.8	+9.1	+9.1	+9.1	—0.3	—0.3	—0.3
7	10 50N	176 32 E	+8.8	+9.1	+9.1	+9.1	—0.3	—0.3	—0.3
7	9 45N	177 19 E	+9.0	+9.1	+9.2	+9.2	—0.1	—0.2	—0.2
8	8 30N	178 09 E	+8.9	+9.2	+9.2	+9.2	—0.3	—0.3	—0.3
8	7 28N	178 51 E	+8.8	+9.2	+9.2	+9.2	—0.4	—0.4	—0.4
10	4 16N	180 17 E	+8.8	+9.1	+9.1	+9.0	—0.3	—0.3	—0.2
10	3 22N	180 39 E	+8.7	+9.1	+9.1	+9.0	—0.4	—0.4	—0.3

Digitized by Google

TABLE II.—Continued.

Manila, Philippine Islands, to Suva, Fiji Islands, March 24 to June 5, 1912.
(Minus indicates west declination, plus east declination.)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1912	° ′	° ′	°	°	°	°	°	°	°
May 11	2 54N	180 48 E	+ 8.6	+ 9.1	+ 9.1	+ 9.0	—0.5	—0.5	—0.4
13	2 16N	180 37 E	+ 8.6	+ 9.1	+ 9.1	+ 8.9	—0.5	—0.5	—0.3
14	1 55N	180 39 E	+ 8.1	+ 9.1	+ 9.1	+ 8.9	—1.0	—1.0	—0.8
15	0 45N	180 16 E	+ 8.5	+ 9.0	+ 9.1	+ 9.0	—0.5	—0.6	—0.5
15	0 03 S	179 56 E	+ 8.8	+ 9.0	+ 9.1	+ 9.0	—0.2	—0.3	—0.2
16	0 36 S	179 16 E	+ 9.1	+ 9.0	+ 9.1	+ 9.0	+0.1	0.0	+0.1
16	1 12 S	178 58 E	+ 8.7	+ 9.0	+ 9.1	+ 9.1	—0.3	—0.4	—0.4
17	2 22 S	178 51 E	+ 8.4	+ 9.1	+ 9.1	+ 9.1	—0.7	—0.7	—0.7
18	2 44 S	178 28 E	+ 9.0	+ 9.1	+ 9.1	+ 9.1	—0.1	—0.1	—0.1
18	3 17 S	177 59 E	+ 8.3	+ 9.1	+ 9.1	+ 9.2	—0.8	—0.8	—0.9
20	5 30 S	176 18 E	+ 9.0	+ 9.1	+ 9.1	+ 9.3	—0.1	—0.1	—0.3
21	7 08 S	174 39 E	+ 9.0	+ 9.1	+ 9.2	+ 9.3	—0.1	—0.2	—0.3
23	9 04 S	174 01 E	+ 9.1	+ 9.2	+ 9.4	+ 9.5	—0.1	—0.3	—0.4
24	9 20 S	174 17 E	+ 9.0	+ 9.2	+ 9.4	+ 9.5	—0.2	—0.4	—0.5
26	14 49 S	172 11 E	+ 9.4	+ 9.6	+ 9.7	+ 9.8	—0.2	—0.3	—0.4
27	16 16 S	171 32 E	+ 9.4	+ 9.8	+ 9.9	+ 9.9	—0.4	—0.5	—0.5
27	17 16 S	171 24 E	+ 9.5	+ 9.8	+10.0	+10.0	—0.3	—0.5	—0.5
28	18 55 S	170 49 E	+10.3	+10.0	+10.1	+10.1	+0.3	+0.2	+0.2
29	21 01 S	169 57 E	+10.4	+10.3	+10.6	+10.6	+0.1	—0.2	—0.2
29	21 20 S	170 05 E	+10.8	+10.3	+10.7	+10.7	+0.5	+0.1	+0.1
30	21 14 S	170 45 E	+10.5	+10.4	+10.7	+10.7	+0.1	—0.2	—0.2
31	21 43 S	171 44 E	+10.5	+10.5	+10.9	+10.9	0.0	—0.4	—0.4
June 1	21 47 S	172 05 E	+12.2	+10.5	+11.0	+11.0	+1.7	+1.2	+1.2
3	21 16 S	174 05 E	+10.8	+10.5	+11.0	+11.0	+0.3	—0.2	—0.2
3	20 43 S	175 09 E	+10.8	+10.5	+10.9	+10.9	+0.3	—0.1	—0.1
4	19 30 S	175 53 E	+10.5	+10.3	+10.7	+10.7	+0.2	—0.2	—0.2
4	18 44 S	176 40 E	+10.4	+10.2	+10.5	+10.6	+0.2	—0.1	—0.2
5	19 23 S	176 22 E	+10.3	+10.3	+10.6	+10.7	0.0	—0.3	—0.4

Digitized by Google

RESULTATE DER INKLINATIONSBEOBACHTUNGEN DER DEUTSCHEN SÜDPOLAR-EXPEDITION, 1901 BIS 1903.

VON FR. BIDLINGMAIER.

Die folgenden Werte sind abwechselnd mit dem Lloyd-Creak-Apparat und mit dem Deviationsmagnetometer Bamberg von Fr. Bidlingmaier a. B. des "Gauss" bestimmt worden. Näheres über Instrumente und Beobachtungsverfahren findet man in Heft 1 der Veröffentlichungen des Instituts für Meereskunde Berlin 1902, pag. 72 ff, über den Einfluss des Schiffseisens und seine Elimination in Heft 2 von Band V des Deutschen Südpolar-Werks. Das in einiger Zeit erscheinende Heft 4 dieses Bandes wird eingehende Darlegungen und Untersuchungen über Entstehung und Genauigkeit der folgenden Werte enthalten. Die Bearbeitung der Beobachtungen rührt von Herrn Dr. Balthasar Hartmann her.

Wilhelmshaven, 23, VII, 12.

Beobachtungstabelle

Datum	Breite	Länge	Inklination	Datum	Breite	Länge	Inklination
1901	° ′	° ′	°	1901	° ′	° ′	°
Aug. 29	35 03N	15 16W	+55.7	Okt. 18	17 46 S	19 54W	—24.1
30	33 44	16 21	+54.8	19	18 43	20 00	—24.5
Sept. 4	27 57	19 56	+49.1	20	19 53	20 09	—26.0
7	24 12	21 55	+45.4		19 58	20 09	—26.2
11	17 44	24 54	+38.8		20 10	20 10	—26.2
Okt. 5	3 32 S	16 47	— 4.9		20 18	20 12	—26.3
6	4 28	16 47	— 6.8		20 28	20 12	—26.6
7	5 57	17 35	— 8.6		20 33	20 13	—26.5
8	7 04	18 04	— 9.4	22	23 33	20 49	—29.6
	7 39	18 09	—10.5	23	24 43	20 38	—31.1
9	8 38	18 13	—11.7	25	26 44	18 42	—34.2
	9 05	18 16	—13.0	26	27 00	17 07	—35.4
10	10 09	18 22	—14.4	28	28 12	17 25	—36.9
	10 30	18 26	—14.9	29	28 52	16 44	—37.6
11	11 13	18 33	—16.6	30	29 50	15 20	—39.3
	11 26	18 36	—16.0	Nov. 1	31 00	12 48	—41.3
12	12 06	18 44	—17.3	2	31 40	11 11	—43.3
	12 27	18 47	—17.3	4	31 22	8 27	—45.3
13	13 08	18 54	—18.3	5	32 06	8 30	—45.4
14	14 02	19 08	—19.7	6	32 48	7 50	—45.8
15	15 07	19 29	—21.0	7	33 20	6 40	—46.6
16	15 56	19 41	—21.6	8	33 39	5 14	—47.2
17	16 53	19 46	—22.8	9	34 02	3 30	—48.4

Digitized by Google

Beobachtungstabelle—Fortsetzung.

Datum	Breite	Länge	Inklination
1901	° ′	° ′	°
Okt. 11	34 29 S	0 42W	—50.4
12	35 04	2 05 O	—52.2
14	35 31	5 45	—54.4
16	35 39	8 13	—56.9
22	34 11	17 16	—58.9
Kapstadt.	Signal Hill.		—58.9
Dez. 10	36 47 S	19 40 O	—60.6
11	37 59	21 19	—61.7
13	39 36	25 19	—63.0
	39 38	25 31	—63.2
14	40 02	27 40	—64.1
17	42 44	34 44	—66.6
18	43 05	36 30	—66.8
19	44 01	39 48	—67.5
20	44 25	42 23	—67.5
21	44 52	44 22	—67.9
23	46 16	49 45	—69.3
24	46 48	50 45	—69.1
Crozet Inseln.	Possession Isl. So-Ecke		—70.9
Dez. 27	47 16 S	58 28 O	—70.1
28	47 44	61 01	—69.7
30	48 01	67 29	—70.1
31	48 48	70 10	—70.3
	48 52	70 18	—70.2
1902			
Jan. 1	Kerguelen Royal Sound		—70.6 —70.6 —70.5
Kerguelen.	Observatory Bay		—70.1
Feb. 5	54 10 S	80 12 O	—73.7
6	55 23	82 50	—74.2
7	55 58	84 30	—74.7
10	58 33	90 03	—76.2
	58 35	90 06	—76.1
11	59 43	91 29	—76.8
13	62 06	95 08	—78.4
14	63 27	96 13	—78.7
15	63 49	95 24	—78.5
17	64 04	91 54	—78.0
18	64 47	89 50	—77.4
20	66 08	90 21	—77.7
21	66 22	90 55	—77.4

Datum	Breite	Länge	Inklination
	° ′	° ′	°
Winterstation.	66 03 S	89 37 O	—77.1
1903			
Feb. 19	65 32	87 23	—77.0
25	65 33	85 19	—76.4
März. 3	65 20	85 06	—76.4
13	64 31	84 33	—76.0
18	64 22	80 40	—75.5
April 10	62 40	80 05	—75.1
11	62 09	75 52	—73.8
12	61 22	72 14	—73.9
13	59 35	70 05	—73.1
15	55 41	72 09	—72.5
17	54 25	70 09	—72.0
18	52 48	69 14	—71.7
20	47 54	71 48	—71.2
22	44 27	73 58	—70.5
23	43 41	75 13	—69.9
24	41 57	76 24	—69.1
25	40 11	76 38	—68.2
St. Paul.			—67.6
April 29	37 13	74 39	—66.6
30	35 56	74 25	—65 8
Mai 1	35 05	74 15	—65.3
2	33 03	73 51	—64.0
3	30 53	73 07	—62.6
4	28 53	71 01	—61.2
5	28 19	69 44	—61.1
6	27 46	67 56	—60.5
7	27 29	66 23	—61.0
8	26 57	64 17	—60.3
9	26 52	63 17	—60.4
10	26 02	63 29	—59.4
11	24 47	62 08	—59.1
12	24 34	60 35	—58.9
13	24 49	58 15	—59.3
14	25 03	56 54	—59.6
15	25 10	55 58	—59.8
16	26 01	53 36	—60.7
17	26 50	50 56	—61.7
18	26 40	50 26	—61.1
20	26 29	48 44	—61.4
21	26 39	46 59	—61.5
22	27 01	46 06	—61.8
23	27 17	44 22	—61.7
24	28 17	42 04	—62.4
25	29 05	40 36	—62.6
26	28 41	39 18	—62.5
27	28 33	37 53	—62.1
28	28 49	36 26	—62.1
29	29 01	34 16	—62.1
30	29 13	32 39	—61.6

Digitized by Google

Beobachtungstabelle—Fortsetzung.

Datum	Breite	Länge	Inklination
1903	° ′	° ′	°
Juni 1	31 50 S	29 53 O	—61.2
2	33 50	28 23	—62.5
3	34 48	25 54	—61.8
5	34 27	24 52	—61.8
7	35 07	22 33	—61.1
Simonstown.	Rifle Range.		—59.4
Aug. 4	33 36 S	16 59 O	—59.2
5	33 16	14 58	—57.3
6	32 42	14 19	—56.7
7	31 08	13 00	—55.5
8	30 45	11 56	—54.9
9	30 24	10 54	—54.4
10	30 10	10 33	—54.0
11	28 44	10 13	—53.6
12	28 46	9 45	—53.3
13	29 08	8 52	—53.1
14	28 55	8 08	—52.4
15	28 25	7 15	—52.1
16	28 04	6 24	—50.9
17	28 23	5 10	—50.8
18	28 23	4 11	—49.9
19	27 24	2 58	—48.6
20	25 47	1 25	—46.8
21	25 17	0 15 O	—46.0
22	24 52	1 29W	—44.7
23	24 30	2 55	—44.1
24	23 11	4 21	—41.2
	23 08	4 22	—41.0
	23 05	4 22	—41.1
25	21 23	4 46	—39.3
26	18 48	5 07	—36.7
27	17 19	5 22	—34.9
St. Helena.	Longwood Plain.		—33.3
Sept. 3	14 02 S	6 02W	—31.0
4	11 57	6 17	—28.2
5	10 56	6 22	—26.8
6	10 07	7 28	—24.7
7	9 40	8 49	—23.2
9	9 03	10 43	—20.3
10	8 38	12 05	—18.4
11	8 10	13 17	—16.1
15	7 52	14 28	—14.7
16	6 53	15 48	—11.4
17	4 59	16 03	— 7.9
	4 05	16 04	— 6.6

Datum	Breite	Länge	Inklination
1903	° ′	° ′	°
Sept. 18	2 46 S	16 03W	— 4.3
	2 15	16 02	— 3.2
19	1 16	16 01	— 1.5
	0 26 S	15 58	— 0.2
20	0 10N	16 27W	+ 2.2
21	0 24N	17 52	+ 4.1
22	0 24 S	18 45	+ 3.4
23	0 21N	18 57	+ 4.9
24	0 28	17 58	+ 4.3
25	0 24	17 22	+ 4.0
26	0 56	19 06	+ 6.3
27	2 06	19 46	+ 8.9
28	3 23	20 17	+12.1
29	4 28	21 11	+15.2
30	5 35	21 45	+17.4
Okt. 1	6 12	21 54	+18.3
2	7 16	22 38	+21.0
3	9 24	24 03	+26.0
4	11 07	25 02	+29.3
5	12 26	25 58	+32.2
6	13 55	26 44	+34.5
7	15 01	27 39	+37.5
8	16 18	28 48	+39.3
9	17 46	29 51	+42.3
10	18 46	30 32	+43.6
12	20 53	32 03	+47.0
14	21 20	31 49	+47.3
15	23 25	31 47	+49.7
16	24 32	32 16	+51.0
17	26 25	33 21	+53.3
18	27 53	34 02	+55.0
20	28 40	34 25	+56.1
22	31 07	33 29	+58.0
23	32 27	32 08	+58.4
24	33 45	31 26	+59.4
26	35 42	29 57	+60.5
27	36 59	27 27	+61.2
Pta. Delgada Azoren.	auf Sao Miguel.		+61.5
Nov. 10	39 43N	25 36W	+62.8
11	41 12	23 48	+63.9
12	42 00	22 44	+63.8
13	43 10	20 26	+64.6
15	46 12	14 43	+65.7
16	46 52	11 49	+65.6
17	47 22	9 03	+65.4
19	48 45	7 00	+66.3
20	50 00	4 46	+66.9
21	50 41	0 18 O	+66.7

Digitized by Google

LETTERS TO THE EDITOR.

REGARDING MAGNETIC RECORDS OBTAINED IN CO-OPERATION WITH CAPTAIN SCOTT'S ANTARCTIC EXPEDITION.

I have been instructed to request Directors of all observatories which have taken part in the programme of special magnetic observations on term hours between May 22, 1911, and January 26, 1912, agreed on with Captain Scott, to kindly send to Kew Observatory, Richmond, Surrey, England, copies (preferably photographic) of the records obtained during all the term hours. Directors are also requested to send information as to any errors in the time marks on the traces, and as to base line values and scale values with explanations of what increasing ordinate signifies in the several curves, and of how the curves should be measured (*c. g.*, whether from the upper edge of base line to upper edge of trace, or otherwise). Information is also desired as to temperature corrections, if these vary sensibly during any of the term hours. It would also be a convenience to have any information that is readily available as to the changes to be expected (from the regular diurnal variation) on a normal day during the respective term hours.

The magnetic records obtained by Captain Scott's Expedition down to the end of January, 1912, have already reached this country, and seem of a very complete and satisfactory character. The year 1911 was exceptionally quiet magnetically, and some even of the Antarctic term-hour curves show only very small movements. It is thus possible that the information derivable from some of the term hours may be wholly of a negative character, and attention may in that case be concentrated on those term hours which show sensible movements. No decision can, however, be reached on this point until the curves received from all stations have been examined. CHARLES CHREE.

Meteorological Office Central Observatory,
Kew Observatory, Richmond, Surrey,
July 31, 1912.

Digitized by Google

MAGNETIC CHARACTER OF DAYS AS OBSERVED AT THE CHELTENHAM MAGNETIC OBSERVATORY, APRIL 1 TO JUNE 30, 1912, G. M. T.

Latitude: 38° 44'. O N; longitude: =75° 50'.5 W

Day	April	May	June
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	1	1	0
6	1	0	0
7	1	0	0
8	0	0	1
9	0	0	0
10	1	0	0
11	0	1	0
12	0	1	0
13	0	1	0
14	1	0	0
15	1	0	0
16	1	0	0
17	1	0	0
18	0	0	0
19	0	0	0
20	1	0	0
21	0	0	0
22	0	0	0
23	0	0	0
24	0	0	0
25	0	0	0
26	0	0	0
27	0	0	0
28	0	0	0
29	0	0	0
30	0	0	0
31	0	0	0

There were no magnetic storms during this period.

O. H. TITTMANN,
Superintendent, Coast and Geodetic Survey.

GEORGE HARTNELL,
Observer-in-Charge.

THE MAGNETIC CHARACTER OF THE YEAR 1911.

The annual review of the "Caractère magnétique de chaque jour" for 1911 has been drawn up in the same manner as for the preceding years. Forty-five observatories contributed to the quarterly reviews; when the tables were dressed, 39 of them had sent complete data. Several observatories have somewhat modified their method of classification on account of a circular sent in March. Table II of the annual review, containing the mean character of each day and of each month and the list of "calm days," are reprinted here. G. VAN DIJK.

Digitized by Google

Table Showing the Magnetic Character of the Year 1911

Dates	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Mean
January	0.6	1.0	1.0	0.8	1.0	0.8	0.2	1.1	1.0	0.9	0.8	0.1	0.9	0.4	0.6	0.9	0.1	0.9	0.4	0.3	0.2	0.4	0.7	1.7	1.4	1.1	0.8	1.1	0.8	0.8	1.2	0.78
February	1.0	1.2	0.5	0.5	0.9	0.9	0.9	0.9	0.5	0.8	0.4	0.3	1.2	1.1	0.3	0.9	0.9	0.8	0.5	0.3	1.8	1.8	1.3	1.1	1.0	0.9	1.1	1.2				0.89
March	1.0	0.4	0.8	0.9	1.2	0.8	0.6	0.8	0.2	0.1	0.1	0.1	0.5	0.9	0.8	0.6	0.1	0.0	0.2	1.9	1.7	1.2	1.4	1.1	1.2	1.2	1.2	1.1	1.0	0.5	0.7	0.78
April	0.6	0.5	0.8	0.8	0.1	0.2	0.5	1.5	1.7	1.2	0.6	0.7	0.2	0.0	0.2	1.7	1.3	1.1	0.9	1.0	1.2	1.1	0.8	0.7	0.3	0.2	0.3	0.8	0.5	1.2		0.76
May	0.1	0.4	0.3	0.0	0.6	0.8	1.4	0.8	0.5	0.6	1.0	0.3	0.1	1.2	1.6	1.1	0.9	0.7	0.9	0.8	1.0	0.3	0.6	0.3	1.0	0.8	0.5	0.6	0.4	1.0	1.1	0.70
June	0.9	0.1	0.1	1.2	1.0	0.7	0.3	0.1	1.1	1.2	1.1	0.8	0.9	0.9	0.6	0.4	0.1	0.0	0.0	0.6	1.0	0.8	0.9	0.2	0.0	0.0	0.1	0.7	0.2	0.3		0.53
July	1.3	0.9	0.8	0.7	0.4	0.6	1.0	1.0	0.2	0.6	0.7	0.4	0.2	0.1	0.1	0.1	1.1	1.0	1.2	0.5	0.3	0.3	0.1	0.8	0.1	0.1	0.4	1.6	1.1	1.0	0.4	0.61
August	0.2	0.1	0.8	0.9	0.8	0.6	0.1	0.1	0.2	0.1	0.1	0.1	0.8	0.1	0.2	0.5	0.5	0.2	1.1	0.8	0.3	0.1	1.8	1.4	1.1	1.1	0.9	0.8	0.1	0.2	0.9	0.53
September	0.2	0.1	0.0	0.1	0.5	0.2	0.3	0.1	0.1	0.6	0.9	0.8	0.5	0.1	0.8	1.1	0.7	0.3	1.0	1.7	1.6	1.1	0.9	0.2	0.0	0.1	0.5	0.2	0.1	0.1		0.50
October	0.1	0.8	0.5	0.3	0.0	0.2	0.5	0.8	0.9	1.6	1.5	0.5	0.6	0.6	0.1	0.7	1.1	1.4	0.9	0.7	0.7	0.6	0.1	0.7	0.7	0.3	0.4	0.0	0.6	0.1	0.1	0.59
November	0.0	0.1	1.0	0.7	0.7	0.3	0.1	0.8	1.1	0.7	0.3	0.8	1.7	1.4	0.9	0.5	0.7	0.1	0.6	0.5	0.2	0.0	0.0	0.1	0.5	0.3	0.1	0.1	0.2	0.2		0.49
December	0.7	0.1	0.2	0.1	0.1	1.5	0.5	0.2	0.1	0.6	1.9	0.5	0.1	0.6	0.1	0.1	1.0	0.6	0.1	0.0	0.1	0.0	0.1	0.1	0.8	1.2	0.8	0.5	0.1	0.3	0.9	0.45

Calm Days

January	7,	12,	17,	20,	21.
April	5,	13,	14,	15,	26.
July	13,	14,	15,	16,	26.
October	1,	5,	15,	23,	28.
February	11,	12,	15,	19,	20.
May	1,	4,	13,	22,	24.
August	7,	8,	10,	11,	29.
November	1,	7,	22,	23,	24.
March	10,	11,	12,	17,	18.
June	3,	17,	18,	19,	25.
September	2,	3,	14,	25,	26.
December	2,	9,	21,	22,	23.

Digitized by Google

LEVÉ MAGNÉTIQUE DU BASSIN DU RIO S. FRANCISCO.

J'ai l'honneur de vous envoyer les résultats obtenus par l'expédition magnétique que j'ai envoyé renouveler le levé effectué, en 1883, par le Dr. van Ryckevorsel, dans le bassin du Rio S. Francisco.

L'expédition était composée de M. Domingos Costa, Assistant de 1e classe, comme chef, et de M. Luiz Rodrigues, Assistant de 3e classe, comme auxiliaire. Elle partit de Rio et commença ses travaux à Barra do Pirahy, le 26 Octobre 1910, et pousuivit ensuite, jusqu'à la fin de Septembre de l'année suivante, l'itinéraire de la Commission Hollandaise. Toutes les localités reconnaissables furent réoccupées, moins quelques unes sur le Rio-das-Velhas, où plusieurs points durent être abandonnés, à cause de l'éloignement de la ligne de chemin de fer actuel, par rapport à l'ancienne route suivie par M. van Ryckevorsel.

Les positions géographiques ont été determinées à nouveau, et très soigneusement, à l'aide d'un excellent instrument universel de C. Bamberg, avec cercles de 17.5 ċm., lunette de 36 mm. d'ouverture et 33 cm. de distance focale, pourvu, à chaque cercle, de deux microscopes donnant 5″. L'heure a été systématiquement observée par la méthode de Zinger, la latitude par celle de Sterneck, et l'azimuth de la mire par les élongations d'étoiles circumpolaires.

Les longitudes ont été determinées télégraphiquement à Sabará, Sete-Lagôas, S. Francisco, Januaria, Carinhanha, Joazeiro et Penedo, et, pour les points intermédiaires, par le transport de l'heure, moyennant dix chronomètres.

Toutefois les positions des stations de Barra do Pirahy, Entre-rios, Palmyra, Barbacena, Lafayette et Curvello avaient déjà été déterminées par le personnel de l'Observatoire, en employant le télégraphe pour les longitudes, à différentes époques antérieures à l'expédition magnétique, et ont été adoptées en les ramenant au lieu d'observation.

Les observations magnétiques ont été executées avec les magnétomètres No. 18 et 20 de Cooke & Sons (Indian Office Pattern), et aussi avec le No. 134 de Negretti & Zambra (Kew Pattern), pendant l'intervalle entre l'abandon forcé du No. 18, occasionné par le décollement spontané des lentilles de l'aimant collimateur, et la mise en service du No. 20. Pour l'inclinaison, le cercle de Barrow No. 114, également construit par Negretti & Zambra, a servi pendant toute la durée du voyage. Tous ces instruments ont eu leurs constantes déterminées à l'Observatoire de Kew. La méthode adoptée pour les observations et leur réduction est celle prescrite dans le "Manual for Scientific Enquiry," publié par l'Amirauté Anglaise.

Des observations comparatives ont été faites, peu avant et après l'expédition, en un point du fond de la Baie de Rio, dénominé "Sitio da Batalha," parce que, depuis plusieurs années; il est devenu impossible

Digitized by Google

Levé magnétique du Bassin du Rio S. Francisco

Resumé Général des Observations

Stations	Positions géographiques						Eléments magnétiques réduits a l'époque 1911.0						Remarques
	Latitude (Sud)			Longitude (Ouest de Greenwich)			Déclinaison (Ouest)			Inclinaison			Intensité horizontale	
	°	′	″	°	′	″	°	′		°	′			
Barra do Pirahy	22	27	57	43	49	41	9	24.6	—	13	58.2		0.24580	
Entre-Rios	22	6	47	43	12	37	10	00.9	—	13	58.5		0.24796	
Palmyra	21	27	20	43	33	12	9	47.0	—	12	36.3		0.24832	
Barbacena	21	13	23	43	46	7	9	29.2	—	12	10.9		0.24878	
Carandahy	20	57	12	43	48	21	9	41.8	—	11	36.7		0.24959	
Lafayette	20	39	12	43	47	13	9	36.7	—	10	54.4		0.25112	
Itabira	20	15	18	43	47	25	10	12.5	—	10	52.5		0.24746	
Sabará	19	53	33	43	48	44	7	49.8	—	7	48.7		0.24723	
S. Luzia	19	46	10	43	52	28	9	48.7	—	9	43.4		0.25276	
Sete-Lagôas	19	27	40	44	14	37	9	24.9	—	9	11.0		0.25187	
Curvello	18	45	39	44	25	46	7	29.3	—	8	08.5		0.25491	
Lassance	17	53	7	44	34	37	9	29.4	—	6	28.5		0.25980	
Pirapóra	17	21	8	44	56	55	7	48.5	—	4	19.5		0.25927	
Guaicuhy	17	12	19	44	49	21	8	21.5	—	2	39.4		0.25517	
Extrema	16	51	39	44	55	1	9	15.4	—	4	55.6		0.25692	
Barra do Paracatú	16	34	15	45	6	31	9	12.6	—	4	24.0		0.25989	
S. Romão	16	22	9	45	4	33	9	10.0	—	3	44.2		0.26087	
Barra do Urucuya	16	7	31	45	5	5	9	17.0	—	3	17.4		0.25981	
S. Francisco	15	56	54	44	52	11	9	31.4	—	3	22.4		0.26057	
Barra do Mangahy	15	41	58	44	32	49	9	35.3	—	2	58.5		0.26208	
Januaria	15	29	26	44	21	34	9	48.0	—	2	51.7		0.26212	
Morro do Angú	15	15	43	44	8	32	10	01.2	—	2	23.1		0.26276	
Praia	14	49	0	43	55	5	10	03.8	—	1	44.9		0.26214	
Carinhanha	14	8	20	43	45	49	10	24.2	—	0	57.3		0.26360	
Barra do Parateca	13	55	31	43	37	12	10	22.4	—	0	29.2		0.26546	
Pituba	13	38	0	43	33	19	10	30.4	+	0	25.2		0.26419	
Lapa	13	15	2	43	25	44	10	33.4	+	0	45.4		0.26761	
Urubú	12	41	18	43	11	39	11	07.9	+	1	16.6		0.26908	
Itahy	12	9	14	43	16	25	10	46.0	+	2	29.1		0.27000	
Joá Novo	11	58	2	43	21	28	10	44.4	+	2	42.1		0.27034	
Torrinha	11	30	21	43	17	14	10	44.6	+	4	15.5		0.27119	
Barra	11	5	26	43	9	22	11	12.0	+	4	25.5		0.27295	Vent frais
Mocambos do Vento	10	49	31	42	53	14	11	34.8	+	4	34.5		0.27443	" "
Pilão Arcado	10	8	59	42	26	17	11	30.8	+	5	25.4		0.27496	" "
Barrocão	9	50	7	42	7	10	11	41.4	+	5	37.9		0.27494	
Remanso	9	40	59	42	4	46	11	53.9	+	5	40.1		0.27632	Vent frais
Trahyras	9	43	37	41	38	21	12	19.3	+	5	26.3		0.27660	
Lagôa	9	39	35	41	20	33	12	19.4	+	5	32.5		0.27649	
Joazeiro	9	24	42	40	30	23	12	36.9	+	5	45.5		0.27254	Vent frais
Capim Grosso	8	59	33	39	54	47	13	21.1	+	5	45.7		0.27629	
S. Felix	8	36	38	39	34	11	13	42.5	+	6	42.0		0.27715	
Rodellas	8	49	58	38	46	17	14	13.8	+	5	23.0		0.27700	Vent frais
Sacco	8	58	0	38	40	57	14	11.6	+	5	09.4		0.27689	
Campinho	8	54	43	38	29	28	14	15.0	+	5	03.7		0.27737	
Jatobá	9	4	10	38	18	58	14	26.6	+	4	34.3		0.27706	
Piranhas	9	37	34	37	45	27	14	41.0	+	4	59.7		0.27570	
Penedo	10	17	24	36	35	8	15	26.6	+	1	12.9		0.27252	
Batalha (Rio)	22	42	38	43	8	58	9	49.2	—	14	33.8		0.24530	

Digitized by Google

de faire aucun travail magnétique à l'Observatoire de Rio, à cause du développement considérable des tramways électriques. Pour la même raison, les enregistreurs d'Eschenhagen, que j'avais espéré pouvoir utiliser, n'ont pas fonctionné, et les observations n'ont pas pu être réduites à la même heure.

La réduction au commencement de l'année 1911 a été obtenue en employant la variation annuelle, déduite de la comparaison avec les observations de van Ryckevorsel. Les détails complets des observations seront publiés aussitôt que possible, mais en attendant, on pourra utiliser, pour des travaux d'ensemble, les données contenues dans le tableau ci-joint, où chaque valeur correspond, au minimum, à deux déterminations indépendantes. H. MORIZE, *Directeur.*

Observatorio Nacional, Rio de Janeiro,
le 15 Juillet, 1912.

OBSERVATION OF THE MAGNETIC DECLINATION AT WARSAW DURING THE SOLAR ECLIPSE OF APRIL 17, 1912.

In want of a magnetograph I employed for this purpose the Kew pattern magnetometer of the Cambridge Scientific Instrument Co. This instrument is used also for my systematic researches in terrestrial magnetism in Poland.

I made my observations during a few days at precisely the same point in the racing field at a distance of about 0.5 km. from the electric tramway line.

On the 13th, 16th, and 19th, I determined the geographic meridian and azimuths of three marks. By means of these azimuths I determined *vice versa* on the 17th and 18th the geographic meridian.

The magnet was suspended by means of four silk fibers (previous experience showed that the double thread was not strong enough and often broke). As customary, the suspension thread was freed from torsion by suspending for some time before each set of observations a brass bar of the same weight as the magnet. After each set of observations the brass bar was again suspended, and in the case of presence of torsion (as happened on the 16th and 18th) a correction was determined.

Using the scale of the collimator magnet, I read the position of the magnet every three minutes, its magnetic axis being determined as usual by suspending the magnet in reversed position at such times when but little changes of the declination occurred.

Digitized by Google

The agreement in determinations of azimuths and the results of two sets of comparisons of the magnetometer with the magnetic instrument in Potsdam, lead me to conclude that the error in determination of declination cannot be greater than 1′.

The declination was observed and corresponding curves were plotted for the day of the eclipse, as also for April 13, 16 and 18. (See Fig. 1.) The value of the base line is 4° 55′ W: 1 mm. ordinate corresponds to 0′.35; a movement upward means increasing west declination.

The dotted part in the curve of April 16 corresponds to the time when the magnet was taken down to enable me to make the afternoon set of observations for determining the geographical meridian.

The dotted lines perpendicular to the time axis of the curve of April 17 mark the beginning, the middle and the end of the eclipse. First the mean time of Warsaw is noted, then directly below the mean time of Greenwich.

Since the accounts thus far published of the solar eclipse of April 17 affirm that no kind of magnetic disturbance took place, it seemed desirable to publish my results, from which the relation between the solar eclipse and the diurnal change of declination can be inferred.

It will be desirable to compare the results obtained with those at points which lie to the west and to the east of Warsaw. From the west there will be plenty material, but I do not know what can be obtained from the east.

At the present moment I am able to compare my curves with those of Potsdam, for which I am obliged to Prof. A. Schmidt. I received also the curves of Beuthen.

It is useful to compare these curves. In all three places the curve of April 17 differs from those of April 16 and 18 by the absence of increasing values of westerly declination, which corresponds generally to the hours under consideration. The general character of the curves is the same in all three places, except that the Potsdam and the Beuthen curves are smoother than mine, which is not difficult to explain, as I did not use any kind of damping.

The part of my curve of April 17 which corresponds to the time of eclipse shows a "dale" and a "mountain." On the curve of Beuthen a like form is evident. On the Potsdam curve this form is difficult to recognize.

S. Kalinowski.

Physical Laboratory,
Museum of Industry and Agriculture,
Warsaw, June 2, 1912.

Digitized by Google

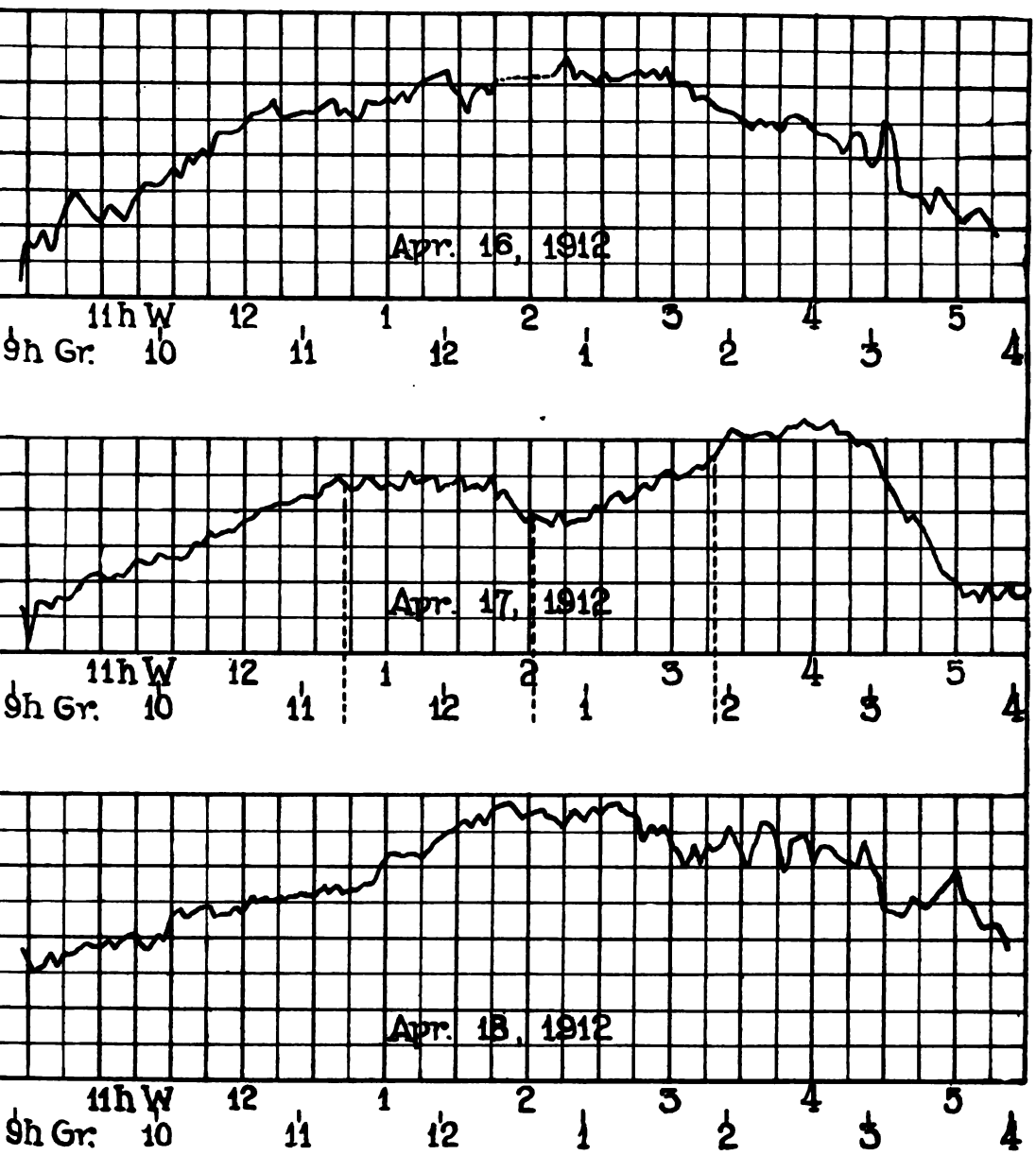


FIG. I. *Variations in the magnetic declination observed at Warsaw, Poland, April 16, 17, and 18, 1912.*

Digitized by Google

ON THE MOMENT OF INERTIA OF LONG MAGNET H26 AT THE CHELTENHAM OBSERVATORY.

On pages 45-46 of the March 1912 number of this Journal, attention was called to the difference between the values of the moment of inertia of a magnet as computed from its dimensions and mass, and as determined experimentally with the aid of an auxiliary inertia weight, and the statement was made that in each of the four cases mentioned where such differences were shown the magnet house of the magnetometer used was of brass, so that any effect due to impurity or damping might affect the experimentally determined results.

At the suggestion of the Editor of this Journal it was decided to test this matter experimentally for the long magnet of Magnetometer No. 26, the standard instrument of the Cheltenham Magnetic Observatory.

In July, 1902, observations were made to test the parts of Magnetometer No. 26 for magnetic impurities. In the course of these observations oscillation experiments were made with the long magnet suspended in a wooden box having a glass suspension tube, and also in its own magnet house. The resulting values of log T^2 were as follows:

In magnet house.....log $T^2 = 1.44106$
In wooden box.....log $T^2 = 1.44180$

For the experiments here considered a wooden oscillation box was constructed in the instrument shop of the Coast and Geodetic Survey, every care being exercised to exclude all magnetic material. This box was securely fixed to a circular wooden base provided with three brass foot-screws. The suspension tube was about 36 inches (91.5 cm.) long and provided with a suitable torsion head on its top end. The lower end of the suspension tube was fitted to the box by means of a wooden collar secured with small brass screws. A small telescope and scale were used in the observations of the oscillations of the magnet, which is a solid cylindrical bar with a mirror on one end. All of the brass used in the construction of the box was tested, and there was no metal within less than about 10 cm. of the suspended magnet except the three small screws which held the suspension tube in place, and these were about 4 cm. from the magnet. The magnet was suspended by a bronze wire which was capable of sustaining slightly more weight than that of the magnet and inertia ring combined, and of the same diameter as that used in the standard declinometer.

In this box the moment of the magnet was determined, by two series of experiments, in July, 1912, using the same auxiliary weights and the same method as was used in the regular magnet house in 1909.

Digitized by Google

The following table includes all of the experimental determinations of the moment of inertia of this magnet since it has been in use at Cheltenham.

Summary of Results.

Date	Inertia Weight	K at 20° C (C. G. S. Units)	Probable Error	Number of Sets	Observer
July, August, 1902.....	PTRII 276	527.28	±0.24	8	Schultz.
April, 1909.....	Ring A	526.96	±0.09	10	Burbank
July, August, 1909.....	Ring 26	527.38	±0.09	10	Burbank
July, 1912, wooden box...	Ring 26	527.02	±0.06	8	Hartnell
July, 1912, wooden box...	Ring A	527.24	±0.12	7	Hartnell
Mean.....	.....	527.18	±0.06	43	.....

The moment of inertia computed from dimensions and mass of magnet is 528.88.

From the above table it will be seen that three different inertia weights were used in the experiments at different times. PTRII 276 is a gold-plated solid bronze cylinder of the same length as the magnet, its moment of inertia being 374.97 at 20° C. Ring *A* is a brass ring made in the Coast and Geodetic Survey instrument shop in 1892 for use with the four magnetometers constructed at the time; its moment of inertia is 295.39. Ring 26 was made in 1909 especially for use with standard magnetometer No. 26, special care being taken in the selection of the material and in the construction of the ring; it is made of brass gold-plated, and has a moment of inertia of 714.60 at 20° C. The dimensions and mass of each of the three inertia weights were determined by the Bureau of Standards.

In the moment of inertia experiments, the oscillations of the magnet were timed by the eye-and-ear method. The probable errors of the several series of results would probably have been smaller had the time of oscillation of the magnet been ascertained with the aid of the chronograph;[1] however, it is not certain that the chronograph method of timing the oscillations would have appreciably changed the resulting mean of all (43) of the sets of determinations.

The results of these moments of inertia experiments indicate that in this instrument the effect of whatever impurities there may be in the metal of the magnet house on the value of the moment of inertia determined in the usual way, is not sufficient to account for the differ-

[1] See Annual Report of the Board of Scientific Advice for India for the year 1910-1911. Calcutta, 1912. - "Personal Error," p. 41.

Digitized by Google

ence of the value from the one computed from the dimensions and mass of the magnet.

While it is evident that the presence of magnetic material in close proximity to the magnet will affect the period of vibration of the magnet, it seems clear also from a consideration of the oscillation formula and the similar conditions under which all of the oscillations are made, that this effect is largely eliminated in the moment of inertia determinations by the combination of oscillations with and without the auxiliary weight, just as the Earth's magnetic force and the magnetic moment of the magnet are eliminated, since the magnet is in the same position relative to the disturbing force in both cases. However, the precise cause of the rather large difference between the observed and computed values of the moment of inertia, as shown on page 157, still remains to be explained; other experiments will be made. R. L. FARIS.

Coast and Geodetic Survey,
Washington, D. C.

Digitized by Google

ABSTRACTS AND REVIEWS

KENNARD, E. H. *Unipolar Induction.*[1]

This is another attempt to obtain a conclusive experimental answer to the old question, "Does a magnet rotating about its axis of magnetic symmetry carry its lines of force with it?"

Since it has been shown that no observations involving closed circuits can answer the question, Mr. Kennard has attacked the problem from the standpoint of open circuits.

If the lines of force rotate with the magnet, then the region around the magnet is an electric field, the intensity at any point being $\frac{1}{c}$ [V H], and the equipotential surfaces are the surfaces traced out by the motion of the lines of H. When a conductor is brought into the field there is a separation of the electric charges in it, part of the conductor becoming charged positively and part negatively, the distribution being such that the electric force set up inside the conductor by the charges is at every point exactly equal and opposite to that due to the motion of the lines of H. If the conductor extends to a point entirely outside of the field of H—*e. g.*, if it is connected to earth—then the portion in the field will have an excess charge of a certain sign. It is this excess which the author desires to measure.

A steel bar, 2 feet long and 2 inches in diameter, was mounted with its axis horizontal, about 4 feet from the floor and inside a magnetizing solenoid. The bar could be rotated about its axis by means of a motor. In order to protect the outside apparatus from any electrostatic disturbance originating in the solenoid, magnet, etc., the solenoid and all connecting parts, including the motor and belt, were surrounded by a continuous metal case K, which was earthed. The portion of the case which surrounded the solenoid and magnet was a cylinder 7.3 cm. in diameter and several centimeters longer than the steel rod. Outside of this cylinder and concentric with it was an insulated metal cylinder (C) 20 cm. in diameter and 49 cm. long. The middle sections of both cylinders, of the solenoid, and of the magnet all lay in the same vertical plane.

The cylinder C was connected to one pair of quadrants of an electrometer, the other pair being put to earth. The two pairs of quadrants were connected, the bar set in rotation and the magnetizing current was sent through the solenoid in a certain direction. Then the connection between the two quadrants was broken, leaving C insulated, and the direction of the magnetizing current was reversed. This should drive the charge from C onto the quadrants of the electrometer and thus produce a deflection. None was observed.

From a computation, necessarily rough, he concluded that the deflection that should have been observed had the lines rotated with the magnet was 40 divisions; the mean of a number of observations was, however, less than one division. Whence he concludes that "a rotating magnet does not carry its force lines around with it." Further investigations are under way.

In this experiment it appears that the solenoid remained at rest while the bar rotated. A closer approximation to what takes place when a permanent

[1] *Phil. Mag.* London, (6) 23, pp. 937–941, 1912.

Digitized by Google

magnet is rotated would be obtained by rotating both the solenoid and the magnet as a single rigid body. It is to be hoped that Mr. Kennard may include this case in the investigation that is now under way. N. E. DORSEY.

ABBOT, C. G. *The Sun.*

Professor Young's admirable book, "The Sun," is now out of print. It is also out of date, for it appeared in 1895, and since that time several important and numerous minor contributions have been made to our knowledge of the sun. Rowland's extensive table of solar spectrum wave-lengths has been published, the infra red spectrum of the sun has been minutely examined, the solar constant closely determined, helium has been discovered, several eclipses have been successfully observed with elaborate apparatus, and even new discoveries of physical phenomena, such as the pressure displacement and the magnetic disintegration of spectrum lines, have given new and almost startling knowledge of the physical condition of our great luminary. At the Yerkes, the Kodaikanal, the Mount Wilson (all constructed since 1895), the Meudon and other observatories, the spectrograph and the spectroheliograph have, in the meantime, analyzed the solar atmosphere and revealed its condition line by line and layer by layer.

It was full time, therefore, that some one of the leading investigators of solar phenomena should tell in English the wonderful story of the sun.

How well Mr. Abbot has told this story, and how wide its scope, may be inferred from the number of chapters and their headings: I. "The Solar System. The Sun's Distance. Its Dimensions;" II. "The Instruments and Methods Used in Solar Investigation;" III. "The Photosphere;" IV. "Eclipses and the Outer Solar Envelopes;" V. "Sun-Spots, Faculæ and Granulation;" VI. "What is the Sun?" VII. "The Sun as the Earth's Source of Heat;" VIII. "The Sun's Influence on Plant Life;" IX. "Utilizing Solar Energy;" X. "The Sun Among the Stars."

The most complete and the most authoritative portions of the book are those that deal with bolometry, pyrheliometry and the general phenomena of radiation—large subjects all of them, and all of them subjects that have been greatly advanced by Mr. Abbot's own investigations.

Chapters VIII. and IX., "The Sun's Influence on Plant Life," and "Utilizing Solar Energy," respectively, deal with questions one would scarcely expect to find discussed in any astronomical book. Nevertheless, they make such interesting reading that one soon ceases to bother about where they belong, and asks no further proof that they have a legitimate place in a popular treatise on the phenomena of the sun and what they mean.

In addition to an occasional obvious typographical error, there are a few things that may be considered with a view to possible modification in a new edition which doubtless will soon be called for, among them:

1. The explanation, on page 50, of why a vibrating tuning-fork, when rotated, waxes and wanes in loudness, might be rewritten to advantage.

2. The use of both prism and grating in Fig. 14 seems to need explanation.

3. Errors found in certain wave length determinations are ascribed (pages 62 and 96) to "deficiencies of the grating as a means of measuring wave lengths." This, however, is a point that may not be entirely settled, and therefore references to original papers in this, and also in many other cases, would be helpful.

[1] New York and London: D. Appleton & Co., 1911. 8°, xxv, 448 pp., with numerous illustrations. Price, $2.50.

Digitized by Google

4. "The astonishing sensitiveness of the bolometer," as described on page 84, refers, of course, to the temperature of the bolometer strip itself, and not to the temperature of the object under investigation. But it might be well to make this important distinction perfectly clear, since the erroneous interpretation may easily be made by those who are familiar with only the thermometer as measurer of temperature.

5. In speaking of "double reversal," page 147, it is stated that "the electric arc often shows this still more finely." This occurs in a long quotation from Young's "The Sun," but it is erroneous. Apparent double reversals are occasionally photographed in arc spectra, but it has been shown that, probably without exception, they are due to successive exposures, to overlapping orders, or to some other simple cause, and never to true double reversal.

6. On page 260 an attempt is made to show, through an ingenious suggestion by W. A. Michelson of Russia, that the Doppler effect may be produced in solar prominences by a slow motion *across* the line of sight. But this explanation will hardly be accepted by the average spectroscopist, and besides, it seems to be very easy of experimental test.

7. The Doppler effect seems also to embarrass the explanation of prominences tentatively suggested on page 261.

But at most these are only trifles that, so far as necessary, can easily be corrected.

The printing is good, the illustrations excellent, and the story, told by one who has done much to make it, thoroughly interesting from start to finish.

W. J. HUMPHREYS.

ANGOT, A. *Observations made during the eclipse of April 17, 1912.*[1]

The article is a brief notice of the observations which were made at the magnetic station of Val Joyeux, at Paris (Bureau Central Météorologique), and at the Observatory at Parc Saint Maur.

The only elements which have been clearly affected were the temperature, the relative humidity, and the intensity of the solar radiation. The drop of temperature during the eclipse was 4½ degrees, 2.9 degrees and 2.4 degrees, as observed at the three stations respectively. The minimum temperature in all cases was observed shortly after the eclipse.

The variation of the relative humidity was small and inverse to that of the temperature.

Measurements of the solar radiation made at Val Joyeux and St. Maur showed that at the instant of totality the value of this quantity had become too small to be measurable. At Paris, where the instruments were slower in reaching a condition of equilibrium, the minimum was not reached until after the eclipse, and was greater than zero.

The variations of the three magnetic elements were almost regular. Towards mid-day the rate of increase of the declination due to the diurnal variation showed a slight diminution, but during the rest of the day other variations of more importance were observed.

C. W. HEWLETT,
Department of Terrestrial Magnetism, Washington, D. C.

[1] *Comptes Rendus*, April 22, 1912, p. 1118.

Digitized by Google

CHAUVEAU, A. B. *Observations of atmospheric electricity during the eclipse of April 17, 1912.*[1]

The article is a brief report of the results of observations made on the atmospheric potential gradient and the positive and negative dissipation coefficients during the total solar eclipse of April 17, 1912. Observations were made continuously from the 9th to the 20th of April.

The curve for the potential gradient on the 17th shows no indication of any direct influence of the diminution of the solar radiation.

In passing, attention is called to the fact that from the 11th to the 16th strong winds were blowing from the north and east, and in spite of the fact that during this time there were great differences in the state of the sky and the insolation, the curve of the potential gradient presented identical characteristics. From the rising to the setting of the Sun there were diurnal variations of great amplitude. On the 17th the wind had passed around by the east and had become a south wind, and was of feeble intensity. The characteristics of the preceding days had completely disappeared, the curve becoming more regular, the variations fewer and of smaller amplitude. These conditions remained the same on the 18th and 19th. Finally the wind changed back by the east, becoming a north wind, when the characteristic disturbances of the previous period reappeared in the curve of the potential gradient.

Observations on the dissipation coefficients gave results somewhat surprising. The positive and negative coefficients were observed alternately in such a way that the instant of totality occurred exactly between two observations. The following table gives the fall in potential, positive and negative, during each interval of 20 minutes during which an observation lasted. The initial potential was the same for all, being between 185 and 195 volts.

Intervals	h m 10 40 to 11 00	h m 11 04 to 11 24	h m 11 27 to 11 47	h m 11 51 to 12 11	h m 12 13 to 12 33	h m 12 37 to 12 57	h m 13 00 to 13 20	h m 13 26 to 13 46	h m 14 07 to 14 27	h m 14 30 to 14 50	h m 14 55 to 15 15
ΔV (+)	21.6		21.7		17.4		24.3			27.4	
(−)		23.1		26.9		22.1		26.0	26.7		25.4

It is thus seen that at the moment of eclipse the positive conductivity increased while the negative decreased, in such a way that the total conductivity underwent no change. An Elster and Geitel apparatus was used in the observations. No explanation of this result was attempted. C. W. H.

BROGLIE, DE. *On the eclipse of the Sun of April 17 and the penetrating radiation measured by the natural ionization of air in a closed vessel.*[2]

Observations were made at Paris on the natural ionization in a galvanized sheet-iron vessel containing .41 cubic meter of air. The results showed a production of 12 ions per cc. per second. This number did not undergo any unusual variation at the time of the eclipse. The author therefore concludes that the eclipse had no apparent influence on the intensity of the penetrating radiation.

C. W. H.

[1] *Comptes Rendus*, June 10, 1912, p. 1652.

[2] *Comptes Rendus*, June 10, 1912, p. 1654.

Digitized by Google

KNOCHE, WALTER. *Determination of the emanation content of sea water and of induced activity of the air, between the coast of Chile and the Easter Island.*[1]

The first of these two articles contains the results of the measurements on the emanation content of sea water made between Talcuhuano, Chile, and the Easter Island, on the ship *General Basquedano*, during the months from March to May, 1911.

The method of measuring the emanation content was to shake a sample of the sea water in a flask and then pump the air which had been shaken with the water through the ionization chamber of a Wulf electrometer. The quantity of emanation per gram of sea water was calculated from the observed rate of fall of the potential. The mean of the results was 0.05 Mache unit per gram, the thirty measurements varying between 0.00 and 0.20 unit.

The emanation content was found to bear no relation to the activity of the air or the nearness to land. The emanation content was found to increase with increase of density of the sea water and with increase of temperature. This latter fact, together with the fact that there is no relation between the value of the activity of the air and the emanation content of sea water leads to the conclusion that there is no direct relation between these two phenomena.

The emanation content of sea water can be due to two causes: Radioactive products brought from the land to the sea by the rivers, or it may be ascribed to exhalations from the sea bottom which, as on land, must show very different activity at different places. The slowness of the vertical currents in the ocean would thus account for the smallness of the emanation content on account of the rapid rate of decay of the latter, even assuming that the sea bottom give out as much emanation as the places on land. The very great variations of the emanation content at different places on the sea would be accounted for by this explanation of its origin.

The second article is a continuation of the preceding article, and contains the results of the measurements of the radioactivity of the atmosphere.

A lead wire 5 meters long and 1 mm. in diameter was stretched over the bridge at a height of 17 meters above the water surface and kept at a potential of -2000 ± 300 volts for two hours. Once the wire was exposed for two hours at a potential of $+2000$ volts and was then tested for radioactivity. The experiment gave negative results. A Wulf electroscope was used and the activity measured in Elster and Geitel units was determined at 3 minutes and at 30 minutes after the potential had been reduced to zero. The time at which the activity of the wire disappeared was also recorded to the nearest quarter of an hour.

There appeared to be no relation between the activity of the atmosphere and the temperature, humidity and air pressure or to the hydrometer readings.

The decay curve fell regularly from 15 to 25 minutes. The rate of decay shows that in the main radium induction was being dealt with, although there was probably another substance present also, as the curve was somewhat steeper than that which corresponded to radium induction. From 25 to 50 or 60 minutes the curve was a little too flat for radium induction. The curve shows, however, that thorium induction was at no time present.

In all 21 determinations of the radioactivity of the atmosphere were carried out. Neglecting two of these, which were clearly influenced by the nearness to

[1] *Physikalische Zeitschrift*, vol. XIII, and p. 152, Feb., 1912, p. 112.

Digitized by Google

the coast of Chile, the mean value of the activity was $A = 3.6$, the separate determinations lying between 0.9 and 8.5. The author regards the rate of fall of the potential, 30 minutes after the introduction of the wire, as a measure of the amount of radium induction present in the atmosphere. The mean value of the activity due to radium induction was 1.8, the separate values lying between 0.4 and 3.9. These values are well in agreement with those found by other observers over the ocean.

The author brings up the question of the origin of the activity of the air over the ocean. On account of the facts brought out in the previous paper, he concludes that it can not be due to the radium and emanation content of the sea water. He then proceeds to discuss the part of the activity due to radium emanation, the object being to find out if the carrying of the emanation from the land to the sea by winds can account for the radium emanation activity of the atmosphere as measured at different points on the sea. From the decay curve of radium emanation he constructs curves showing the per cent of the original radium emanation in the wind when it has traveled various distances for different wind velocities. He then shows by actual calculation that under ordinary conditions the radioactivity (that part due to radium emanation) over the ocean can be explained very well as arising from winds off land. These winds do not necessarily come from the nearest land nor even from the coasts, but on account of vertical air currents the winds may bring emanations from places far inland where emanation is rich. This would explain the wide variations found in the radioactivity of the atmosphere over the ocean. C. W. H.

STADE, H. *Determination of the radioactivity of the atmosphere over the Atlantic Ocean.*[1]

This article contains the results of measurements of the radioactivity of the atmosphere on the Atlantic Ocean during a trip from Rio de Janeiro to the mouth of the Kaiser Wilhelm Canal (Kiel). The idea underlying the work was that where there is great friction between the wind and water, a great activity of the air is to be expected. He therefore made measurements of the activity in the trade winds.

A very remarkable result of the work was that considerable activity was measured by charging the wire to a high positive potential for two hours. Thus, in one case an activity as high as $A = 5.8$ (Elster and Geitel units) was found. Moreover, using a negatively charged wire exceedingly high values of A were obtained. At times A was more than 400. There also appeared to be a regular variation of the activity, the values being high at mid-day and falling off very rapidly as afternoon advanced. C. W. H.

SIMPSON, G. C., AND WRIGHT, C. S. *Atmospheric electricity over the ocean.*[2]

The article contains an account of observations made on atmospheric electricity during a voyage from England to New Zealand of Captain Scott's Antarctic ship *Terra Nova*. The quantities investigated were: 1. The electric potential gradient. 2. The quantity of radioactive products in the air. 3. The number of free ions over the ocean. 4. The ionization of air in a closed vessel.

[1] *Meteorologische Zeitschrift*, Oct., 1910, page 469.

[2] *Proceedings Royal Society*, A, vol. 85, page 175. 1911.

Digitized by Google

A Wulf electroscope was used in all the work, the advantage being that its reading is unaffected by the rolling of the ship. A short copper rod coated with ionium served as collector.

Potential gradient.—The sign of the potential gradient was found to be without exception positive on fine days. Great difficulty was experienced in obtaining the form of the curve representing the daily variation of the potential gradient, because of the constant change in the surroundings on deck. A few series of observations were obtained, however, which gave satisfactory representations of the diurnal variation. From the data given it appears that within 40 degrees of the equator the normal potential gradient has its chief maximum during the evening and the early hours of the night, and its chief minimum at about mid-day. This is in general agreement with the results found on land in the same latitudes. This mid-day minimum does not accord well with the theory advanced to explain the appearance of the afternoon minimum in hot countries, viz., that the minimum is due to the disturbances caused by the large amounts of dust raised by ascending currents where solar energy is intense. The author suggests that the variation may depend in some more fundamental way upon latitude and that the simple curve with the mid-day minimum may be characteristic of tropical countries, while the simple curve with the four-hour minimum may be characteristic of the polar regions. In between these two extremes we have the two minima occurring together, but with varying relative intensities. The absolute value of the potential gradient was found to be very similar to that over land. The method of finding the reduction factor is very interesting. At Melbourne, within 1000 yards of the ship at anchor, was a level beach over which the equipotential surfaces were horizontal. Here two rods with insulators on their ends were erected at a short distance apart (about 18 meters) so that the field midway between them was undisturbed. A fine wire was stretched from insulator to insulator as tightly as possible so that it was practically horizontal. To the middle of the wire an ionium collector was attached. This brought the wire to the potential in its neighborhood, and as the wire was horizontal, it lay in an equipotential surface and had no tendency to disturb the field. While observations were being made on shore measurements were made on the ship with the conditions of yards and ropes as similar as possible to those during three of the sets of observations at sea.

Radioactivity.—Elster and Geitel's method of measuring the radioactivity of the air was used. A wire 810 cm. long and 0.4 cm. in diameter was stretched from one side of the ship to the other above the bridge. This wire was kept charged to a negative potential of from 2000 to 2500 volts for two hours and was then coiled up and placed within a cylinder fixed on a Wulf electroscope. The activity was expressed in the same units proposed by Elster and Geitel.

From England to Melbourne the mean value of the radioactivity of the air was found to be 6. Observations on land made by Elster and Geitel at Wolfenbüttel gave 19; by Gockel at Freiburg 84, and by Simpson at Karasjok 93. To clinch the result indicated by these observations measurements were made 200 miles in from the sea when the boat reached Cape Town. The mean value of radioactivity was found to be 124.

Various observations were made to determine the influence on the radioactivity of the direction of the wind. In every case land winds gave high values and sea winds gave low values. From observations made in different latitudes

Digitized by Google

results were obtained which are readily explained by considering the course of the trade winds.

The radioactivity over the sea appeared to be very constant in amount and no diurnal variation could be detected.

Ionization.—About 35 determinations of the ionization were made in the southern hemisphere with one of the newest types of the Ebert instruments.

The ionization increased with distance from the equator from 10 degrees to 40 degrees south. Radioactivity decreased at the same time. This is explained by the fact that at 40 degrees the trade winds which have recently descended are very pure and dust-free and hence the ionization may be great even with a very small amount of radioactivity. As the winds approach the equator they become loaded with salt, and hence the ionization may decrease, even though the radioactivity may increase.

The ionization increased with temperature up to 20 degrees centigrade. Measurements made in the trade winds at temperatures greater than 20 degrees in latitudes from 10° to 27° south gave smaller values. The salt content of the air may here again be responsible.

During strong winds the ionization appeared to be smaller than during winds of less intensity.

The ionization on the sea appeared to be smaller than that on land during similar meteorological conditions. The sea values were compared with those at Karasjok, Lapland, with the following result:

	I+	I—	q
Land.....	0.45	0.42	1.08
Sea.....	0.37	0.31	1.21

Ionization of air in a closed vessel.—The object of this part of the work was: 1. To determine the factors governing the variations of the ionization in closed vessels, and particularly to investigate a possible variation due to greater or smaller quantities of radium products in the atmosphere. 2. To throw further light on the origin of this natural ionization and if possible to determine its absolute value.

A large zinc cylinder, capacity 26,500 cc., filled with dry, dust-free air, was used as the ionization chamber. The electrode of a Wulf electrometer projected into this chamber.

In the open sea the ionization measured was about 6½ ions per cc. per second. There was no evidence of a regular daily variation or of any connection between the observed ionization and the radium content of the air. No certain connection, moreover, was found with air temperature, sea water temperature, rainfall, or quantity of coal in the adjacent bunkers. On the whole, a low barometer pressure was associated with a high value of the ionization.

Barometer reading	Ions per cc. per second
Over 30.2 in.	6.36
Under 30.2	6.70

On several occasions when ship was near land the ionization rose considerably. At the same time the radioactivity of the atmosphere increased much above the values found in the open sea. Upon receding from land the radioactivity would decrease very rapidly, and the ionization would not return to its normal sea value till some time after the radioactivity had decreased to its usual

Digitized by Google

low value. In some cases this lag amounted to as much as 24 hours. This lag was attributed to the deposit of radioactive products on the ship while near the land. From the slow rate of decrease of this radioactivity it appears that in some cases thorium products played an important part. If the value of the ionization measured over the sea be subtracted from that measured over the land, there is a considerable remainder, amounting sometimes to a value as great as that observed over the sea. If the ionization due to the penetrating radiation from the sea (calculated from the radium content of sea water) be added to this remainder the result is the ionization due to the penetrating radiation from the soil at the place where the land observation was made, neglecting the difference in the penetrating radiation from the atmosphere at the two places.

It appears that there is in all cases a residual ionization of something like 4 ions per cc. per second which must be due to causes other than penetrating rays from terrestrial sources.

Three possible contributors to this residual ionization are: 1. Emanation within the vessel and admitted with the gas. 2. Intrinsic radioactivity of the metal and radioactive impurities in it. 3. Spontaneous ionization or breakdown of the enclosed gas. Each of these possibilities is discussed, and the author concludes that either intrinsic radioactivity of the metal or spontaneous ionization is the explanation. He apparently rather inclines to the view of spontaneous ionization of the gas. C. W. H.

Magnetic Survey of Egypt.

Values of the Magnetic Elements at 81 Stations.[1]

The report of the survey of Egypt covers the period from the autumn of 1908, to the spring of 1910, and gives the values of the magnetic elements at 81 stations. These stations are mainly along the Nile Valley as far south as latitude 22°, but a few are distributed along the Suez Canal and others along routes to oases in the Western Desert; the observations having been made by H. E. Hurst and C. B. Middleton.

Those in the Nile Valley and Delta have an average distribution distance of about 40 kilometers, while those in Nubia and the Western Desert are more widely separated. Repeat stations were examined for local disturbance, and were in many cases occupied more than once.

The instruments in use were found, from comparisons at Helwan, to be in practical agreement with the International Magnetic Standard, provisionally adopted by the Carnegie Institution, of Washington.

The survey is being extended into Sudan, and will soon be completed, with a distance between stations of about 100 kilometers.

The values obtained are presented in a table showing position of each station. An outline map accompanies the report which indicates the positions of the points of observation and the distribution of the repeat stations. The results are reduced to 1910.0, and are given to the fourth place C. G. S. in intensity, and to nearest minute for declination and inclination. H. W. Fisk.

[1] Ministry of Finance, Egypt; Survey Department; Cairo, 1911.

Digitized by Google

SKEY, H. F., AND FARR, C. C. *Magnetic Observations in the Subantarctic Islands of New Zealand.*[1]

This is an account of the magnetic observations made in the Campbell and Auckland Islands during the Canterbury Philosophical Institute expedition in 1907. A station was occupied at Stewart Island, one at Snares Island, eight in the Auckland Islands, and three on Campbell Island, hence a total of 13 stations. Two sets of instruments were employed, both of which were standardized by reference to Christchurch Observatory, which, in turn, has been compared with the International Magnetic Standard of the Carnegie Institution of Washington.

Two sketch maps, one of Campbell Island and one of the Auckland Islands, both based on the Admiralty charts, show the approximate positions of the stations. In most cases permanent marks were left at the points of observation, and precise descriptions noted to make possible their recovery at a later time. The Christchurch Observatory will furnish these descriptions in detail to any one desiring them.

It appears that very large local disturbances exist, especially in Auckland Island, doubtless due to the proximity of magnetic rocks. Unfortunately, the authors have used two sets of Roman numerals referring to the stations, so that the notes on the geological character of the neighborhood of the stations are unintelligible, without the use of other articles, not given in this publication.

H. W. F.

FARIS, R. L. *Distribution of the Magnetic Declination in the United States for January 1, 1910 (with isogonic chart and secular change tables).*[2]

This publication is devoted to a new isogonic chart of the United States for the epoch 1910.0, accompanied by explanatory notes and secular variation tables. The previous chart by Dr. Bauer was for 1905. The secular changes during the intervening five years were well determined, special attention having been given to the reoccupation of well distributed repeat stations. No considerable changes are made in the general appearance of the isogonics over those of 1905, except that with the increased accumulation of data the regions of local disturbance become more definitely outlined, causing the lines to become even more irregular than before.

In the lines of equal annual change, however, several striking differences appear, due to the rapid secular changes disclosed by the recent data of the Coast and Geodetic Survey and the Department of Terrestial Magnetism, and because of additional material. Thus the line of no annual change has been shifted one degree westward at the head of Lake Superior, about two degrees westward in latitude 32°, and almost nine degrees eastward in the latitude of Havana, so that whereas it was found west of Yucatan in 1905, on the new chart it passes about one degree east of Havana. In general, in the eastern part of the United States, west declination is decreasing and in the western part, east declination is increasing more rapidly than disclosed by the records of previous observers. In Bermuda it is stated the change is now seven minutes, and in Porto Rico nine minutes annually. The author says, "There appear to be two waves of

[1] Reprint Art. XXXIII Subantarctic Islands of New Zealand, Wellington, N. Z., 1909.

[2] Special Publication No. 9, U. S. Dept. Com. & Lab., Coast and Geodetic Survey, Washington, D. C., 1911, Pp. 14, 1 chart.

Digitized by Google

secular motion progressing across the country in opposite directions, and it is impossible to state which will predominate, but interesting developments may be expected in the next few years." H. W. F.

HAZARD, D. L. *Results of Observations Made at the Coast and Geodetic Survey Magnetic Observatories.*[1]

The issuance of the several series of publications, of which those enumerated in the foot-note are the latest volumes, was begun early in 1909, and the numbers have followed in rapid order, until at present the work of each observatory begun in 1901 or 1902, has been brought down practically to date. Each volume covers two years' work at a given observatory; the earlier ones contain detailed descriptions and illustrations of buildings and instrumental installations, while subsequent issues make reference to additions or changes.

Necessarily, the bulk of any volume of this character must consist mainly of the tabulations of hourly values of declinations, horizontal intensity and vertical intensity scaled and reduced from the three photographic traces. The tabulations in these publications include also the maximum value, and the minimum value of the measured elements, with the hour of occurrence for each day. The means of the hourly values are taken, first, for all the days of the month, and second, for ten selected least disturbed days. These two series of means show very inconsiderable divergencies, rarely amounting to more than 0.5 in declination or than 5 γ in horizontal intensity. In the case of months in which several storms occur near the same time of day, the selected day means would doubtless form the smoothest curves, but in general it may be questioned whether any improvement in values for diurnal variation reductions result from such selections.

For general reference, the summaries preceding the detailed tabulations are most useful. Tables are presented showing the mean hourly variation from the mean of day for each month, in D, H, Z, I, X, and Y. These are prepared from the means of the ten selected days. An additional feature is the plan of grouping the results by months into three groups—winter, equinoctial, and summer; thus: I, January; February, November, December; II, March, April, September, October; III, May, June, July, August. This gives a compact diurnal variation table, readily available for general use, and has proved to be very convenient. The final summaries show the monthly means in each of the six quantities mentioned above with total intensity added, and their mean annual values for each year of operation of the observatories.

Reproductions of the photographic traces made on days of magnetic storms are appended in each volume. The later volumes show these to a scale of one hour to about 15 mm., the time, for convenience in comparing with other observatories, being shown in Greenwich mean time.

At four of the observatories seismographs are kept in operation, and a record of the earthquakes registered is given in the report. The observatory

[1] Results of observations made at the Coast and Geodetic Magnetic Observatories: at Cheltenham, Maryland, 1907–1908; Sitka, Alaska, 1907–1908; Vieques, Porto Rico, 1909–1910; Honolulu, Hawaii, 1909–1910; Baldwin, Kansas, 1907–1909. Daniel L. Hazard, Government Printing Office, Washington, D. C.

Digitized by Google

located until October, 1909, at Baldwin, Kansas, was at that time discontinued and the instruments transferred to new buildings at Tucson, Arizona.

The general supervision of the work of the observatories is in the hands of Mr. R. L. Faris, Inspector of Magnetic Work; the preparation for publication of the reports under consideration was entrusted to Mr. Hazard, computer. The Coast and Geodetic Survey is to be congratulated on the promptness and regularity with which the results of its valuable magnetic work are being published.

H. W. F.

Hess, V. F. *Measurements of the penetrating radiation during two balloon ascents.*[1]

The apparatus used for measuring the intensity of the penetrating radiation was a thick-walled vessel containing an electrode designed by Wulf. The vessel was air-tight and capable of withstanding any differences of pressure occasioned by balloon ascents.

The object of the investigation was to determine the variation of the intensity of the penetrating radiation with the height. Some previous investigations have shown a decrease in intensity with the height. The author criticises these determinations, for the reason that the walls of the ionization chamber were not sufficiently thick to cut off the beta radiations.

Two balloon ascents were made, the first on the forenoon of August 28, 1911, and the second the night of October 12, 1911. During the first trip, measurements were made at altitudes ranging from 150 to 1070 meters, and during the second trip by night, at altitudes from 150 to 300 meters. None of the measurements show any clear indication of the decrease of natural ionization with height; moreover, the ionization appears to be of the same value day and night. During the night trip several fog regions were passed over. In all of these instances the ionization appeared a little greater. The author explains this as due to the fact that more radium C products exist in the fog (as condensation centers) than in the clear space without fog. The penetrating radiation was found to produce about 10 ions per cc. per second.

The author concludes that there are effective in the atmosphere sources of penetrating radiation, and that their effectiveness increases with the height, since the decrease of intensity of the radiation from the earth with height was compensated for in some way.

The possibility of induced activity on the surface of the balloon was excluded by making observations in its immediate vicinity just after a trip, when the ionization was the same as before the journey.

C. W. H.

[1] Sitzber. der Kaiser. Acad. Wiss. in Wien, Band CXX, Abt. IIA, No. 1911.

Digitized by Google

LIST OF RECENT PUBLICATIONS

Terrestrial Magnetism

CHREE, C. Studies in Terrestrial Magnetism. MacMillan and Co., London, 1912. 8°. Pp. XII + 206.

DEUTSCHE SEEWARTE. Vierunddreissigster Jahresbericht über die Tätigkeit der deutschen Seewarte für das Jahr 1911. Hamburg, H. W. Köbner & Co., 1912 (38) 26 1-2 cm. P.

DUBINSKIJ, V. Verteilung der Elemente des Erdmagnetismus im Bottnischen Meerbusen. (In Russian.) Denkschr. d. Hydrogr. Petersburg, Hft. 33.

HAZARD, DANIEL L. Results of Observations made at the Coast and Geodetic Survey Magnetic Observatory at Cheltenham, Maryland, 1909 and 1910. U. S. Dept. Com. and Lab., C. & G. S., 1912 (94 and 1 plate) 29 cm. P.

INTERNATIONAL METEOROLOGICAL COMMITTEE. Report of the ninth meeting of the International Meteorological Committee, and of the sixth meeting of the Commission for Terrestrial Magnetism and Atmospheric Electricity. Berlin, 1910. London, Wyman & Sons, 1912 M. O. No. 208 (144) 24 1-2 cm. P.

KLOTZ, OTTO. Magnetic Observations from the Dominion Observatory. Repr. Ottawa, J. R. Astr. Soc. Can. Jan.-Feb. 1912 (53-55). (During the season of 1911, 58 stations between Winnipeg and the Rocky Mountains were occupied, besides Ottawa and the base stations at Agincourt, in continuation of the systematic magnetic survey of Canada, undertaken by the Dominion Observatory, and in the charge of Dr. Klotz.)

KÖNIGLICH PREUSSISCHEN METEOROLOGISCHEN INSTITUTS. Bericht über die Tätigkeit des Königlich Preussischen meteorologischen Instituts im Jahre 1911. (Veröffentlichungen des Königlich Preussischen meteorologischen Instituts. G. Hellmann, Director.) Mit einem Anhang enthaltend wissenschaftliche Mitteilungen. No. 244. Berlin, 1912, Behrend & Co., (190 and figs.) 27 cm. P.

KONINKLIJK NEDERLANDSCH METEOROLOGISCH INSTITUUT. Annuaire, Soixante-deuxième année, 1910, No. 98, B, Magnétisme terrestre. Utrecht, Kemink & Zoon, 1911 (44) 32 cm. P.

KÜHL, W. Versuche betreffend den Einfluss der Luftfeuchtigkeit und anderer Fehlerquellen auf eine magnetische Wage. Veröff. König. Preuss. Meteor. Inst., Berlin, No. 244, 1912 (121-129).

PALAZZO, LUIGI. Misure Magnetiche fatte in Sardegna nel 1892. [Describes in detail the magnetic instruments, constants, and observations, a digest of which has already been published in this Journal by Prof. Palazzo, Vol. 14, pp. 147-152.] Reprint Rome, Ann. Uff. centr. meteor, geodin. v. XXIV, Part 1, 1902 (103 con 3 tavole).

RIES, CHR. Das Licht in seinen elektrischen und magnetischen Wirkungen. Versuchsergebnisse, Theorien und Literatur. Wissen und Können. (11 Band.) Leipzig. J. A. Barth, 1909 (VIII + 262). [Chap. IV. Effect of light on the electric conductivity of bodies and production of an electromotive force. Chap. V. The magnetic effects of light, including solar radiation and terrestrial magnetic effects and literature on same.]

Digitized by Google

Royal Magnetical and Meteorological Observatory. Observations made at the Royal Magnetical and Meteorological Observatory at Batavia. Containing meteorological, magnetical, and seismometric observations made in 1908. (Published by order of the Government of Netherlands East-India by Dr. W. van Bemmelen, Director.) Batavia, Roy. Mag. & Meteor. Obsy., v. 31, 1911 (XLVIII + 174 and 4 pl.) 36 1-2 cm. P.

San Fernando. Anales del Instituto y Observatorio de Marina de San Fernando, publicados de orden de la superiordad por el Director Don Tomás de Azcárate. Seccion 2ª. Observaciones meteorológicas, magnéticas, y sísmicas. Año 1910. San Fernando. 1911 (164 with two pl.) 35 cm. P.

Schmidt, Ad. Vorläufige Mitteilungen über die Ergebnisse der magnetischen Beobachtungen in Potsdam und Seddin im Jahre 1911. Veröff. König. Preuss. Meteor. Inst., Berlin, No. 244, 1912 (115-121).

Schuster, Arthur. A Critical Examination of the Possible Causes of Terrestrial Magnetism. Review: Elec. Rev. Chicago, Ill., v. 60, No. 16, April 20, 1912 (763-764).

Stonyhurst College Observatory. Results of meteorological and magnetical observations, 1911. With report and notes of the Director, Rev. W. Sidgreaves. Liverpool, 1912 (XVII + 57) 18 1-2 cm. P.

Störmer, Carl. Photographie des Aurores Boréales et Nouvelle Méthode pour Mesurer leur Altitude. Bruxelles, C. R. Congrès International de Radiologie et d'Électricité, Sciences Physiques, Tome I, 1911 (27-32).

Störmer, Carl. Bericht über eine Expedition nach Bossekop zwecks photographischer Aufnahmen und Höhenmessungen von Nordlichtern. Reprint—Kristiania, Skr. Vid. selsk. 1911, No. 17.

Störmer, Carl. Sur les trajectoires des corpuscles électrisés dans l'espace sous l'action du magnétisme terrestre avec application aux aurores boréales. Second mémoire. Troisième partie. Arch. Sci. Phys. Genève, v. 33, No. 1, Jan. 15, 1912, (51-69); No. 2, Feb. 15, 1912, (113-150).

Atmospheric Electricity

Baldit, A. Nouvelles observations sur les charges électriques de la pluie en 1911, au Puy-en-Velay. Paris, Radium, V. 9, No. 3, Mar., 1912 (92-101).

Benham, Charles E. A Simple Recording Device for Atmospheric Electricity. London: Knowledge, V. XXXV., No. 525. April 1912, p. 125.

Hurmuzescu, A. et Patriciu. La radioactivité de quelques eaux minérales de Roumanie. Bruxelles, C. R. Congrès International de Radiologie et d' Électricité, Sciences, Physiques, Tome I., 1911 (77-78).

McClelland, J. A. and Nolan, J. J. The Electric Charge on Rain. Dublin, Proc. R. Irish Acad., V. 29, Sec. A. No. 5, Feb. 1912, (81-91).

Rudge, W. A. Douglas. A Note on the Electrification of the Atmosphere and the Surface of the Earth. Phil. Mag., London, V. 23, No. 137, May, 1912 (852-855).

Schweidler, E. R. Atmospheric Electricity, XLIV. Observations at Seeham in the summer of 1910. (Akad. Wiss. Wien, Sitz. Ber. 119, 2a., pp. 1069-1072, July, 1910.) Abstr. 190. London, Sci. Abstr., A. V. 15, part 2, Feb. 26, 1912, p. 61.

Digitized by Google

Digitized by Google

EMINENT GEOGRAPHERS AT WASHINGTON, D. C., OCTOBER 13, 1912.

[PLATE III]

Digitized by Google

Terrestrial Magnetism
and
Atmospheric Electricity

VOLUME XVII DECEMBER, 1912 NUMBER 4

PRELIMINARY NOTE ON AN ATTEMPT TO DETECT THE GENERAL MAGNETIC FIELD OF THE SUN.

By George E. Hale.

In 1908, shortly after magnetic fields had been found in Sunspots, an attempt was made to observe the Zeeman effect due to the general magnetic field of the Sun with the 60-foot tower telescope and 30-foot spectrograph of the Mount Wilson Solar Observatory. Although slight shifts of certain solar lines, such as might be caused by a magnetic field, were found on measuring the photographs, the quantities were too small to command confidence, and the work was postponed until more perfect polarizing apparatus could be provided.

The investigation was resumed in October, 1911, but no certain evidences of the Zeeman effect were detected. On the completion of the 150-foot tower telescope and 75-foot spectrograph, observations were begun with this instrument in January of the present year. The objective of 150 feet focal length now used with this telescope was not then ready, and, accordingly, the 12-inch objective of 60 feet focal length, belonging to the 60-foot tower telescope, was temporarily transferred to the new tower. This forms an image of the Sun 6.7 inches in diameter on the slit of the spectrograph. After passing through the slit the light descends vertically into a well, excavated in the earth beneath the tower, to a depth of 75 feet, where it falls upon the objective which serves for both collimator and camera of the auto-collimating spectrograph. We were fortunate in having a large grating of the finest quality, ruled by Michelson, which gives almost theoretical resolution in the third order spectrum. Here the scale is 4.9 mm to the Ängström unit, corresponding to a distance of 29 mm between the D lines.

Let us assume the Sun's magnetic field to be similar to that of

Digitized by Google

a magnetized sphere, with magnetic poles coinciding in position with the poles of rotation. If the field were strong enough, and if the observer could look along the Sun's axis, and form an image of one of the magnetic poles on the slit of a powerful spectrograph, he would find certain solar lines split into doublets, with components circularly polarized in opposite directions. If a Nicol prism were placed in front of the slit of the spectrograph with its long axis parallel to the slit, in combination with a quarter-wave plate set with its principal section at an angle of 45°, one of the components of the magnetic doublets would be extinguished, while the other would be transmitted by the Nicol. If we assume the red component to be transmitted, rotation of the quarter-wave plate through an angle of 90° would cause this to be extinguished and the violet component to be transmitted. Consequently, if the quarter-wave plate were built up of mica strips 2 mm wide, mounted so that the principal sections of successive strips make an angle of 45° with the slit and 90° with each other, the Nicol would transmit (say) the red component for the odd strips and the violet component for the even strips. In a photograph of the spectrum the lines would have a zigzag appearance, the magnitude of the separation of the components shown on successive strips varying directly with the strength of the field. If the slit of the spectrograph were directed, not at the Sun's pole, but at a point in 45° latitude, the effect would still be clearly observable, though the transformation of the circularly polarized light of the components into elliptically polarized light would result in less complete extinction by the Nicol.

In practice, on account of the weakness of the Sun's magnetic field, in comparison to the fields of Sun-spots, complete separation into doublets is not to be expected. The superposition of the components would result in a slightly widened line, with edges elliptically polarized in opposite directions. It was hoped that a spectrograph of very high dispersion might reveal slight shifts of the lines, caused by the extinction of the red and violet edges by the successive strips of the quarter-wave plate. The increasing ellipticity of the light, in passing from high to low latitudes, would involve less and less complete extinction. This, coupled with the decrease in the total magnetic intensity toward the equator, would be indicated by decreasing displacements of the line. The sign of the displacements should be reversed in passing from the northern to the southern hemisphere.

Digitized by Google

With the aid of the polarizing apparatus described above, a series of photographs of the region near the D lines in the third order spectrum was made with the slit at various latitudes on the central meridian of the Sun. The photographs were measured by Miss Lasby, who had acquired much experience in the measurement of extremely small displacements of solar lines. The measures were made without possible bias, as the latitude corresponding to the plate was not known to the observer. With very few exceptions, the displacements were found to be positive in the northern hemisphere and negative in the southern hemisphere of the Sun, the magnitude of the displacements decreasing from a maximum near 50° north or south latitude, to zero near the equator.

The conditions were extremely favorable for the detection of very small shifts of the solar lines, as the spectrograph is very massive and stable, the grating is mounted in a region of practically constant temperature about 75 feet below the level of the ground, and the method of observation is a purely differential one, the displacements measured being those of sections of the same line corresponding to adjoining strips of the quarter-wave plate. As a check upon the results, atmospheric lines were measured, and found to give shifts rarely exceeding 0.0002 Ängström, while some of the solar lines gave shifts as great as 0.003 Ängström. Nevertheless, there were certain peculiarities about the results which make it advisable to take every possible precaution to check their reliability.

The most marked of these peculiarities lay in the fact that only a few of the solar lines seemed to be affected, the others showing no shifts, although they were known to give large Zeeman effects in the laboratory. In our work on the spectra of Sun-spots, it is true, we had found that the magnetic field is very strong at low levels, but decreases rapidly in passing upward through the spot vapors. If some analogous condition exists in the case of the general field, certain low-level lines may show it, while those corresponding to higher levels may fail to do so.

As a check on the question whether the displacements of the solar lines might safely be attributed to a magnetic field, a half-wave plate was inserted between the compound quarter-wave plate and the Nicol. Leaving the quarter-wave plate and Nicol untouched, it should be possible to reverse the sign of the displacement in any given case by turning the half-wave plate through an angle of 45°. A number of tests were made in this way, and in all

Digitized by Google

cases Miss Lasby's measurements showed shifts such as would occur if the edges of the lines were circularly or elliptically polarized. A similar test was applied by omitting the half-wave plate, and inverting the compound quarter-wave plate, so as to bring its lower face uppermost. This also reversed the sign of the displacement, and proved so satisfactory a means of checking the observations that the plan of taking all plates in pairs, corresponding to the two positions of the quarter-wave plate, was subsequently adopted.

Up to this time all of the measures had been made by Miss Lasby, except for a few check measures by Mr. Adams. As it seemed desirable to have a complete series of independent measures of the plates, several members of the Observatory staff undertook the work. It was found, however, that while in many cases the measures showed excellent internal agreement, there was no general harmony among the observers. It was evident that with the lines in question systematic errors were likely to occur, frequently of such a magnitude as to reverse the sign of the measured displacement. Two attempts to avoid this difficulty were accordingly made. A parallel plate micrometer, in which the line corresponding to one of the strips of the compound mica plate is displaced to right or left by inclining a plate of plane parallel glass, mounted immediately above the negative, was substituted for the ordinary measuring machine used in the previous work. With this simple device, the use of which for this purpose was suggested by Mr. Pease, the line is shifted until it coincides with the line on an adjoining strip of the spectrum, and the magnitude of the displacement is read off on a graduated arc, which measures the angle through which the plate is turned.

All of the plates of the first series have been measured with this instrument by Dr. Van Maanen, with results agreeing well with those of Miss Lasby, except that there appears to be a systematic tendency toward smaller values for the displacements on the part of Dr. Van Maanen. Thus, while the true magnitude of the displacement for a given latitude is still open to further investigation, there is no question as to the reversal of the sign of the displacements in the two hemispheres, and the decrease in their magnitude from middle latitudes toward the equator.

In order to obtain further light on the question of systematic errors, the simple expedient was adopted of displacing a line by a known amount, and then measuring the displacement by settings

Digitized by Google

on the line itself. In this way it is possible to obtain the personal equation of any observer for lines of any width or intensity. The results indicate that a slight peculiarity in the plates may often introduce systematic errors, capable of reversing the sign of the displacement. This probably accounts for some of the discrepancies encountered in the measurement of the plates.

In spite of the fact that the results obtained are precisely such as the existence of a magnetic field would lead us to expect, and the apparent impossibility of attributing them to any other cause, some difficulties remain unexplained. A long series of photographs, made in the second order in May and June, has been measured by Dr. Van Maanen with the parallel plate micrometer. The second order was chosen because of the greater sharpness of the solar lines, but it is doubtful whether this policy was a wise one. The displacements on these photographs are much smaller than those of the first series, though it is extremely improbable that the strength of the Sun's field would change materially between the two sets of observations. As all of the photographs were made in pairs (quarter-wave plate inverted), it is possible to test the magnetic nature of the displacements. Thirty-eight pairs of plates give reversed signs for the displacements, while 10 pairs do not. In view of the very small magnitude of the displacements, which rarely exceed 0.003 mm, the evidence is decidedly in favor of an actual reversal of sign produced by inversion of the quarter-wave plate.

I am unable to account for the greatly decreased magnitude of the displacements on these plates, unless it is due to the use of the second order spectrum instead of the third. On the average, the displacements are only about one-quarter as great (when reduced to the third order) as those obtained by Dr. Van Maanen for the first series of plates. In this connection, and as a necessary check on all of the observations, a careful study has been made of the elliptical polarization due to the reflection of the sunlight on the two silvered mirrors of the coelostat. The polarization phenomena vary with the declination and hour angle of the Sun, but in no case do they appear to be of such a character as to influence appreciably the magnitude of the displacements.

Another series of observations is now in progress, using the third order of the grating and the 16½-inch image of the Sun given by the object-glass of 150 feet focal length. The displacements, as far as measured, are much larger than those on the

Digitized by Google

second order plates, and the results clearly confirm those of the first series.

As the matter now stands, further observations will be required to prove conclusively whether or not the observed displacements are due to the Sun's magnetic field. It may be interesting to inquire, however, as to the sign of the charge and the corresponding polarity indicated by the observations already obtained. Knowing the direction of the Sun's rotation, and assuming, for example, that the field is produced by the rotation with the Sun of two opposite and equal charges distributed through the Sun's mass and contained within spheres whose radii differ slightly, we may deduce the sign of the outer charge required to produce such line displacements as have been observed.[1] This outer charge is found to be negative, which would imply that the north and south poles of the Sun agree in magnetic polarity with those of the Earth. As for the strength of the field, a knowledge of the Zeeman effect for the lines in question is necessary to determine this. It happens that all of these lines are too faint in the spark to appear in our photographs, but another effort is being made by Mr. Babcock to observe their behavior in the magnetic field.

The investigation is being pushed forward as rapidly as possible, in view of the quiet condition of the Sun, since the appearance of Sun-spots, with their very powerful magnetic fields, will tend to introduce troublesome perturbations.

Pasadena, Cal., December 5, 1912.

[1] Schuster, A., A Critical Examination of the Possible Causes of Terrestrial Magnetism, *Proc. Phys. Soc. London*, April 15, 1912; Swann, W. F. G., The Earth's Magnetic Field, *Phil. Mag.*, July, 1912, v. 24, p. 80; Bauer, L. A., On the Origin of the Earth's Magnetic Field, *Terr. Mag.*, September, 1912, p. 136.

Digitized by Google

MAGNETIC DECLINATIONS AND CHART CORRECTIONS OBTAINED BY THE *CARNEGIE* FROM SUVA, FIJI, TO PAPEETE, TAHITI, JULY TO SEPTEMBER, 1912.[1]

By L. A. Bauer and W. J. Peters.

(*Minus sign indicates west declination, and plus, east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1912	° ′	° ′	°	°	°	°	°	°	°
July 2	19 33 S	177 04 E	+10.2	+10.4	+10.5	+10.8	—0.2	—0.3	—0.6
5	26 26 S	174 50 E	+11.7	+11.6	+12.2	+12.2	+0.1	—0.5	—0.5
7	28 57 S	177 56 E	+12.8	+12.2	+12.8	+12.8	+0.6	0.0	0.0
7	29 09 S	179 03W	+12.3	+12.3	+12.7	+12.8	0.0	—0.4	—0.5
7	29 20 S	178 06W	+12.6	+12.3	+12.7	+12.7	+0.3	—0.1	—0.1
8	29 28 S	176 35W	+12.7	+12.2	+12.7	+12.7	+0.5	0.0	0.0
9	29 55 S	175 21W	+12.9	+12.2	+12.7	+12.7	+0.7	+0.2	+0.2
11	31 31 S	170 09W	+13.5	+12.2	+12.7	+12.6	+1.3	+0.8	+0.9
12	32 05 S	168 38W	+13.7	+12.2	+12.7	+12.5	+1.5	+1.0	+1.2
14	31 35 S	161 41W	+13.3	+11.8	+12.0	+12.0	+1.5	+1.3	+1.3
15	30 58 S	158 23W	+13.3	+11.5	+11.7	+11.8	+1.8	+1.6	+1.5
16	30 14 S	154 40W	+13.0	+11.3	+11.3	+11.4	+1.7	+1.7	+1.6
16	30 09 S	153 30W	+13.0	+11.3	+11.2	+11.5	+1.7	+1.8	+1.5
17	30 07 S	152 00W	+13.0	+11.3	+11.2	+11.3	+1.7	+1.8	+1.7
21	31 38 S	146 12W	+13.5	+11.5	+11.5	+11.6	+2.0	+2.0	+1.9
22	31 12 S	144 33W	+13.4	+11.5	+11.5	+11.6	+1.9	+1.9	+1.8
23	30 07 S	142 27W	+13.3	+11.4	+11.3	+11.4	+1.9	+2.0	+1.9
23	29 33 S	141 05W	+13.0	+11.3	+11.2	+11.3	+1.7	+1.8	+1.7
24	28 59 S	139 22W	+12.9	+11.3	+11.2	+11.3	+1.6	+1.7	+1.6
25	28 36 S	137 41W	+12.7	+11.3	+11.2	+11.3	+1.4	+1.5	+1.4
25	28 35 S	137 21W	+12.6	+11.3	+11.2	+11.3	+1.3	+1.4	+1.3
26	27 49 S	136 24W	+12.5	+11.2	+11.1	+11.2	+1.3	+1.4	+1.3
31	32 01 S	125 22W	+14.0	+13.6	+13.7	+13.7	+0.4	+0.3	+0.3
Aug. 1	31 29 S	121 40W	+13.6	+14.0	+14.1	+14.2	—0.4	—0.5	—0.6
1	31 10 S	120 06W	+14.0	+14.2	+14.4	+14.5	—0.2	—0.4	—0.5
2	30 36 S	118 14W	+14.0	+14.4	+14.6	+14.8	—0.4	—0.6	—0.8
2	30 01 S	117 10W	+14.5	+14.4	+14.6	+14.9	+0.1	—0.1	—0.4
3	28 52 S	116 08W	+14.4	+14.3	+14.4	+14.8	+0.1	0.0	—0.4
3	28 05 S	115 34W	+14.2	+14.2	+14.3	+14.6	0.0	—0.1	—0.4
4	26 26 S	114 40W	+13.7	+13.8	+14.0	+14.1	—0.1	—0.3	—0.4
5	23 30 S	114 16W	+11.9	+12.8	+13.1	+13.1	—0.9	—1.2	—1.2
5	22 28 S	114 09W	+12.0	+12.5	+12.8	+12.8	—0.5	—0.8	—0.8
6	20 18 S	114 10W	+11.4	+11.7	+12.1	+11.9	—0.3	—0.7	—0.5
8	16 44 S	114 13W	+10.7	+10.7	+11.0	+10.9	0.0	—0.3	—0.2
8	15 16 S	114 02W	+10.1	+10.2	+10.7	+10.5	—0.1	—0.6	—0.4

[1] For previous tables see *Terr. Mag.* v. 15, pp. 57–82, 129–144; v. 16, pp. 133–136; v. 17, pp. 31–32, 97–101, 141–144.

Digitized by Google

(*Minus sign indicates west declination, and plus, east declination.*)

Date	Position		Carnegie	Chart Values			Chart Corrections		
	Latitude	Longitude		Brit.	Ger.	U. S.	Brit.	Ger.	U. S.
1912	° ′	° ′	°	°	°	°	°	°	°
Aug. 9	13 29 S	113 56W	+10.0	+ 9.9	+10.3	+10.0	+0.1	—0.3	0.0
9	12 26 S	113 47W	+ 9.6	+ 9.8	+10.1	+ 9.8	—0.2	—0.5	—0.2
10	10 45 S	113 35W	+ 9.6	+ 9.6	+ 9.8	+ 9.6	0.0	—0.2	0.0
10	9 40 S	113 40W	+ 9.3	+ 9.4	+ 9.6	+ 9.4	—0.1	—0.3	—0.1
11	8 18 S	114 00W	+ 9.2	+ 9.2	+ 9.2	+ 9.2	0.0	0.0	0.0
11	7 19 S	113 52W	+ 9.2	+ 9.1	+ 9.0	+ 9.1	+0.1	+0.2	+0.1
12	6 20 S	113 35W	+ 8.9	+ 9.0	+ 9.0	+ 9.0	—0.1	—0.1	—0.1
12	5 12 S	113 14W	+ 8.9	+ 8.9	+ 8.9	+ 8.9	0.0	0.0	0.0
13	4 09 S	113 02W	+ 8.8	+ 8.8	+ 8.8	+ 8.8	0.0	0.0	0.0
13	3 03 S	113 02W	+ 8.6	+ 8.7	+ 8.7	+ 8.7	—0.1	—0.1	—0.1
14	1 41 S	112 57W	+ 8.7	+ 8.6	+ 8.6	+ 8.5	+0.1	+0.1	+0.2
14	0 22 S	112 36W	+ 8.5	+ 8.4	+ 8.4	+ 8.4	+0.1	+0.1	+0.1
15	1 15 N	113 07W	+ 8.4	+ 8.3	+ 8.4	+ 8.3	+0.1	0.0	+0.1
16	1 50 N	114 08W	+ 8.4	+ 8.2	+ 8.1	+ 8.1	+0.2	+0.3	+0.3
16	2 18 N	115 18W	+ 8.2	+ 8.0	+ 8.1	+ 8.0	+0.2	+0.1	+0.2
17	2 48 N	116 48W	+ 8.5	+ 8.0	+ 8.1	+ 8.0	+0.5	+0.4	+0.5
17	3 22 N	118 29W	+ 8.2	+ 7.9	+ 8.1	+ 8.0	+0.3	+0.1	+0.2
18	4 04 N	120 33W	+ 8.3	+ 7.8	+ 8.1	+ 7.9	+0.5	+0.2	+0.4
18	4 40 N	122 09W	+ 8.2	+ 7.7	+ 8.1	+ 7.9	+0.5	+0.1	+0.3
19	6 17 N	125 03W	+ 8.3	+ 7.8	+ 8.1	+ 8.0	+0.5	+0.2	+0.3
20	7 07 N	127 42W	+ 8.4	+ 7.8	+ 8.1	+ 7.9	+0.6	+0.3	+0.5
21	7 15 N	130 51W	+ 8.4	+ 7.7	+ 7.9	+ 7.7	+0.7	+0.5	+0.7
22	7 59 N	132 01W	+ 8.6	+ 7.7	+ 7.9	+ 7.8	+0.9	+0.7	+0.8
22	8 38 N	132 53W	+ 8.5	+ 7.8	+ 8.0	+ 7.8	+0.7	+0.5	+0.7
23	8 23 N	133 52W	+ 8.7	+ 7.7	+ 7.8	+ 7.8	+1.0	+0.9	+0.9
24	7 56 N	130 22W	+ 8.5	+ 7.7	+ 8.0	+ 7.8	+0.8	+0.5	+0.7
26	7 11 N	126 05W	+ 8.4	+ 7.8	+ 8.2	+ 8.0	+0.6	+0.2	+0.4
27	6 58 N	125 51W	+ 8.4	+ 7.8	+ 8.1	+ 8.0	+0.6	+0.3	+0.4
28	6 26 N	126 27W	+ 8.5	+ 7.7	+ 8.1	+ 7.9	+0.8	+0.4	+0.6
29	5 55 N	126 47W	+ 8.4	+ 7.6	+ 8.0	+ 7.8	+0.8	+0.4	+0.6
30	5 02 N	129 51W	+ 8.3	+ 7.5	+ 7.7	+ 7.6	+0.8	+0.6	+0.7
Sept. 1	4 20 N	134 16W	+ 8.3	+ 7.3	+ 7.2	+ 7.2	+1.0	+1.1	+1.1
2	3 32 N	135 26W	+ 8.0	+ 7.2	+ 7.0	+ 7.0	+0.8	+1.0	+1.0
2	2 20 N	136 29W	+ 8.0	+ 7.1	+ 6.8	+ 6.8	+0.9	+1.2	+1.2
3	0 41 N	137 14W	+ 7.9	+ 7.0	+ 6.8	+ 6.7	+0.9	+1.1	+1.2
3	0 49 S	137 48W	+ 8.2	+ 7.0	+ 6.7	+ 6.6	+1.2	+1.5	+1.6
4	2 26 S	138 27W	+ 7.9	+ 7.0	+ 6.8	+ 6.7	+0.9	+1.1	+1.2
4	3 52 S	139 09W	+ 8.2	+ 7.0	+ 6.8	+ 6.8	+1.2	+1.4	+1.4
5	5 11 S	139 53W	+ 8.3	+ 7.2	+ 6.9	+ 7.0	+1.1	+1.4	+1.3
5	6 26 S	140 32W	+ 8.2	+ 7.3	+ 7.0	+ 7.1	+0.9	+1.2	+1.1
6	7 38 S	141 13W	+ 8.6	+ 7.4	+ 7.2	+ 7.4	+1.2	+1.4	+1.3
6	8 44 S	141 40W	+ 8.7	+ 7.7	+ 7.5	+ 7.6	+1.0	+1.2	+1.0
8	11 14 S	142 32W	+ 8.8	+ 8.1	+ 8.0	+ 8.2	+0.7	+0.8	+0.6
9	13 55 S	144 04W	+ 9.4	+ 8.5	+ 8.4	+ 8.7	+0.9	+1.0	+0.7
9	14 35 S	145 08W	+ 9.3	+ 8.6	+ 8.5	+ 8.8	+0.7	+0.8	+0.5
10	16 02 S	147 24W	+ 9.2	+ 8.8	+ 8.8	+ 9.0	+0.4	+0.4	+0.2
11	17 33 S	148 52W	+10.3	+ 9.0	+ 9.0	+ 9.3	+1.3	+1.3	+1.0

Digitized by Google

UEBER DIE GEGENSEITIGE EINWIRKUNG ZWEIER MAGNETE IN BELIEBIGER LAGE.

Von AD. SCHMIDT *in Potsdam.*

Die Berechnung der von zwei Magneten auf einander ausgeübten mechanischen Wirkungen bildet eine grundlegende Aufgabe der erdmagnetischen Beobachtungsmethoden und ist daher seit jeher eingehend behandelt worden. Die grosse Ausdehnung der dabei auftretenden Formeln und die Weitläufigkeit der zu ihrer Ableitung dienenden Rechnungen hat indessen dazu geführt, dass man sich vorwiegend auf die einfachsten Fälle, die praktisch allerdings auch die wichtigsten sind, beschränkt hat. In Bezug auf die gegenseitige Lage hat man fast stets angenommen, dass sich beide Magnete in ein und derselben Ebene befinden, die zugleich zur Drehungsachse des einen (der Nadel, wie man zu sagen pflegt) senkrecht ist. Nur B ö r g e n[1] ist darüber hinausgegangen, indem er den Fall einer beliebigen räumlichen Lage (allerdings unter Beschränkung auf die Berechnung des Drehungsmoments, das der eine Magnet in Bezug auf eine bestimmte Achse erfährt) untersucht hat. In einer im ersten Bande dieser Zeitschrift veröffentlichten Arbeit führt er die Rechnung bis zu den Gliedern 9. Potenz der reziproken Entfernung wenigstens so weit durch, dass man davon ausgehend durch einfache, wennschon freilich recht mühsame Substitutionen zu praktisch verwendbaren Schlussformeln gelangen kann. Eine solche, bis zur 7. Potenz ausgedehnte explizite Formel hatte er selbst schon früher an anderer Stelle mitgeteilt.[2] Was andrerseits die Magnete betrifft, so hat man diese fast ausschliesslich als linear und zugleich als symmetrisch (oder vielmehr, richtiger gesagt, antisymmetrisch) in Bezug auf ihren Mittelpunkt vorausgesetzt, indem man sich damit begnügte, gelegentlich den Einfluss der Querdimensionen unter vereinfachenden Annahmen durch Korrektionen zu berücksichtigen.

Will man sich von allen diesen Beschränkungen frei machen, also beliebige Magnete in beliebiger Lage betrachten, so gestaltet sich die Entwickelung der reziproken Entfernung zweier Punkte,

[1] C. BÖRGEN: Allgemeiner Ausdruck für die Coefficienten der Formel für die Ablenkung einer Magnetnadel durch einen Ablenkungsstab in beliebiger Lage. *Terrestrial Magnetism*, Vol. I, S. 176, (1896).

[2] C. BÖRGEN: Ableitung des Ausdrucks für die Ablenkung einer Magnetnadel durch einen Magnet, dessen Lage im Raume eine beliebige sein kann. *Aus dem Archiv der Deutschen Seewarte*, XIV. Jahrgang (1891), Nr. 2.

Digitized by Google

auf die ja die Aufgabe im wesentlichen hinauskommt, nach dem üblichen Verfahren (mit Hilfe der binomischen Reihe) so umständlich und unübersichtlich, dass ihre Durchführung kaum noch möglich ist. Es liegt nahe, zu versuchen, sie durch Benutzung von Kugelfunktionen zu vereinfachen, die ja gerade als Hilfsmittel zur Lösung jener Aufgabe in die Analysis eingeführt worden sind. Dass man dies trotzdem früher nicht getan hat, liegt vielleicht daran, dass einige zur allgemeinen Durchführung der Rechnung nötige Beziehungen in den üblichen Darstellungen der Theorie dieser Funktionen nicht fertig vorliegen, wennschon es sich dabei nur um Identitäten von ganz elementarer Natur handelt.[3]

Vor einigen Jahren habe ich nun die hiermit bezeichnete Aufgabe in einem Aufsatze behandelt, der in den Berliner Sitzungsberichten erschienen ist.[4] Die Betrachtung konnte aber dort aus Mangel an Raum nur bis zur Aufstellung einer allgemeinen Formel durchgeführt werden, die zwar jede Anwendung auf beliebige Einzelfälle gestattet, aber das Ergebnis doch nicht unmittelbar ablesen lässt. Die dazu nötige Spezialisierung und damit die Aufstellung fertiger Gebrauchsformeln soll nun hier im Folgenden gegeben werden. Es erscheint am zweckmässigsten, dies im Rahmen einer systematischen Gesamtdarstellung der ganzen Frage zu tun.

Das Potential eines Magnets.

Jede magnetische Wirkung eines Magnets in seinem Aussenraume kann durch ein Potential dargestellt werden, und dieses lässt sich ausserhalb einer Kugel, die ihn vollständig umschliesst, nach Kugelfunktionen entwickeln. Die Koeffizienten der Kugelfunktionenreihe, die in erster Linie von der Beschaffenheit des Magnets, dann aber auch von der Wahl des Koordinatensystems abhängen, mögen die *Parameter* des Magnets heissen. Es ist danach klar, dass das gegenseitige Potential und damit die gegen-

[3] Das gilt, wie bemerkt, für die allgemeine Aufgabe. In den üblicherweise betrachteten speziellen Fällen besteht gar keine Schwierigkeit. Besonders einfach gestaltet sich dabei die Rechnung bei der ersten Hauptlage, wie sich schon deshalb vermuten lässt, weil hier die in der Schlussformel auftretenden goniometrischen Ausdrücke $(1 - 5 \sin \phi^2, 1 - 14 \sin \phi^2 + 21 \sin \phi^4$, u. s. w., vgl. z. B. Formel (12) bei Börgen, *Terr. Magn.* I, S. 186) selbst Kugelfunktionen sind. Für diesen Fall hat in der Tat auch bereits Chwolson in einer anscheinend wenig bekannt gewordenen (auch von Börgen nicht zitierten) Arbeit die Entwickelung nach Kugelfunktionen durchgeführt. O. CHWOLSON: Ueber die Wechselwirkung zweier Magnete mit Berücksichtigung ihrer Querdimensionen. *Mémoires de l'Acad. Imp. des Sciences de St. Pétersbourg*, VIIe Série, Tome XXXI, Nr. 10, S. 7 (1883).

[4] AD. SCHMIDT: Ueber die Bestimmung des allgemeinen Potentials beliebiger Magnete und die darauf begründete Berechnung ihrer gegenseitigen Einwirkung. *Sitzungsberichte der Kgl. Pr. Akademie d. Wiss. Math.-Phys. Classe.* Jahrgang 1907, Nr. XVI, S. 306.

Digitized by Google

seitige Einwirkung zweier Magnete auf einander eine Funktion der Parameter beider und ihrer relativen Lage sein muss. Diese Funktion zu bestimmen, ist die hier gestellte Aufgabe. Dazu ist zunächst das Potential des einzelnen Magnets (d. h. sein Potential in Bezug auf einen irgendwo gelegenen Einheitspol) zu ermitteln.

Mit dem Magnet sei ein rechtwinkliges Koordinatensystem (x, y, z) fest verbunden. Die Achsen sind so zu definieren und festzulegen, dass man ihre Lage im Raume bei allen Operationen, die man mit dem Magnet vornimmt, exakt bestimmen kann. Ist dieser regelmässig gestaltet, so empfiehlt es sich, die Achsen so zu wählen, dass sie in möglichst einfacher Beziehung zur Gestalt stehen. Besitzt der Körper einen geometrischen Mittelpunkt, so wird man diesen zum Koordinatenursprung wählen; hat er Symmetrieebenen, so ist es zweckmässig, in diese die Koordinaten-Hauptebenen fallen zu lassen. Bei Magneten von gestreckter Gestalt, wie sie fast ausschliesslich in Betracht kommen, wird die magnetische Achse im allgemeinen in die Längsrichtung des Körpers und zugleich annähernd auf eine der Koordinatenachsen fallen. Diese letztere möge dann stets als x-Achse bezeichnet werden.

Es sei nun A irgend ein Punkt im Innern des Magnets mit den Koordinaten $x=a$, $y=b$, $z=c$, ferner μ die räumliche Dichte der an dieser Stelle anzunehmenden freien Magnetisierung, und P ein Punkt im Aussenraume mit den Koordinaten x, y, z. Das Potential des Magnets hat dann im Punkte P den Wert

$$(1) \quad \Pi = \int\int\int \mu \, [\, (x-a)^2 + (y-b)^2 + (z-c)^2 \,]^{-\frac{1}{2}} \, da \, db \, dc$$

wobei die Integration über den ganzen vom Magnet eingenommenen Raum auszuführen ist. Es liege nun P ausserhalb einer den Magnet vollständig umschliessenden Kugel, deren Zentrum der Koordinatenursprung O sei, und es werde

$$x = r \cos\sigma \qquad y = r \sin\sigma \cos\tau \qquad z = r \sin\sigma \sin\tau$$

gesetzt, so dass $r = OP$ und σ der Winkel zwischen OP und der positiven Achse der x ist. Alsdann lässt sich nach den bekannten Eigenschaften der Kugelfunktionen der vorstehende für Π angegebene Ausdruck in eine konvergente unendliche Reihe von der Gestalt

$$(2) \quad \Pi = \sum_{1}^{\infty} n \sum_{0}^{n} m \; r^{-n-1} \, P_m^n (\cos\sigma) \, (M_{nm} \cos m\tau + N_{nm} \sin m\tau)$$

Digitized by Google

entwickeln,[5] deren Koeffizienten M und N ausschliesslich von dem Betrage und der räumlichen Verteilung von μ abhängen, also durch die Beschaffenheit des Magnets vollständig bestimmt sind.

Bei der üblichen Entwickelung nach der Binomialreihe treten nun bekanntlich lauter Koeffizienten von der Form

$$(3) \quad F_{\alpha\beta\gamma} = \int\int\int \mu\, a^{\alpha}\, b^{\beta}\, c^{\gamma}\, da\, db\, dc$$

auf.[6] Es ist klar, dass in die Entwickelung nach Kugelfunktionen (die ja im Grunde nichts anderes ist, als eine eigenartige Anordnung und Zusammenfassung der einzelnen Glieder der binomischen

[5] Die Bezeichnung, deren ich mich hier bediene, weicht etwas von derjenigen, die G a u s s enutzt hat, ab. Dieser setzt

$$P^{n,\,m}(\cos\sigma) = \sin\sigma^m\Big[\cos\sigma^{n-m} - \frac{(n-m)\,(n-m-1)}{2.\,(2n-1)}\cos\sigma^{n-m-2} + \frac{(n-m)\,(n-m-1)\,(n-m-2)\,(n-m-3)}{2.\,4.\,(2n-1)\,(2n-3)}\cos\sigma^{n-m-4} - \ldots.\Big]$$

Damit verglichen ist hier

$$P_m^n(\cos\sigma) = 1.\,3.\,5\ldots.\,(2n-1)\sqrt{\frac{\epsilon_m}{(n+m)\,!\,(n-m)\,!}}\,P^{n,\,m}(\cos\sigma)$$

mit $\epsilon_0 = 1,\ \epsilon_1 = \epsilon_2 = \ldots. = 2$

Man hat also beispielsweise, wenn kurz P_m^n für $P_m^n(\cos\sigma)$, ferner c für $\cos\sigma$ und s für $\sin\sigma$ geschrieben wird:

$$P_0^0 = 1 \qquad P_0^1 = c \qquad P_1^1 = s \qquad P_0^2 = \tfrac{3}{2}(c^2 - \tfrac{1}{3}) \qquad P_1^2 = \sqrt{3}\,sc$$
$$P_2^2 = \tfrac{1}{2}\sqrt{3}\,s^2 \qquad P_0^3 = \tfrac{5}{2}(c^3 - \tfrac{3}{5}c) \qquad P_1^3 = \tfrac{5}{4}\sqrt{6}\,s\,(c^2 - \tfrac{1}{5})$$
$$P_2^3 = \tfrac{1}{2}\sqrt{15}\,s^2 c \qquad P_3^3 = \tfrac{1}{4}\sqrt{10}\,s^3$$

ferner

$$P_0^4 = \tfrac{35}{8}(c^4 - \tfrac{6}{7}c^2 + \tfrac{3}{35}) \qquad P_0^5 = \tfrac{63}{8}(c^5 - \tfrac{10}{9}c^3 + \tfrac{5}{21}c)$$
$$P_0^6 = \tfrac{231}{16}(c^6 - \tfrac{15}{11}c^4 + \tfrac{5}{11}c^2 - \tfrac{5}{231})$$
$$P_0^7 = \tfrac{429}{16}(c^7 - \tfrac{21}{13}c^5 + \tfrac{105}{143}c^3 - \tfrac{35}{429}c)$$
$$P_0^8 = \tfrac{6435}{128}(c^8 - \tfrac{28}{15}c^6 + \tfrac{14}{13}c^4 - \tfrac{28}{143}c^2 + \tfrac{7}{1287})$$
$$P_0^9 = \tfrac{12155}{128}(c^9 - \tfrac{36}{17}c^7 + \tfrac{126}{85}c^5 - \tfrac{84}{221}c^3 + \tfrac{63}{2431}c)$$

Lässt man überall den zu Anfang stehenden Zahlenfaktor weg, so erhält man die Gaussischen Funktionen. Fügt man andererseits den Faktor $\sqrt{2n+1}$ hinzu, so ergeben sich die früher einmal von mir unter der Bezeichnung R_m^n verwendeten Funktionen. (Vgl. *Terr. Magn.* Vol. I, S. 73.)

[6] Nimmt man Flächenbelegungen, speziell eine solche der Oberfläche, an, so ist das Integral natürlich dementsprechend zu bilden. Statt $\mu\, da\, db\, dc$ ist $\mu'\, ds$ zu setzen, wenn μ' die Flächendichte und ds das Flächenelement bezeichnet. Denkt man die Beschaffenheit des Magnets, was physikalisch sachgemässer ist, durch die Intensität seiner Magnetisierung definiert, deren Komponenten nach den Achsen A, B, C sein mögen, so ist es doch zweckmässig, zur Bestimmung der F erst auf die äquivalente Verteilung von freiem Magnetismus überzugehen. Diese besitzt im Innern die Raumdichte $-\left(\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z}\right)$ und an der Oberfläche, wenn die Richtungskosinus der nach aussen gehenden Normale α, β, γ sind, die Flächendichtigkeit $\mu' = (A\alpha + B\beta + C\gamma)$.

Digitized by Google

Entwicklung) dieselben Ausdrücke eingehen müssen, und dass somit die M und N lineare Aggregate der F bilden. Diese lassen sich auch leicht angeben. Setzt man für den Augenblick

$$a = R\cos s \qquad b = R\sin s\cos t \qquad c = R\sin s\sin t$$

so ist, wenn dv das Volumenelement $R^2\sin s\,dR\,ds\,dt$ bezeichnet und die Integration sich wieder über den ganzen Magnet erstreckt,

$$(4)\qquad M_{nm} = \int \mu\, R^n P_m^n(\cos s)\cos n\,t\,dv$$

$$N_{nm} = \int \mu\, R^n P_m^n(\cos s)\sin n\,t\,dv$$

Die Ausdrücke unter dem Integrationszeichen gehen durch die umgekehrte Substitution in ganze homogene Funktionen n. Grades von a, b, c über und liefern damit ohne weiteres die gesuchte Darstellung. So wird z. B.

$$(5)\quad \begin{cases} M_{10} = F_{100} & \\ M_{11} = F_{010} & N_{11} = F_{001} \\ M_{20} = F_{200} - \frac{1}{2}(F_{020} + F_{002}) & \\ M_{21} = \sqrt{3}\,F_{110} & N_{21} = \sqrt{3}\,F_{101} \\ M_{22} = \frac{1}{2}\sqrt{3}\,(F_{020} - F_{002}) & N_{22} = \sqrt{3}\,F_{011} \\ M_{30} = F_{300} - \frac{3}{2}(F_{120} + F_{102}) & \\ M_{31} = \frac{1}{4}\sqrt{6}\,(4F_{210} - F_{030} - F_{012}) & \\ \qquad N_{31} = \frac{1}{4}\sqrt{6}\,(4F_{201} - F_{021} - F_{003}) & \\ M_{32} = \frac{1}{2}\sqrt{15}\,(F_{120} - F_{102}) & N_{32} = \sqrt{15}\,F_{111} \\ M_{33} = \frac{1}{4}\sqrt{10}\,(F_{030} - 3F_{012}) & \\ \qquad N_{33} = \frac{1}{4}\sqrt{10}\,(F_{021} - F_{003}) & \end{cases}$$

Insbesondere ist allgemein

$$(5a)\quad \begin{cases} M_{n0} = F_{n00} - \dfrac{n(n-1)}{2\,.\,2}(F_{n-2,2,0} + F_{n-2,0,2}) \\ \qquad + \dfrac{n(n-1)(n-2)(n-3)}{2.2.4.4}(F_{n-4,4,0} + 2F_{n-4,2,2} \\ \qquad\qquad + F_{n-4,0,4}) + \cdots\cdots \end{cases}$$

Digitized by Google

Da das Koordinatensystem zunächst willkürlich gelassen wurde, so ist es von Interesse, die Wirkung einer daran vorgenommenen Aenderung zu untersuchen. Im allgemeinen werden dadurch natürlich sämtliche Parameter M und N gleichfalls geändert. Betrachtet man aber $M_{10}\ M_{11}\ N_{11}$ als Komponenten eines Vektors nach den drei Koordinatenachsen, so zeigt sich, dass dieser Vektor gegenüber jeder orthogonalen Transformation invariant ist, dass er also eine vom Koordinatensystem unabhängige, charakteristische Eigenschaft des Magnets darstellt. Der absolute Betrag dieses Vektors heisst nach der von Gauss gegebenen Definition das *Moment des Magnets* $M = \sqrt{M_{10}\ M_{10} + M_{11}\ M_{11} + N_{11}\ N_{11}}$; seine Richtung wird als die der *magnetischen Achse* bezeichnet. Gibt man der x-Achse diese Richtung, so wird demnach $M_{10} = M$ und $M_{11} = 0$, $N_{11} = 0$. Ueberdies kann man dann offenbar noch durch Drehung des Systems um diese Achse einen der vier Werte M_{21}, N_{21}, M_{22}, N_{22} zu Null machen. Es empfiehlt sich, einen der beiden letzten zu wählen oder allgemeiner $M_{21} \cos 2\tau + N_{22} \sin 2\tau = N \cos (2\tau - \alpha)$ mit einem beliebig angenommenen α zu setzen. Verlegt man nun ohne Änderung der Achsenrichtungen den Anfangspunkt, so kann man es (ausser bei einem astatischen Magnetsystem, bei dem $M = 0$ ist) stets erreichen, dass gleichzeitig M_{20}, M_{21}, N_{21} verschwinden, während neben M auch M_{22} und N_{22} ungeändert bleiben. Der neue Anfangspunkt, für den dies gilt, heisst nach Thomson[7] und Riecke[8] der *magnetische Mittelpunkt* des Magnets. Nennt man seine Koordinaten im alten System a_0, b_0, c_0, so leitet man aus den vorhergehenden Formeln leicht ab, dass im allgemeinen Falle (d. h. wenn die Achsenrichtungen noch beliebig sind)

$$(6)\quad a_0 = \frac{M_{10}\ M_{20} + \frac{1}{2}\sqrt{3}\ (M_{11}\ M_{21} + N_{11}\ N_{21})}{2\ M_{10}\ M_{10} + M_{11}\ M_{11} + N_{11}\ N_{11}}$$

$$b_0 = \frac{\frac{1}{2}\sqrt{3}\ M_{21} - a_0\ M_{11}}{M_{10}} \qquad c_0 = \frac{\frac{1}{2}\sqrt{3}\ N_{21} - a_0\ N_{11}}{M_{10}}$$

ist. Hat man das Koordinatensystem bereits vorher nach der magnetischen Achse orientiert, so wird demnach einfacher

$$(6a)\quad a_0 = \frac{M_{20}}{2\ M} \qquad b_0 = \frac{\sqrt{3}\ M_{21}}{3\ M} \qquad c_0 = \frac{\sqrt{3}\ N_{21}}{3\ M}$$

[7] SIR W. THOMSON: *Reprint of Papers on Electrostatics and Magnetism.* § 494. London, 1872. (Zusatz aus dem Jahre 1871 zu der Arbeit: A Mathematical Theory of Magnetism. *Proc. Roy. Soc.* 1849.)

[8] E. RIECKE: Zur Lehre von den Polen eines Stabmagnets. *Annalen der Physik und Chemie*, Neue Folge, 8. Bd. 1879, S. 299.

Digitized by Google

In bezug auf das neue System lautet alsdann der Potentialausdruck:

$$(7)\quad \Pi = \frac{M}{r^2} P_0^1(\cos\sigma) + \frac{N}{r^3} P_2^2(\cos\sigma)\cos(2\tau - \alpha) + \ldots\ldots\ldots$$

Es möge dies die *kanonische Form des Potentials* heissen, und ebenso möge das zugehörige Koordinatensystem als *kanonisch* bezeichnet werden.

So wertvoll die geschilderte Transformation in mancher Hinsicht auch ist (besonders durch die Einführung und scharfe Definition der Begriffe des Moments, der Achse und des Mittelpunkts), so muss doch betont werden, dass sie gerade für exakte Messungen nur sekundäre Bedeutung besitzt. Das kanonische Achsensystem lässt sich praktisch nur durch seine Beziehung auf ein anderes, vorher irgendwie angenommenes festlegen, es ist im allgemeinen nicht bequem gelegen und mit der Zeit möglicherweise ebenso wie das Moment etwas veränderlich. Deswegen soll im Folgenden stets ein an sich beliebiges, aber in der früher angegebenen Weise sicher festgelegtes System vorausgesetzt werden. Die so erhaltenen allgemeinen Resultate können natürlich nachträglich beliebig spezialisiert, also auch auf die kanonische Form angewendet werden.

Wichtiger, als bei dem Koordinatensystem, erweist sich die Betrachtung einiger besonderen Fälle bei dem Magnet. Es sind deren hauptsächlich zwei zu nennen, die bei weiterer Spezialisierung beide auf den freilich nicht mehr streng zu verwirklichenden linearen Magnet mit seinem wichtigsten Sonderfalle, dem schematischen Magnet, führen, der endlich seinerseits im Grenzfall zum Elementarmagnet wird. Diese zwei Fälle sind der prismatische und der rings um die Achse symmetrische Magnet.

A. Der prismatische Magnet ist ein solcher, bei dem in allen Querschnitten die magnetische Dichte nach demselben Gesetz verteilt ist. Ebensowohl kann man sagen, dass sie in allen Längslinien in gleicher Weise verteilt sei. Es ist also $\mu = \bar{\mu}.\nu$ zu setzen, worin ν eine Funktion von b und c allein ist, während $\bar{\mu}$ nur von a abhängt. Am zweckmässigsten ist es, unter $\bar{\mu}$ die gesamte Magnetisierung des Querschnitts (genauer gesagt, unter $\bar{\mu}\,da$ die zwischen den Querschnitten bei a und $a + da$ vorhandene Magnetismusmenge) zu verstehen. Aus dem allgemeinen Ansatz $\mu = \bar{\mu}.\nu$ folgt

$$(8)\quad F_{\alpha\beta\gamma} = \int \bar{\mu}\, a^\alpha\, da \,.\, \int \nu\, b^\beta\, c^\gamma\, db\, dc = M_\alpha \,.\, \phi_{\beta\gamma}$$

Digitized by Google

so dass sämtliche M_{nm} und N_{nm} lineare Aggregate der Grössen M_n, M_{n-1} . . . werden. Dabei ist speziell $\phi_{00}=1$, wenn $\bar{\mu}$ in der angegebenen Weise definiert wird.

B. Der achsensymmetrische Magnet ist ein solcher, bei dem μ nur von a und dem Abstande $\rho = \sqrt{b^2 + c^2}$, nicht aber von dem Winkel τ abhängt. Aus den allgemeinen Formeln ergibt sich hier ohne weiteres

$$(9)\quad M_{n0} = f_{n,0} - \frac{n\,(n-1)}{2\,.\,2}\, f_{n-2,2} + \frac{n\,(n-1)\,(n-2)\,(n-3)}{2\,.\,2\,.\,4\,.\,4}$$

mit $f_{n-4,4} - \ldots .$

$$(9a)\quad f_{n-\alpha,\alpha} = 2\pi \int\int \mu\, a^{n-\alpha}\, \rho^{\alpha+1}\, da\, d\rho$$ und speziell

$$f_{n,0} = \int \bar{\mu}\, a^n\, da = M_n \ .$$

wenn $\bar{\mu} = \int \mu\, d\,(\pi\rho^2)$ gesetzt wird, so dass es dieselbe Bedeutung hat, wie im vorigen Falle.

Von den weiteren, bereits oben erwähnten Sonderfällen verdient vor allem

C. der lineare Magnet schon deswegen Interesse, weil er, wie bemerkt, bisher fast ausschliesslich Beachtung gefunden hat. Er geht aus *A)* wie aus *B)* hervor, wenn man den Querschnitt unendlich klein annimmt. Es verschwinden dann dort alle $\phi_{\beta\gamma}$ ausser ϕ_{00}, das allgemein zu 1 wird, und hier alle $f_{n-\alpha,\alpha}$ ausser $f_{n,0}$. Damit ergibt sich übereinstimmend

$$(10)\quad M_{n0} = \int \bar{\mu}\, a^n\, da = M_n$$

und das Potential wird

$$(11)\quad \Pi = \sum_1^\infty{}_n\ M_n\, r^{-n-1}\, P_0^n\ (\cos\sigma)$$

Die Koeffizienten M_n in dieser Reihe sind, wie man aus der sie definierenden Gleichung ersieht, identisch mit den von Lamont unter derselben Bezeichnung eingeführten Grössen. Insbesondere wird auch hier $M_1 = M$, dem magnetischen Moment. Durch weitere Spezialisierung ergibt sich

D. der schematische Magnet, wie ich einen solchen nennen will, der nur aus zwei Polpunkten besteht.[9] Ist deren Abstand gleich $2l$ und die in ihnen konzentriert gedachte Magnetismus-

[9] Von Lamont einfacher Magnet, von Riecke Elementarmagnet genannt. Der Ausdruck schematischer Magnet findet sich bei Fr. Kohlrausch: Ueber die Berechnung der Fernwirkung eines Magnets. *Sitz. Ber. Akad. München*, 1887, wiederabgedruckt in den *Ann. der Phys. u. Chem.* N. F. 31. Bd. 1887. S. 609.

Digitized by Google

menge $+\bar{\mu}$ und $-\bar{\mu}$ (zum Unterschiede von der als μ bezeichneten Dichte des Magnetismus so genannt), so erhält man durch einfachen Grenzübergang aus der für M_n abgeleiteten Formel

$$(12) \quad M_{2n+1} = 2\bar{\mu}\, l^{2n+1} = M\, l^{2n} \text{ mit } M = 2\bar{\mu}\, l\,;\; M_{2n} = 0$$

Die Annahme $2l = 0$ liefert dann endlich als Grenzfall

E. den Elementarmagnet, dessen Potential sich auf

$$(13) \quad \Pi = \frac{M}{r^2} P_0^1 (\cos\sigma) = \frac{M}{r^2} \cos\sigma$$

reduziert.

Die Bedeutung von *D)* und *E)* liegt im wesentlichen darin, dass man sie an Stelle wirklicher Magnete gesetzt denken kann, um die von diesen ausgehenden Kräfte in grösserer oder geringerer Annäherung zu berechnen.

Noch grössere Annäherung lässt sich natürlich mit dem linearen Magnet erreichen, der unendlich viele und, wie man leicht einsieht, unabhängige Parameter zur Verfügung stellt. Ja, hieraus folgt weiter, dass er den achsensymmetrischen Magnet sogar in aller Strenge vertreten kann. Vergleicht man die Fälle *B)* und *C)* mit einander, so sieht man, dass in beiden der Potentialausdruck von derselben Form ist, indem er allein von den zonalen Kugelfunktionen P_0^n abhängt. Allerdings sind die Koeffizienten M_{no} und M_n verschieden gebildet, wenn man sie aus der Magnetisierungsverteilung ableitet. Da diese aber tatsächlich unbekannt ist (ganz abgesehen davon, dass ihr gar keine physikalische Wirklichkeit zukommt), so ist dieser Unterschied ohne Bedeutung. Die Parameter sind individuelle Konstanten, die für jeden Magnet besonders bestimmt werden müssen.

Aus dem Gesagten ergibt sich, dass jeder achsensymmetrische Magnet restlos durch einen linearen vertreten werden kann, nämlich durch denjenigen, der mit ihm identische Parameter besitzt. Dies ist von grosser praktischer Wichtigkeit, denn es findet auf alle zylindrischen und hohlzylindrischen Magnete Anwendung. Die für diese gelegentlich berechneten Korrektionen zur Berücksichtigung ihrer Querdimensionen sind daher im Grunde überflüssig. Eine sachliche Bedeutung hätten sie nur dann, wenn wir etwas Genaues über die sogenannte Verteilung des Magnetismus im Magnet wüssten. Ja, jene Aequivalenz kann sogar mit beliebiger Annäherung auch bei nicht achsensymmetrischen Magneten herbeigeführt werden, wenn man diese durch Drehung um ihre Achse in eine hinreichende Anzahl von äquidistanten Lagen bringt

Digitized by Google

und das Mittel der in diesen einzelnen Lagen beobachteten Wirkungen bestimmt. (Weicht die magnetische Achse von der Drehungsachse merklich ab, ist mit andern Worten der Winkel zwischen beiden, die Kollimation σ', nicht sehr klein, so ist das für jene mittlere Wirkung massgebende M_1 nicht gleich dem Moment M, sondern gleich $M \cos \sigma'$. Das ist beispielsweise bei der Kombination von Ablenkungs- und Schwingungsbeobachtungen zu beachten, da unter diesen Umständen im allgemeinen nicht bei beiden dasselbe Moment wirksam ist.)

Eine weitere Vereinfachung ist bei jedem Magnet durch Polvertauschung, d. h. durch halbe Umdrehung um eine zur Längsachse senkrechte, den Mittelpunkt enthaltende Linie zu erzielen. Im Durchschnitt der mit entgegengesetztem Vorzeichen genommenen Wirkungen in den beiden so erhaltenen Lagen fallen offenbar alle Glieder fort, in denen gerade Funktionen auftreten, und es bleibt als Potential

$$(14) \quad \Pi = \frac{M_1}{r^2} P_0^1(\cos\sigma) + \frac{M_3}{r^4} P_0^3(\cos\sigma) + \frac{M_5}{r^6} P_0^5(\cos\sigma) + \ldots$$

Ein Magnet, dessen Potential diese Gestalt hat, möge *regulär* heissen, und es gilt nach dem Gesagten der Satz, dass sich im Mittel der bezeichneten Lagen jeder beliebige Magnet wie ein regulärer verhält. Der Wert des ersten Koeffizienten M_1 möge, wenn es nötig erscheint, ihn ausdrücklich von M zu unterscheiden, als das zu dem speziell gewählten Achsensystem gehörige oder kurz als das wirksame (oder effektive) Moment bezeichnet werden.

In dem Spezialfalle des schematischen Magnets ist nun $M_3 : M_1 = l^2$, mit l als dem halben Polabstande. In Anlehnung hieran pflegt man dieselbe Beziehung allgemein zur Definition des *Polabstandes* zu benutzen. Auch hier wird man, wenn die Kollimation nicht zu vernachlässigen ist, genauer von der zu dem speziellen Achsensystem gehörigen oder von der wirksamen Poldistanz sprechen. Freilich verliert dieser Ausdruck seine anschauliche Bedeutung, wenn (M_1 als absolute Grösse betrachtet) M_3 negativ ist; denn in diesem, wohl zuerst von R i e c k e (a. a. O.) hervorgehobenen und eingehend erörterten Falle ergibt sich natürlich die Poldistanz, wenn man diesen Begriff dann noch beibehalten will, als imaginär.

Setzt man nun weiter $M_{2n+1} : M_1 = \kappa_{n-1} \cdot l^{2n}$, wobei sämtliche κ reine Zahlen (speziell $\kappa_0 = 1$) sind, so nimmt der Potentialausdruck die Form an:

Digitized by Google

$$(15)\quad \Pi = \frac{M_1}{r^2}\left(P_0^1(\cos\sigma) + \frac{l^2}{r^2}P_0^3(\cos\sigma) + \kappa_1\frac{l^4}{r^4}P_0^5(\cos\sigma) + \kappa_2\frac{l^6}{r^6}P_0^7(\cos\sigma) + \ldots\right)$$

Die Koeffizienten κ_1, κ_2, . . . , die bei einem schematischen Magnet sämtlich gleich 1 sind, wie Formel (11) zeigt, mögen die *Verteilungsfaktoren* heissen.[10] Sie sind bestimmend für die Gestaltung des magnetischen Feldes. In der Tat ändert sich diese, solange die κ ungeändert bleiben, nicht bei wechselndem M und l. Die dabei entstehenden Felder sind ähnliche, nur im Längen- und Kraftmasstab verschiedene Gebilde. Auch das Feld des Elementarmagnets stellt nur den Grenzfall der unendlichen Verkleinerung dar. Dagegen beeinflusst jede Aenderung der κ den geometrischen Charakter des Feldes.

Wie M und l hängen auch die κ von dem gewählten Koordinatensystem ab. Die mitgeteilten Formeln (5) ermöglichen ohne weiteres die Ableitung der durch eine Koordinatentransformation bedingten Veränderungen dieser Grössen. Um den Zusammenhang nicht zu sehr zu unterbrechen, gehe ich auf diese Entwickelungen, die geringe praktische Bedeutung haben, hier nicht näher ein. Von einiger Wichtigkeit sind höchstens die Beziehungen zwischen den kanonischen und den effektiven Konstanten, in Bezug auf die es genügen mag, ein besonders einfaches Resultat zu erwähnen. Stimmen die Achsenrichtungen beider Systeme überein, während ihre Anfangspunkte (der magnetische und der geometrische Mittelpunkt) in der Richtung der magnetischen Achse um die Strecke a_0 von einander entfernt sind, so gilt für die zugehörigen Polabstände $2l_0$ und $2l$ die Beziehung $l^2 = l_0^2 + 3a_0^2$, während das Moment M, wie schon früher bemerkt wurde, dadurch nicht berührt wird. Besteht gleichzeitig eine seitliche Verschiebung vom Betrage b_0, so wird $l^2 = l_0^2 + 3a_0^2 - \frac{3}{2}b_0^2$, eine Gleichung, die eine einfache und interessante geometrische Deutung zulässt. Beim Vergleich dieser Angabe mit dem entsprechenden von Riecke a. a. θ. abgeleiteten Ausdruck ist zu beachten, dass dort von S. 312 an die Bedeutung von α und β gegenüber der Festsetzung auf S. 304 vertauscht ist. Auch sind die zuerst auf S. 313 in den Formeln für l^2 auftretenden Faktoren 6 und 6 in $\frac{3}{2}$ und 4 zu ändern.

[10] Die Einführung der Verteilungsfaktoren setzt voraus, dass l von 0 verschieden sei und wird natürlich schon dann unzweckmässig, wenn l im Verhältnis zur Länge des Magnets sehr klein ist. Da dies indessen ein kaum vorkommender Ausnahmefall ist, so wird dadurch die Bedeutung dieser Faktoren als charakteristischer Magnetkonstanten nicht geschmälert. Der Faktor κ_1 tritt bei Chwolson (S. 17, 18) unter der Bezeichnung $\bar{a}^4 : a^4$ auf; dabei ist a das, was hier l heisst.

Digitized by Google

Die Pole eines Magnets.

Der im Vorhergehenden benutzte Begriff der Poldistanz setzt den der *Pole* bereits voraus. Bei einem regulären Magnet kann kein Zweifel darüber bestehen, welche Punkte man als seine Pole zu bezeichnen hat. Es sind die beiden Punkte, die auf der durch den magnetischen Mittelpunkt parallel zur magnetischen Achsenrichtung gehenden Geraden nach der einen und der anderen Seite um den halben Polabstand vom Mittelpunkt entfernt liegen.

Damit ergibt sich aber auch ungezwungen eine einfache und zweckmässige Definition für die Pole eines beliebigen Magnets. Man wird als solche diejenigen des regulären Magnets wählen, der ihm im Mittel der früher bezeichneten Lagen äquivalent ist. Da dieser von der Wahl des Koordinatensystems abhängt, so ist diese Festsetzung allerdings nicht eindeutig; soll sie es sein, so muss man die Bedingung hinzufügen, dass das kanonische System gewählt werde. Wenn man schlechtweg von den Polen eines bestimmten Magnets spricht, so muss dies strenggenommen immer vorausgesetzt werden. Die so getroffene Festsetzung hat den Vorteil, dass die danach bestimmten Pole für alle Wirkungen des Magnets gelten. Denkt man in ihnen die magnetischen Massen $+ M : 2 l_0$ und $- M : 2 l_0$ angebracht, so ersetzt der dadurch gebildete schematische Magnet den tatsächlich vorhandenen (soweit dieser als regulär gelten darf) mit weitgehender Annäherung. Lässt man ihn für diesen eintreten, so bewirkt dies im Potential nur den Fehler

$$(\kappa_1 - 1)\,\frac{M l_0^4}{r^6} + (\kappa_2 - 1)\,\frac{M l_0^6}{r^8} + \ldots .$$

während die beiden ersten Glieder, die r^{-2} und r^{-4} als Faktor enthalten, vollkommen dargestellt werden.

Bei bestimmten Anwendungen (z. B. wenn der Magnet als Ablenkungsstab dient) empfiehlt es sich aber, ebenso wie bei dem Moment und dem Polabstand, daneben den Begriff der für diesen speziellen Fall wirksamen Pole einzuführen. Als solche sind offenbar in notwendiger Folge der früheren Festsetzungen diejenigen Punkte zu bezeichnen, die die Koordinaten $+ l$, 0, 0 und $- l$, 0, 0 in bezug auf das gerade gewählte Koordinatensystem besitzen. Bei regelmässig gestalteten Magneten wird man dabei natürlich stets, wenn nichts anderes ausdrücklich gesagt wird, das durch die Gestalt ausgezeichnete System voraussetzen. Aber es darf nicht übersehen werden, dass auch dann die effektiven Pole mit den

Digitized by Google

vorher definierten eigentlichen Polen (die man, wenn es nötig sein sollte, sie ausdrücklich hervorzuheben, die kanonischen nennen könnte) wegen der Kollimation und der Exzentrizität des Magnets im allgemeinen nicht identisch sind.

Dagegen bleibt den effektiven Polen die wichtige Eigenschaft erhalten, dass sie zur Darstellung des Potentials im ganzen Raume und daher auch zur Ableitung der Feldkomponenten in jedem Punkte mit der zuvor angegebenen Näherung ausreichen.

Abweichend hiervon hat man manchmal die Pole so definiert, dass sich der von ihnen gebildete schematische Magnet dem gegebenen an einer bestimmten Stelle des Raumes noch besser anschliesst, was natürlich für den übrigen Raum umgekehrt zu einer Verschlechterung der Darstellung führt.[11] Es ist dies teils ganz bewusst geschehen (so in den Arbeiten von Riecke und Kohlrausch), teils liegt es mehr oder weniger klar der von Börgen und anderen erörterten Frage zu Grunde, ob die Poldistanz von der Entfernung abhängig sei.

Der Vorteil, den man bei diesen Darstellungen gewinnt, einen Koeffizienten im Potentialausdruck zu ersparen, hat natürlich nur bei mässigen Ansprüchen an Genauigkeit Bedeutung, denn andernfalls muss man ja von der Einführung eines äquivalenten schematischen Magnets überhaupt absehen. Aber auch in diesem Falle ist er im allgemeinen geringer, als der Nachteil, dass die beiden verbleibenden charakteristischen Grössen (M und l) aufhören, im strengen Sinne Konstanten zu sein. Das angedeutete Verfahren empfiehlt sich höchstens im Falle der Hauptlagen, auf den ich mich deshalb hier beschränken will.

Im allgemeinen Falle lautet die Aufgabe für einen achsensymmetrischen Magnet offenbar so: welche Werte müssen M und l erhalten, damit in einem bestimmten Punkte $(r_0\ \sigma_0)$

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \Pi^1}{\partial r} \qquad \frac{\partial \Pi}{r \partial \sigma} = \frac{\partial \Pi^1}{r \partial \sigma^1}$$

gilt, wenn

$$\Pi = \frac{M_1}{r^2} P_0^1 (\cos \sigma) + \frac{M_3}{r^4} P_0^3 (\cos \sigma) + \frac{M_5}{r^6} P_0^5 (\cos \sigma)$$

und

$$\Pi' = \frac{M}{r^2} P_0^1 (\cos \sigma) + \frac{M l^2}{r^4} P_0^3 (\cos \sigma) + \frac{M l^4}{r^6} P_0^5 (\cos \sigma)$$

ist. Beschränkt man sich auf die Hauptlagen, in denen je eine

[11] Man könnte die vorhergehende Festsetzung als speziellen Fall hiervon betrachten, nämlich als denjenigen, bei dem die Stelle des engsten Anschlusses im Unendlichen liegt.

Digitized by Google

Komponente identisch verschwindet, so kann man noch ein weiteres Glied in der Entwickelung hinzunehmen oder, was vorzuziehen ist, ein für allemal $M = M_1$ setzen und nur l als Unbekannte einführen, wie man es in der Tat auch bisher stets getan hat. Berücksichtigt man, dass in der 1. Hauptlage $\sigma_0 = 0$, in der 2. dagegen $\sigma_0 = \frac{\pi}{2}$ ist, so erhält man mit Benutzung der in Anmerkung (5) gegebenen Ausdrücke durch eine einfache Rechnung die hinreichend genauen Näherungsformeln:

$$(16)\begin{cases}\text{für die 1. Hauptlage: } l_1^2 = (M_3 + \frac{3}{2} r_0^{-2} M_5) : (M_1 + \frac{3}{2} r_0^{-2} M_3) \\ \text{für die 2. Hauptlage: } l_2^2 = (M_3 - \frac{5}{4} r_0^{-2} M_5) : (M_1 - \frac{5}{4} r_0^{-2} M_3)\end{cases}$$

oder in anderer Form mit $l_0^2 = M_3 : M_1$

$$(16a)\quad l_1 = l_0\left(1 + \frac{3}{4}(\kappa_1 - 1)\left(\frac{l_0}{r_0}\right)^2\right) \quad l_2 = l_0\left(1 - \frac{5}{8}(\kappa_1 - 1)\left(\frac{l_0}{r_0}\right)^2\right)$$

woraus nebenbei, wenn l_1 und l_2 experimentell bestimmt werden,

$$l_0 = (5 l_1 + 6 l_2) : 11 \qquad \kappa_1 = 1 + \frac{8}{11}\left(\frac{r_0}{l_0}\right)^2 \frac{l_1 - l_2}{l_0}$$

abgeleitet werden kann.

Von grösserem Interesse ist ein anderer, von Riecke behandelter Fall, bei dem das Potential des gegebenen Magnets zu

$$\Pi = \frac{M_1}{r^2} P_0^1(\cos\sigma) + \frac{M_{30}}{r^4} P_0^3(\cos\sigma) + \frac{M_{32}}{r^4} P_2^3(\cos\sigma)\cos 2\tau$$

und unter der stillschweigenden Annahme von $M = M_1$ die Bestimmung von l so vorgenommen wird, dass am Orte $(r_0\, \sigma_0\, \tau_0)$

$$\Pi' = \frac{M}{r^2} P_0^1(\cos\sigma) + \frac{Ml^2}{r^4} P_0^3(\cos\sigma)$$

an die Stelle von Π treten kann. Man erkennt ohne weiteres, dass die Lösung hier nur von σ_0 und τ_0, nicht aber von der Entfernung r_0 abhängt. (Dasselbe sagt der von Riecke a. a. O. S. 312 bewiesene Satz aus.) Es ist dies der einfachste und im allgemeinen der einzige praktisch wichtige Fall eines nicht rings um die Achse symmetrischen Magnets, beispielsweise eines solchen von rechteckigem Querschnitt.[12]

[12] Insbesondere ist hier das in der Praxis übliche Verfahren einzureihen, wonach der Ablenkungsstab durch Polvertauschung und einfaches Umlegen (Vertauschen von oben und unten bei horizontaler Lage) in vier Stellungen benutzt wird.

Digitized by Google

Beschränkt man sich wieder auf die Hauptlagen und bei der 2. überdies auf die durch $\tau_0 = 0$ (oder $= \pi$) und $\tau_0 = \frac{\pi}{2}$ (oder $= \frac{3\pi}{2}$) definierten Symmetrieebenen, so findet man

$$(17)\begin{cases}\text{für die 1. Hauptlage:} & l_1^2 = M_{30} : M_1 \\ \text{für die 2. Hauptlage:} & l_2^2 = (M_{30} \mp \frac{1}{3}\sqrt{15}\, M_{32}) : M_1\end{cases}$$

wobei das obere Zeichen für die 1. und das untere für die 2. Ebene gilt.

Drückt man die Parameter M_{30} und M_{32} mittels der früher angegebenen Formeln (5) durch die Grössen $F_{\alpha\beta\gamma}$ aus, so erhält man die folgenden Werte, die mit den von R i e c k e abgeleiteten übereinstimmen:

$$(17a)\quad \begin{aligned} l_1^2 &= [F_{300} - \tfrac{3}{2}(F_{120} + F_{102})] : M \\ l_2^2 &= [F_{300} - 4\ F_{120} + F_{102}\ ] : M \end{aligned}$$

Ein Umstand verdient noch besondere Beachtung. Findet der Ersatz des gegebenen Magnets durch einen schematischen, wie in den beiden behandelten Fällen, nur für einen bestimmten Punkt statt, so gilt er auch nur für die Wirkung auf einen unendlich kleinen Magnet. Es ist nicht ohne weiteres zu übersehen, ob bei der später zu behandelnden Einwirkung zweier Magnete von endlicher Ausdehnung auf einander jeder durch einen schematischen ersetzt werden kann, der nach den hier gefundenen Formeln zu bestimmen ist, der also nur von seinen eigenen Parametern, nicht auch von denen des andern abhängt.

Berechnung der Parameter unter besonderen Annahmen über die magnetische Verteilung.

Wie schon betont wurde, sind die Parameter eines jeden Magnets, ganz abgesehen von ihrer Abhängigkeit von der Temperatur und anderen äusseren Bedingungen, durchaus als individuelle Konstanten zu betrachten, die wenigstens bei scharfen Beobachtungen stets eigens zu bestimmen oder zu eliminieren sind. Ihre Berechnung aus der hypothetischen Verteilung der Magnetisierung führt zu blossen Näherungsformeln, da diese Verteilung nicht genau bekannt ist. Trotzdem sind die in dieser Richtung bisher vielfach ausgeführten Berechnungen keineswegs ohne Wert, da bequeme und exakte Methoden zur experimentellen Bestimmung jener Konstanten noch fast ganz fehlen, so dass man im allgemeinen darauf angewiesen bleibt, sich mit derartigen Näherungen zu

Digitized by Google

begnügen. Und so lange nicht sehr hohe Ansprüche an die Genauigkeit der Messungen gestellt werden, reichen diese Näherungen auch wohl aus; denn es zeigt sich, dass alle mit den bisherigen Erfahrungen verträglichen Annahmen über die magnetische Verteilung in Magneten der üblichen Formen zu nahe übereinstimmenden Ergebnissen führen. Es sollen deshalb hier die Formeln für die wichtigsten Fälle zusammengestellt werden, im wesentlichen unter Beschränkung auf diejenigen, die bisher schon vorzugsweise behandelt worden sind.

Der einfachste Fall ergibt sich aus der Annahme von Polpunkten. Hierauf hat sich Lamont in seinem *Handbuch des Magnetismus* im wesentlichen beschränkt, wenn er auch die allgemeineren Entwickelungen kurz andeutet, die er früher, so besonders in seinem *Handbuch des Erdmagnetismus*, mitgeteilt hatte. Dieselbe Beschränkung üben durchaus Chwolson und Börgen in ihren bereits erwähnten Arbeiten, ebenso neuerdings Leyst[13] in einer sehr eingehenden und sorgfältigen Untersuchung, in der er sich noch ein besonderes Verdienst durch die Verbesserung mancher Ungenauigkeiten erwirbt, von denen einige der bisher veröffentlichten Formeln nicht ganz frei sind.

A) Die spezielle Annahme von zwei Polpunkten führt auf den Fall des schematischen Magnets, über den bereits alles Nötige gesagt ist. Das Ergebnis der Parameterberechnung ist hier dadurch charakterisiert, dass die Koeffizienten κ sämtlich gleich 1 werden.

B) Einen Schritt weiter geht Chwolson, der 8 gleichstarke Polpunkte in den Ecken eines rechtwinkligen Parallelepipedons annimmt, um einigermassen die Querdimensionen der wirklichen Magnete zu berücksichtigen. Sind die Seiten der Figur $2a$, $2b$, $2c$,[14] wobei $2a$ parallel der magnetischen Achse sein soll, und nennt man die Magnetismusmenge in jeder Ecke $+\bar{\mu}$ oder $-\bar{\mu}$, so sieht man ohne weiteres, dass diejenigen $F_{\alpha\beta\gamma}$ verschwinden, in denen ein gerades α oder ein ungerades β oder γ vorkommt. Ferner ergibt sich allgemein

$$F_{2\lambda+1,\,2\mu,\,2\nu} = 8\,a^{2\lambda+1}\,b^{2\mu}\,c^{2\nu}\,\bar{\mu} = M\,a^{2\lambda}\,b^{2\mu}\,c^{2\nu}$$

[13] E. Leyst: Ueber erdmagnetische Ablenkungsbeobachtungen. Moskau, 1910. (Sonderabdruck). Aus einigen Bemerkungen (S. 81 u. 113) könnte man schliessen, dass L. die erwähnte Beschränkung, die auf die Annahme hinauskommt, sämtliche κ seien gleich 1, als einen Fortschritt ansehe.

[14] So mag zur Vereinfachung überall statt $2a_0$, $2b_0$, $2c_0$ geschrieben werden, wo eine Verwechselung mit den in den allgemeinen Formeln auftretenden variabeln Koordinaten a, b, c nicht zu befürchten ist. Ebenso soll, soweit es unbedenklich ist, l für l_0 geschrieben werden.

Digitized by Google

und danach

$$(18)\quad \begin{cases} M_{10} = M = 8\,a\,\bar{\mu} \qquad M_{30} = M\left(a^2 - \frac{3}{2}(b^2+c^2)\right) \\ M_{32} = \frac{1}{2}\sqrt{15}\,M\,(b^2-c^2) \qquad M_{50} = M\left(a^4 - 5\,a^2\,(b^2+c^2) + \frac{15}{8}(b^2+c^2)^2\right) \text{ u. s. w.} \end{cases}$$

Das Quadrat des halben Polabstands wird

$$l^2 = a^2 - \frac{3}{2}(b^2+c^2)$$

Setzt man $(b^2+c^2) : a^2 = \nu^2$, so wird, wenn ν^2 eine kleine Grösse ist, genähert

$$(18a)\quad l = a\,(1 - \tfrac{3}{4}\nu^2 \ldots) \qquad \kappa_1 = M_{50} : M\,l^4 = 1 - 2\,\nu^2 \ldots$$
$$\kappa_2 = M_{70} : M\,l^6 = 1 - 6\,\nu^2 \ldots$$

Für die Aequivalentpole erhält man

$$(18b)\quad \begin{cases} l_1^2 = a^2 - \frac{3}{2}(b^2+c^2) \\ l_2^2 = a^2 - 4\;b^2 + c^2 \end{cases}$$

Die Formeln ändern sich nur wenig, wenn man mit Börgen zur Berücksichtigung der Querdimensionen Polflächen mit einer gesetzmässigen Verteilung der Magnetisierung annimmt.[15] Da er in seiner Abhandlung die in Betracht kommenden Integrale, aus denen ohne weiteres die $F_{\alpha\beta\gamma}$ folgen, ausführlich für alle Fälle angibt, so lassen sich seine Resultate unmittelbar zur Berechnung der M_{nm} verwenden und damit in die hier gewählte Form bringen. Es ist deshalb überflüssig, sie alle anzuführen; es mag genügen, diejenigen anzugeben, die für den Fall homogener Flächenmagnetisierung, d. h. für μ = const gelten. Darüber hinaus möge noch, wie im vorhergehenden Beispiel, überall M_{50} abgeleitet wer-

[15] C. Börgen: Ueber den Einfluss der körperlichen Dimensionen eines Magnets auf die durch denselben aus beliebiger Lage hervorgebrachte Ablenkung einer Nadel. *Aus dem Archiv der Deutschen Seewarte*, XVIII, Nr. 5. (1895). Die Frage wird hier sehr eingehend sowohl für eine homogene Magnetisierung der Endflächen (μ = const) als auch für eine solche nach dem Gesetz $\mu = \mu_0\left[1 + \gamma\,(y:b)^{2n} + \gamma\,(z:d)^{2n}\right]$ behandelt. (d ist die halbe Dicke des Stabs, also das, was hier c genannt ist.) Die entsprechenden Fälle werden dann auch für zylindrische und hohlzylindrische Stäbe erledigt. Die Berechnung unter der Annahme (μ = const) gibt auch Chwolson am Schlusse seiner in Anm. 3 erwähnten Arbeit.

Will man die in diesen Untersuchungen zu findenden Ergebnisse mit den hier abgeleiteten vergleichen, so muss man beachten, dass dort die Einwirkung des Stabs auf eine gleichfalls ausgedehnte Nadel, hier zunächst diejenige auf einen Elementarmagnet dargestellt wird. Eine vollständige Vergleichung kann daher erst an den weiterhin folgenden Ausdrücken durchgeführt werden.

Digitized by Google

den, damit wenigstens der erste der Verteilungsfaktoren, κ_1, bestimmt werden kann. (μ bezeichnet hier die Flächendichte.)

C) Sind die Polflächen Rechtecke mit den Seiten $2b$ und $2c$, so folgt sofort $M = F_{100} = 8abc\mu$ und allgemein für $\alpha - 1 \equiv \beta \equiv \gamma \equiv 0 \bmod 2$

$$F_{\alpha\beta\gamma} = M\, a^{\alpha-1}\, b^{\beta}\, c^{\gamma} : (\beta+1)\,(\gamma+1)$$

also

$$(19)\quad \begin{cases} M_{20} = M\left(a^2 - \frac{1}{2}(b^2+c^2)\right) \qquad M_{22} = \frac{1}{6}\sqrt{15}\, M\,(b^2-c^2) \\ M_{40} = M\left(a^4 - \frac{5}{3}a^2(b^2+c^2) + \frac{1}{24}(9b^4 + 10b^2c^2 + 9c^4)\right) \end{cases}$$

Danach ist $l^2 = l_1^2 = a^2 - \frac{1}{2}(b^2+c^2) \qquad l_2^2 = a^2 - \frac{4}{3}b^2 + \frac{1}{3}c^2$

(19a) und $l = a\,(1 - \frac{1}{4}\nu^2 \ldots)$, $\kappa_1 \doteq 1 - \frac{2}{3}\nu^2 \ldots$, wenn wieder $(b^2+c^2) : a^2 = \nu^2$ gesetzt wird.

D) Sind die Polflächen Kreise vom Radius ρ, so ergibt sich $M = 2\pi\rho^2 a\mu$ und es bietet keine Schwierigkeit, die bei der Berechnung der F auftretenden Integrale auszuwerten. Doch ist dies nicht nötig; die M_{nm} lassen sich einfacher durch sinngemässe Anwendung der Formeln (9) gewinnen, die hier in

$$f_{n-\alpha,\,\alpha} = 4\pi\mu\, a^{n-\alpha}\, \rho^{\alpha+2} : (\alpha+2) = 2M\, a^{n-\alpha-1}\, \rho^{\alpha} : (\alpha+2)$$

übergehen. Es ist also

$$f_{30} = Ma^2 \quad f_{12} = \frac{1}{2}M\rho^2 \quad f_{50} = Ma^4 \quad f_{32} = \frac{1}{2}Ma^2\rho^2 \quad f_{14} = \frac{1}{3}M\rho^4$$

(20) mithin $M_{20} = M\,(a^2 - \frac{3}{4}\rho^2) \quad M_{40} = M\,(a^4 - \frac{5}{2}a^2\rho^2 + \frac{5}{8}\rho^4)$

und, wenn $\rho : a = \nu$ gesetzt wird, für kleine ν

(20a) $\quad l^2 = a^2 - \frac{3}{4}\rho^2 \qquad l = a\,(1 - \frac{3}{8}\nu^2 \ldots) \qquad \kappa_1 = 1 - \nu^2 \ldots$

E) Aus diesen Resultaten lassen sich auch ohne weiteres die entsprechenden Formeln für einen Hohlzylinder ablesen. Ist in diesem der äussere Durchmesser $2\rho_1$, der innere $2\rho_2$, so folgt

$$(21)\quad \begin{cases} M = f_{10} = 2\pi\mu a\,(\rho_1^2 - \rho_2^2) \qquad f_{30} = Ma^2 \qquad f_{12} = \frac{1}{2}M\,(\rho_1^2 + \rho_2^2) \quad \text{u. s. w.} \\ M_{20} = M\left(a^2 - \frac{3}{4}(\rho_1^2 + \rho_2^2)\right) \\ M_{40} = M\left(a^4 - \frac{5}{2}a^2(\rho_1^2+\rho_2^2) + \frac{5}{8}(\rho_1^4 + \rho_1^2\rho_2^2 + \rho_2^4)\right) \end{cases}$$

Damit sind wieder l_0 und κ_1 gegeben.

Digitized by Google

F) Denkt man sich, wie es Leyst tut, die magnetisierte Ringfläche unendlich schmal, so dass sie sich auf einen Kreis vom Radius ρ konzentriert, so gilt mithin, $\rho : a = \nu$ gesetzt,

$$(22) \quad M_{30} = M\,(a^2 - \frac{3}{2}\rho^2) \qquad M_{50} = M\,(a^4 - 5\,a^2\rho^2 + \frac{15}{8}\rho^4)$$

$$(22a) \quad l^2 = a^2 - \frac{3}{2}\rho^2 \qquad l = a\,(1 - \frac{3}{4}\nu^2 \ldots) \qquad \kappa_1 = 1 - 2\,\nu^2 \ldots$$

In einem gewissen Gegensatz zu den im Vorhergehenden behandelten Fällen, in denen die Magnetisierung auf zwei getrennte Polregionen beschränkt erscheint, steht die Annahme einer Verteilung über die ganze Länge des zunächst als Linie betrachteten Magnets. Schon Lambert und Hansteen sind von dieser Anschauung ausgegangen; dann hat sich Lamont ihrer bei seinen grundlegenden Untersuchungen bedient, und ihm ist Fritsche[16] in einer weiter durchgeführten Darstellung gefolgt.

Da die allgemeinen Formeln für die Parameter eines linearen Magnets, um den es sich hier handelt, schon im ersten Abschnitt unter *C*) angegeben worden sind, so bleibt nur noch die von der speziellen Annahme über das Verteilungsgesetz abhängige Auswertung zu erledigen. Zwei solche Annahmen über die Abhängigkeit der linearen Dichte $\bar{\mu}$ von dem (wenn $2a_0$ die Länge des Magnets ist, zwischen $+a_0$ und $-a_0$ schwankenden) Abstande a vom Mittelpunkt sind fast ausschliesslich untersucht worden und sollen daher auch hier allein betrachtet werden.

Einerseits hat man, mit k als einer positiven Konstanten, $\bar{\mu} = Ca^k$ gesetzt, wofür übrigens korrekter $\bar{\mu} = Ca\,\mathbf{a}^{k-1}$, unter $\mathbf{a}$ den absoluten Wert von a verstanden, geschrieben werden müsste. Dies haben Hansteen (der sich die Aufgabe stellte, k, bei ihm r genannt, aus den Fundamentalversuchen zugleich mit dem Exponenten des Kraftwirkungsgesetzes zu bestimmen) und Fritsche getan, während sich Lambert und Lamont auf die Betrachtung des Falles $k = 1$ beschränken. Andrerseits wurde nach Biot's Versuchen für μ die Gleichung $\bar{\mu} = C\,(e^{qa} - e^{-qa})$ angenommen. Für diesen Ansatz hat Lamont[17] die Ausdrücke für die ersten vier Parameter und einige daraus folgende Re-

[16] H. Fritsche: Ueber die Bestimmung der geographischen Länge und Breite und der drei Elemente des Erdmagnetismus durch Beobachtung zu Lande sowie erdmagnetische und geographische Messungen an mehr als tausend verschiedenen Orten in Asien und Europa, ausgeführt in den Jahren 1867-1891. St. Petersburg, 1893.

[17] In den entsprechenden Formeln Lamont's finden sich (ausser leicht zu erkennenden Druckfehlern) mehrere nicht im Verzeichnis der Verbesserungen angeführte Fehler. Auf S. 43 ist im Faktor von *B* überall das vorletzte Glied mit $\frac{1}{2}$ zu multiplizieren so dass die Koeffizienten $\frac{1}{2}$, 30, 1260 an Stelle von 1, 20, 840 auftreten. Auf S. 44 muss im Zähler der Formel für $M_n : M$ der Koeffizient von $c^4 l^4$ den Nenner 21120 statt 4120 haben.

Digitized by Google

sultate in seinem *Handbuch des Erdmagnetismus* (§§ 32 und 33) ausführlich entwickelt.

G) Ist (im oben präzisierten Sinne) $\bar{\mu} = C a^k$ oder, in physikalischer Hinsicht zweckmässiger, $\bar{\mu} = m\,(a:a_0)^k = m\,x^k$, so liefert Formel (10)

$$(23)\quad M = M_1 = \frac{2}{k+2}\, m a_0^2 \qquad M_{2n+1} = \frac{k+2}{2n+k+2}\, M a_0^{2n}$$

$$(23a)\quad l^2 = \frac{k+2}{k+4} a_0^2 \qquad \varkappa_1 = \frac{(k+4)^2}{(k+2)\,(k+6)} = 1 + \frac{4}{(k+2)\,(k+6)}$$

Man kann erfahrungsgemäss annehmen, dass im allgemeinen k zwischen 1 und 2 liegt. Für $k=1$ ergibt sich $M = \frac{2}{3}\, m\, a_0^2$, $l = 0.77\ a_0$ und $\varkappa_1 = 1.19$, für $k=2$ nur wenig davon verschieden $M = \frac{1}{2}\, n\, a_0^2$, $l = 0.82\ a_0$ und $\varkappa_1 = 1.12$.

H) Geht man von dem Verteilungsgesetze $\bar{\mu} = C\,(e^{sa} - e^{-sa})$ aus, das man wiederum besser mit $a:a_0 = x$ in der abgeänderten Form $\bar{\mu} = \frac{m}{2s}(e^{sx} - e^{-sx})$ schreibt, so ergibt sich, wenn noch $e^s + e^{-s} = A$ und $e^s - e^{-s} = B$ gesetzt wird,[17]

$$M = M_1 = m a_0^2 s^{-3}\,(A\,s - B)$$
$$M_3 = m a_0^4 s^{-5}\,[A\,(s^3 + 6s) - B\,(3s^2 + 6)]$$
$$M_5 = m a_0^6 s^{-7}\,[A\,(s^5 + 20 s^3 + 120 s) - B\,(5 s^4 + 60 s^2 + 120)]$$

oder u. s. w.

$$(24)\quad \begin{cases} M_1 = \frac{2}{3} m a_0^2 (1 + \frac{1}{16} s^2 + \frac{1}{280} s^4 \ldots) \\ M_3 = \frac{2}{5} m a_0^4 (1 + \frac{5}{42} s^2 + \frac{1}{216} s^4 \ldots) \\ M_5 = \frac{2}{7} m a_0^6 (1 + \frac{7}{54} s^2 + \frac{7}{1320} s^4 \ldots) \end{cases}$$

und somit

$$(24a)\quad l = \sqrt{\tfrac{1}{3}}\, a_0 \left(1 + \frac{1}{105} s^2 \ldots\right) \qquad \varkappa_1 = \frac{25}{21}\left(1 - \frac{8}{945} s^2 \ldots\right)$$

Nach den vorliegenden Erfahrungen darf man annehmen, dass im allgemeinen s ein echter Bruch ist und sogar meistens unter $\frac{1}{2}$ liegt. Für sehr kleine Werte von s wird $\bar{\mu} = mx$, was denselben Spezialfall ergibt, wie die Annahme $k = 1$ bei der vorigen Hypothese.

Es ist von Interesse und kann als typisch angesehen werden, dass die Annahme endlicher Polflächen statt blosser Polpunkte

Digitized by Google

wie in den Fällen *A*) . . . *F*), den Faktor κ_1 unter 1 herabdrückt, während ihn die Annahme einer Längsmagnetisierung, wie bei *G*) und *H*), über 1 wachsen lässt. Man kann danach schon vermuten, dass κ_1 bei der tatsächlich stattfindenden räumlichen Magnetisierung im allgemeinen nicht sehr von 1 verschieden sein wird.

Die einfachsten Fälle einer derartigen Verteilung haben Chwolson und Fritsche behandelt. Die von ihnen betrachteten parallelepipedischen und zylindrischen Stäbe ordnen sich dem hier unter *A*), im ersten Abschnitt, definierten prismatischen Magnet unter. Die für sie geltenden Ausdrücke lassen sich ohne weiteres aus den dort angegebenen Formeln (8) und (9) gewinnen, indem man für M_α die hier in den Fällen *G*) und *H*) abgeleiteten Werte einsetzt. Zur Berechnung der $\phi_{\beta\gamma}$ hat man nur zu beachten, dass bei dem Prisma $\nu = 1 : 4 b_0 c_0$, bei dem Zylinder $\nu = 1 : \pi \rho^2$ gilt. Noch einfacher kommt man zum Ziele, wenn man in den Resultaten der Fälle *C*) . . . *F*), die dieselben $\phi_{\beta\gamma}$ enthalten, während in ihnen $M_\alpha = M a_0^{\alpha-1}$ ist, statt dieses letzteren Wertes den in *G*) oder *H*) berechneten einführt. Beschränkt man sich, wie die genannten Forscher, auf homogene Quermagnetisierung und die Annahme $\bar{\mu} = m\,(a : a_0)^k$, so findet man auf diese Weise, wenn wieder einfach a, b, c statt a_0, b_0, c_0 geschrieben wird, Folgendes:

I) Im Falle eines Magnets von rechteckigem Querschnitt

$$(25)\quad \begin{cases} M = \dfrac{2}{k+2} m a^2 \qquad M_{30} = M\left(\dfrac{k+2}{k+4} a^2 - \dfrac{1}{2}(b^2 - c^2)\right) \\ M_{32} = \dfrac{1}{6}\sqrt{15}\, M\,(b^2 - c^2) \\ M_{50} = M\left(\dfrac{k+2}{k+6} a^4 - \dfrac{5}{3}\dfrac{k+2}{k+4} a^2 (b^2 + c^2) + \dfrac{1}{24}(9 b^4 + 10 b^2 c^2 + 9 c^4)\right) \end{cases}$$

und danach, wenn wieder $(b^2 + c^2) : a^2 = \nu^2$ gesetzt wird,

$$(25a)\quad \begin{cases} l^2 = l_1^2 = \dfrac{k+2}{k+4} a^2 - \dfrac{1}{2}(b^2 + c^2) \qquad l_2^2 = \dfrac{k+2}{k+4} a^2 - \dfrac{4}{3} b^2 + \dfrac{1}{3} c^2 \\ l = a\sqrt{\dfrac{k+2}{k+4}}\left(1 - \dfrac{1}{4}\dfrac{k+4}{k+2}\nu^2 \ldots\right) \\ \kappa_1 = \dfrac{(k+4)^2}{(k+2)(k+6)}\left(1 - \dfrac{2}{3}\dfrac{k^2 + 8k + 6}{(k+2)(k+4)}\nu^2 \ldots\right) \end{cases}$$

Speziell $k = 1$ gibt

$$l = a\sqrt{\tfrac{3}{5}}\,(1 - \tfrac{5}{12}\nu^2 \ldots) \qquad \kappa_1 = \tfrac{25}{21}(1 - \tfrac{2}{3}\nu^2 \ldots)$$

Digitized by Google

K) Bei einem zylindrischen Magnet ist

$$26\quad \begin{cases} M=\dfrac{2}{k+2}\,m\,a^2 \qquad M_3=M\left(\dfrac{k+2}{k+4}a^2-\dfrac{3}{4}\rho^2\right) \\ M_5=M\left(\dfrac{k+2}{k+6}a^4-\dfrac{5}{2}\dfrac{k+2}{k+4}a^2\rho^2+\dfrac{5}{8}\rho^4\right) \end{cases}$$

und für $\rho:a=\nu$

$$(26a)\quad \begin{cases} l=a\sqrt{\dfrac{k+2}{k+4}}\left(1-\dfrac{3}{8}\dfrac{k+4}{k+2}\nu^2\ldots\right) \\ \kappa_1=\dfrac{(k+4)^2}{(k+2)\,(k+6)}\left(1-\dfrac{k^2+8\,k+6}{(k+2)\,(k+4)}\nu^2\ldots\right) \end{cases}$$

Ist insbesondere $k=1$, so gibt dies

$$l=a\sqrt{\tfrac{3}{5}}\,(1-\tfrac{5}{8}\nu^2\ldots) \qquad \kappa_1=\frac{25}{21}\,(1-\nu^2\ldots)$$

Diese Resultate bestätigen, dass sich in κ_1 die Wirkungen der longitudinalen und der transversalen Verteilung mehr oder weniger kompensieren. In l wirken beide dagegen in gleicher Richtung, nämlich im Sinne einer Verkürzung des Polabstandes.

L) Zum Schluss möge nun noch der theoretisch interessante Fall des gleichmässig magnetisierten Ellipsoids behandelt werden. Sind dessen Halbachsen a, b, c, und bezeichnen A, B, C die entsprechenden Komponenten der Intensität der Magnetisierung, so findet man, wenn das Volumen $\frac{4\pi}{3}\,a\,b\,c=V$ und $VA=\alpha M$, $VB=\beta M$, $VC=\gamma M$ gesetzt wird, so dass M das Moment und α, β, γ die Richtungskosinus der magnetischen Achse sind,

$$F_{100}=\alpha M \quad F_{300}=\frac{3}{5}\alpha M a^2 \quad F_{120}=\frac{1}{5}\alpha M b^2 \quad F_{500}=\frac{3}{7}\alpha M a^4$$

$$F_{320}=\frac{3}{35}\alpha M a^2 b^2 \quad F_{140}=\frac{3}{35}\alpha M b^4 \quad F_{122}=\frac{1}{35}\alpha M b^2 c^2$$

und entsprechend $F_{010}=\beta M$ $F_{030}=\frac{3}{5}\beta M b^2$ $F_{210}=\frac{1}{5}\beta M a^2$ u. s. w.

Daraus ergibt sich

$$M_{10}=\alpha M \quad M_{11}=\beta M \quad N_{11}=\gamma M$$

$$M_{30}=\frac{3}{5}\alpha M\left(a^2-\frac{1}{2}(b^2+c^2)\right)$$

$$(27)\quad M_{31}=\frac{1}{20}\sqrt{6}\beta M\,(4a^2-3b^2-c^2) \quad N_{31}=\frac{1}{20}\sqrt{6}\gamma M\,(4a^2-b^2-3c^2)$$

$$M_{32}=\frac{1}{10}\sqrt{15}\,\alpha M\,(b^2-c^2) \qquad N_{32}=0$$

$$M_{33}=\frac{3}{20}\sqrt{10}\,\beta M\,(b^2-c^2) \qquad N_{33}=\frac{3}{20}\sqrt{10}\,\gamma M\,(b^2-c^2)$$

Digitized by Google

Bei einem Rotationsellipsoid ist $b = c$ und man erhält

$$(28) \quad M_{30} = \frac{3}{5}\alpha M(a^2 - b^2) \qquad M_{31} = \frac{1}{5}\sqrt{6}\beta M(a^2 - b^2)$$

$$N_{31} = \frac{1}{5}\sqrt{6}\gamma M(a^2 - b^2)$$

während M_{32}, M_{33} und N_{33} zu Null werden. Durch Drehung des Achsensystems kann man natürlich auch noch M_{31} oder N_{31} zum Verschwinden bringen.[18] Wird schliesslich noch $b = a$, also das Ellipsoid zur Kugel, so wird bekanntlich das Potential mit demjenigen eines Elementarmagnets identisch, und in der Tat gehen dann auch die vorstehenden Ausdrücke in Null über, und es bleiben nur die Parameter der ersten Ordnung übrig.

Fällt die Achse der Magnetisierung mit der einen Achse des zunächst wieder als dreiachsig angenommenen Ellipsoids zusammen, ist also etwa $\beta = 0$, $\gamma = 0$, so ergeben sich die spezialisierten Formeln

(29)

$$M_{10} = M \quad M_{30} = \frac{3}{5}M\left(a^2 - \frac{1}{2}(b^2 + c^2)\right) \quad M_{32} = \frac{1}{10}\sqrt{15}\,M(b^2 - c^2)$$

$$M_{50} = \frac{3}{7}M\left(a^4 - a^2(b^2 + c^2) + \frac{1}{8}(3b^4 + 2b^2c^2 + 3c^4\right) \text{u.s.w.}$$

die zu den folgenden, bereits von Riecke berechneten äquivalenten Poldistanzen führen:

$$(29a) \quad l^2 = l_1^2 = \frac{3}{5}\left(a^2 - \frac{1}{2}(b^2 + c^2)\right) \qquad l_2^2 = \frac{3}{5}\left(a^2 - \frac{4}{3}b^2 - \frac{1}{3}c^2\right)$$

Der besondere Fall des Rotationsellipsoids gibt mit $b = c = \rho$ und $\rho : a = \nu$

$$l^2 = \frac{3}{5}(a^2 - \rho^2) \quad l = a\sqrt{\frac{3}{5}}\left(1 - \frac{1}{2}\nu^2 \ldots\right) \quad \kappa_1 = \frac{25}{21}(1 + \nu^2 \ldots)$$

Je nachdem a grösser oder kleiner als ρ, das Ellipsoid also gestreckt oder abgeplattet ist, wird l^2 positiv oder negativ, l selbst mithin reell oder imaginär. In den beiden durch $\rho = 0$ und $a = 0$ definierten Grenzfällen erhält man $l = a\sqrt{\frac{3}{5}}$ und $l = \rho\sqrt{-\frac{3}{5}}$. Bei einem Kreiszylinder, für den eine homogene Magnetisierung mit einer Belegung der Endflächen gleichwertig ist, so dass Formel

[18] Diesen Ergebnissen kommt eine gewisse praktische Bedeutung zu, da sich Rotationsellipsoide ohne grössere technische Schwierigkeiten herstellen lassen. Vgl. darüber die Angaben von Benedicks, *Ann. der Phys.* 4. Folge, Bd. 6, S. 730 (1901). Allerdings beziehen sich diese auf Stäbe aus weichem Eisen; bei Stahlmagneten wird man wohl damit rechnen müssen, dass der Erfolg nicht selten durch die beim Härten eintretenden Gestaltsveränderungen vereitelt werden wird.

Digitized by Google

(20a) Anwendung findet, ergeben die entsprechenden Annahmen $l = a$ und $l = \rho\sqrt{-\frac{1}{4}}$. Die manchmal vorgenommene Uebertragung der für das Rotationsellipsoid geltenden Resultate auf dünne Drähte und flache kreisförmige Scheiben gibt hiernach, wenigstens bei dem Polabstand, nur eine recht grobe Annäherung.

Zum Schluss sei noch bemerkt, dass sich natürlich jede starre Verbindung mehrerer Magnete (wie z. B. eine Kompassrose oder ein astatisches Nadelpaar) als ein einheitliches System betrachten lässt, das ebenso wie ein einfacher Magnet durch bestimmte Parameter charakterisiert ist. Diese sind lineare Aggregate der Parameter seiner einzelnen Bestandteile und aus ihnen leicht mit Hilfe der Gleichungen (3) und (5) zu berechnen, in einigen besonders einfachen Fällen auch aus einzelnen späteren Formeln unmittelbar abzulesen.

Das Potential von Stromkreisen und Stromspulen.

Verhältnismässig selten verwendet man bei erdmagnetischen Messungen stromdurchflossene Spulen an Stelle von permanenten Magneten. In manchen Fällen sind sie in der Tat weniger geeignet; aber es gibt eine Reihe von Aufgaben, bei denen sie durchaus mit Vorteil benutzt werden könnten, und wenn es nicht geschieht, so liegt der Grund wohl hauptsächlich an äusseren, technischen Schwierigkeiten oder Unbequemlichkeiten, die zu überwinden sich verlohnen würde.

Gegenüber einem Stahlmagneten besitzt eine Stromspule zwei wichtige Vorzüge. Ihr Potential ist, von seinem absoluten Betrage abgesehen, vollkommen durch die Gestalt und Anordnung der Strombahnen bestimmt, also durch rein geometrische Ausmessung zu ermitteln, und die bei allen exakten Messungen zur Elimination systematischer Fehler erwünschte Umkehrung der Wirkung lässt sich auf die einfachste und zugleich vollkommenste Weise durch die Umkehr der Stromrichtung erzielen. Dem steht als ein wesentlicher Nachteil gegenüber, dass der absolute Betrag des Potentials und damit aller Wirkungen von einer Grösse, der Stromstärke, abhängt, die nicht leicht mit der nötigen Schärfe absolut zu messen oder auch nur längere Zeit hindurch konstant zu halten ist.

Die Anwendung von Stromspulen wird daher vorzugsweise bei relativen Messungen angezeigt sein; insbesondere eignet sie sich vorzüglich zu Nullmethoden.

Digitized by Google

Im Anschluss an die vorausgehenden Formelübersichten für Magnete sollen deshalb hier auch die wichtigsten entsprechenden Zusammenstellungen für Stromspulen gegeben werden. Indessen wird es genügen, dabei nur Kreisströme zu berücksichtigen, die praktisch weitaus die grösste Bedeutung haben. Von den verschiedenen Formen der analytischen Darstellung, die dabei möglich sind, kommt hier allein diejenige durch Kugelfunktionen in Betracht, wie sie Maxwell[19] gegeben hat.

A) Es handle sich zunächst um einen einfachen Kreisstrom. Sein Radius sei gleich ρ, die Drahtdicke sei zu vernachlässigen. Auf der im Zentrum *O* auf der Ebene des Kreises senkrecht stehenden Achse liege im Abstand a von *O* der Punkt *A*. Positiv sei a auf derjenigen Seite des Raumes, von der aus gesehen der positive Strom *I* (in Ampères gemessen $10I$) gegen den Uhrzeiger läuft. *A* werde nun zum Ursprung eines Polarkoordinatensystems (r, σ, τ) gewählt, in dem speziell die Punkte der Strombahn die Koordinaten $r = c, \sigma = \alpha$ haben mögen. Es ist also $\cos \alpha = -a : c$ und $\sin \alpha = \rho : c$.

Die durch die Strombahn gehende Kugelfläche mit dem Mittelpunkt *A* teilt nun den Raum in zwei Gebiete, in deren jedem eine besondere Entwickelung für das Potential gilt.

Im Aussenraum, d. h. für $r > c$ ist

$$(30)\quad \Pi = 2\pi I \sin\alpha^2 \Big[\frac{1}{2}\frac{c^2}{r^2} P_1'(\alpha)\, P_1(\sigma) + \frac{1}{3}\frac{c^3}{r^3} P_2'(\alpha)\, P_2(\sigma) + \ldots . + \frac{1}{n+1}\frac{c^{n+1}}{r^{n+1}} P_n'(\alpha)\, P_n(\sigma) + \ldots\Big]$$

im Innenraum, d. h. für $r < c$ dagegen (bis auf eine hier gleichgültige, nur beim Vergleich beider Entwickelungen auf der Kugelfläche selbst eine Rolle spielende Konstante)

$$(31)\quad \Pi = -2\pi I \sin\alpha^2 \Big[\frac{r}{c} P_1'(\alpha)\, P_1(\sigma) + \frac{1}{2}\frac{r^2}{c^2} P_2'(\alpha)\, P_2(\sigma) + \ldots . + \frac{1}{n}\frac{r^n}{c^n} P_n'(\alpha) P_n(\sigma) + \ldots\Big]$$

Dabei ist $P_n(\sigma) = P_0^n(\cos\sigma) \qquad P_n'(\alpha) = \dfrac{d\,P_0^n(\cos\alpha)}{d\cos\alpha}$

[19] J. Cl. Maxwell: *Treatise on Electricity and Magnetism.* 1873. Auch in deutscher Uebersetzung durch Weinstein unter dem Titel: *Lehrbuch der Electricität und des Magnetismus.* 1883. Die hier benutzten Formeln finden sich im 2. Bande, § 695.

Digitized by Google

Lässt man A mit O zusammenfallen, sodass $a = \frac{\pi}{2}$ und $c = \rho$ wird, so gehen diese Ausdrücke in die folgenden über:

$$(32)\quad \mathrm{II} = 2\pi I \left[\frac{1}{2}\frac{\rho^2}{r^2} P_0^1(\cos\sigma) - \frac{1.3}{2.4}\frac{\rho^4}{r^4} P_0^3(\cos\sigma) + \ldots\ldots + (-1)^s \frac{1.3\ldots\ldots(2s+1)}{2.4\ldots\ldots(2s+2)}\frac{\rho^{2s+2}}{r^{2s+2}} P_0^{2s+1}(\cos\sigma) + \ldots\right]$$

für $r > \rho$

$$(33)\quad \mathrm{II} = -2\pi I \left[\frac{r}{\rho} P_0^1(\cos\sigma) - \frac{1}{2}\frac{r^3}{\rho^3} P_0^3(\cos\sigma) + \ldots\ldots + (-1)^s \frac{1.3\ldots\ldots(2s-1)}{2.4\ldots\ldots 2s}\frac{r^{2s+1}}{\rho^{2s+1}} P_0^{2s+1}(\cos\sigma) + \ldots\right]$$

für $r < \rho$

Nur der erste Fall findet im allgemeinen sein Analogon bei Stahlmagneten, während bei ihnen der zweite, der bei Strömen gerade von besonderer Bedeutung ist, überhaupt nur ausnahmsweise (z. B. bei Hohlmagneten) vorkommt und auch dann kaum jemals in Betracht kommt. Indessen gilt diese Bemerkung nur, wenn das zu Grunde liegende Koordinatensystem, wie hier, zentriert ist.

Vergleicht man nun den ersten Ausdruck mit dem entsprechenden früher für reguläre Magnete aufgestellten kanonischen Potential (vgl. Formel (14)), so ergeben sich ohne weiteres die Beziehungen

$$(34)\quad M_1 = \pi\rho^2 I \quad M_3 = -\frac{3}{4}\pi\rho^4 I \quad M_5 = \frac{5}{8}\pi\rho^6 I \quad M_7 = -\frac{35}{64}\pi\rho^8 I$$

u. s. w.

oder

$$(34a)\quad M = \pi\rho^2 I \quad l^2 = -\frac{3}{4}\rho^2 \qquad \kappa_1 = \frac{10}{9} \qquad \kappa_2 = \frac{140}{81}$$

Die Poldistanz ist also imaginär, $l = \rho\sqrt{-\frac{3}{4}}$, und die den Verteilungsfaktoren analogen Grössen κ sind grösser als 1 und steigen mit wachsendem Index stark an. Diese Ergebnisse stimmen natürlich mit denen bei einem unendlich niedrigen Zylinder überein.

Um ihrer praktischen Wichtigkeit willen mögen einige Folgerungen aus den vorstehenden Formeln angeführt werden. Für die Feldstärke im Punkte $(r, 0)$ der Achse ergibt sich, da dort allgemein $P_0^n(\cos\sigma) = 1$ ist,

Digitized by Google

$$(35)\quad \begin{cases} X = -\dfrac{\partial \Pi}{\partial r} = 2\pi I\left[\dfrac{\rho^2}{r^3} - \dfrac{3}{2}\dfrac{\rho^4}{r^5} + \dfrac{3.5}{2.4}\dfrac{\rho^6}{r^7} \ldots\ldots\right] & \text{für } r > \rho \text{ und} \\ X = -\dfrac{\partial \Pi}{\partial r} = 2\pi I\left[\dfrac{1}{\rho} - \dfrac{3}{2}\dfrac{r^2}{\rho^3} + \dfrac{3.5}{2.4}\dfrac{r^4}{\rho^5} \ldots\ldots\right] & \text{für } r < \rho \end{cases}$$

Beide Ausdrücke lassen sich offenbar in der Form

$$X = 2\pi\rho^2 I : (r^2+\rho^2)^{\frac{3}{2}}$$

schreiben und liefern damit den bekannten Wert der Feldstärke in der Achse eines Kreisstromes.

Für einen Punkt $\left(r, \frac{\pi}{2}\right)$ in der Ebene des Stromes andrerseits findet man die dort gleichfalls der Achse parallele Feldstärke ausgedrückt durch

$$(36)\quad \begin{cases} X = \dfrac{\partial \Pi}{r d\sigma} = -\dfrac{2\pi\rho^2 I}{r^3}\left[\dfrac{1}{2} + \dfrac{9}{16}\dfrac{\rho^2}{r^2} + \dfrac{75}{128}\dfrac{\rho^4}{r^4} + \ldots\ldots \right. \\ \qquad\qquad \left. + (2s+2)\, c_{s+1}^2 \dfrac{\rho^{2s}}{r^{2s}} + \ldots\ldots\right] & \text{für } r > \rho \text{ und} \\ X = \dfrac{\partial \Pi}{r d\sigma} = \dfrac{2\pi I}{\rho}\left[1 + \dfrac{3}{4}\dfrac{r^2}{\rho^2} + \dfrac{45}{64}\dfrac{r^4}{\rho^4} + \ldots\ldots \right. \\ \qquad\qquad \left. + (2s+1)\, c_s^2 \dfrac{r^{2s}}{\rho^{2s}} + \ldots\ldots\right] & \text{für } r < \rho. \end{cases}$$

Zur Abkürzung ist hier c_s an Stelle von

$$(-1)^s \frac{1.3 \ldots\ldots (2s-1)}{2.4 \ldots\ldots 2s}$$

geschrieben worden. Für $s = 0$ hat c_s, das nichts anderes, als der zum Exponenten $\left(-\frac{1}{n}\right)$ gehörige s-te Binomialkoeffizient ist, den Wert 1.

Die vorstehenden Ausdrücke lassen sich leicht in die folgenden umwandeln, die wesentlich schneller als sie, und im Gegensatz zu ihnen auch noch für $r = \rho$ konvergieren.

$$(37)\quad \begin{cases} X = -\dfrac{2\pi\rho^2 I}{r(r^2-\rho^2)}\left[\dfrac{1}{2} + \dfrac{1}{16}\dfrac{\rho^2}{r^2} + \dfrac{3}{128}\dfrac{\rho^4}{r^4} + \ldots\ldots \right. \\ \qquad\qquad \left. + \dfrac{1}{2s+2}\, c_s^2 \dfrac{\rho^{2s}}{r^{2s}} + \ldots\right] & \text{für } r \geq \rho \text{ und} \\ X = \dfrac{2\pi\rho I}{\rho^2 - r^2}\left[1 - \dfrac{1}{4}\dfrac{r^2}{\rho^2} - \dfrac{3}{64}\dfrac{r^4}{\rho^4} - \ldots\ldots \right. \\ \qquad\qquad \left. - \dfrac{1}{2s-1}\, c_s^2 \dfrac{r^{2s}}{\rho^{2s}} - \ldots\right] & \text{für } r \leq \rho\,. \end{cases}$$

Digitized by Google

Ist $r = \rho$, so gehen beide Reihen in den Klammern in den Wert $(2 : \pi)$ über.

B) Durch Kombination mehrerer von demselben Strome durchflossenen Kreise lassen sich Felder von beliebig vorgeschriebener Beschaffenheit herstellen, die wertvolle Anwendung finden können. Beschränkt man sich, was fast immer das Gegebene sein wird, auf konaxiale Anordnung, also auf achsensymmetrische Felder, so reichen die vorstehenden Formeln zur Berechnung aus.

In dem einfachsten und wichtigsten Falle einer Spule mit gleichmässigen und so engen Windungen, dass man diese als Kreise betrachten und die auftretenden Summen durch Integrale ersetzen darf, findet man so für den Aussenraum, d. h. für $r^2 > l^2 + \rho^2$,

$$(38)\quad \mathrm{II} = \frac{2\pi\rho^2 N l}{r^2}\left[P_0^1(\cos\sigma) + \frac{l^2}{r^2}\left(1 - \frac{3}{4}\frac{\rho^2}{l^2}\right) P_0^3(\cos\sigma) + \frac{l^4}{r^4}\left(1 - \frac{5}{2}\frac{\rho^2}{l^2} + \frac{5}{8}\frac{\rho^4}{l^4}\right) P_0^5(\cos\sigma) + \ldots \right]$$

wenn N die auf die Längeneinheit entfallende Stromstärke (also $10N$ die sogenannte Ampèrewindungszahl), $2l$ die Länge und ρ der Radius der Spule ist. Der Ausdruck stimmt, wie es sein muss und wie es der Vergleich mit Formel (20) bestätigt, vollständig mit demjenigen für einen geraden Zylinder von denselben Abmessungen überein, dessen Endflächen eine magnetische Belegung von der Flächendichte N tragen, dessen Moment also $M = 2\pi\rho^2 Nl$ ist.

Für den Innenraum d. h. für $r < \rho$, ergibt sich der etwas kompliziertere Ausdruck

$$(39)\quad \mathrm{II} = -4\pi N l\left[\frac{r}{s} P_0^1(\cos\sigma) - \frac{1}{2}\frac{\rho^2 r^3}{s^5} P_0^3(\cos\sigma) + \frac{1}{8}\frac{\rho^2 r^5}{s^7}\left(3 - 7\frac{l^2}{s^2}\right) P_0^5(\cos\sigma) + \ldots\ldots + (-1)^\nu \frac{\rho^2 r^{2\nu+1}}{s^{2\nu+3}}\left(a_0 + a_1\frac{l^2}{r^2} + a_2\frac{l^4}{r^4} + \ldots\right) P_0^{2\nu+1}(\cos\sigma) + \ldots \right]$$

in dem zur Abkürzung $l^2 + \rho^2 = s^2$ gesetzt ist. Die Koeffizienten $a_0, a_1 \ldots$ ergeben sich aus denjenigen von $P_0^{2\nu+1}(\cos\sigma) = A_0 \cos\sigma + A_1 \cos^3\sigma + \ldots$ durch die Rekursionsformeln

$$a_0 = A_0 \qquad a_1 = A_1 + \frac{2\nu+3}{3} a_0 \qquad a_2 = A_2 + \frac{2\nu+5}{5} a_1 \ldots .$$

Digitized by Google

oder, vom Fall $\nu = 0$ abgesehen, durch die independente Formel

$$a_q = A_q \left[\binom{\nu}{q} - \binom{\nu}{q-1} + \binom{\nu}{q-2} - \ldots\ldots \pm \binom{\nu}{0} \right] : \binom{\nu}{q}$$

Bei einer unendlich langen Spule wird $s = l$ und man erhält $\Pi = -4\pi N r P_0^1 (\cos\sigma) = -4\pi N x$, wobei $x = r \cos\sigma$ längs der Achse gemessen ist. Für das Feld im Innern ergibt sich daraus der bekannte Wert $X = 4\pi N$.

Bei einer unendlich niedrigen Spule andrerseits geht (39), in dem dann $2Nl = I$ zu setzen ist, in den Ausdruck (33) über.

Das gegenseitige Potential zweier Magnete in beliebiger Lage.

Nunmehr wende ich mich zu der eigentlichen Aufgabe. Es seien irgend zwei Magnete durch ihre Parameter bestimmt und es sei ihre relative Lage gegeben. Aus diesen Daten soll das Potential des einen Magnets auf den andern berechnet werden.

Auf die Ableitung des gesuchten Ausdrucks gehe ich hier nicht ein. Er ergibt sich aus den Entwickelungen meiner früheren, in Anm. 4) erwähnten Arbeit. Ich ersetze aber hier die dort ganz allgemein gewählte Lagenbezeichnung durch eine etwas spezialisierte, die dem besonderen Zweck der erdmagnetischen Beobachtungstechnik angepasst ist und die deshalb auch von den früher genannten Autoren stets benutzt worden ist. Bei allen erdmagnetischen Messungen handelt es sich so gut wie ausschliesslich um das Drehungsmoment, das der eine Magnet (die Nadel) in Bezug auf eine bestimmte Achse erfährt. Die Drehungsmomente um andere, insbesondere um die dazu senkrechten Achsen zieht man nicht in Betracht, ebenso wenig die translatorischen Kräfte. (Allerdings sind diese Grössen nicht immer ganz gleichgültig; sie sind aber in den praktisch vorkommenden Fällen meistens sehr klein und können hinreichend genau gefunden werden, wenn man die gegebenen Magnete durch die ihnen entsprechenden schematischen ersetzt.) Die Berechnung jenes allein interessierenden Drehungsmoments vereinfacht sich nun etwas, wenn man bei der Definition der gegenseitigen Lage der Magnete von der betreffenden Achse ausgeht. Dabei empfiehlt es sich um des bequemeren und deutlicheren Ausdrucks willen, diese Achse als vertikal zu bezeichnen, was sie ja in der grossen Mehrzahl aller Fälle auch ist. Die Bedeutung der Ergebnisse wird dadurch offenbar nicht eingeschränkt; sie gelten unter entsprechender Aenderung der Ausdrucksweise ganz allgemein. Während nun durch die Anknüpfung

Digitized by Google

an die Hauptdrehungsachse die Berechnung des für diese geltenden Drehungsmoments etwas erleichtert wird, gestaltet sich dabei diejenige der übrigen Grössen z. T. weniger bequem; sie ist deshalb, ausser in den allereinfachsten Fällen, bisher unbeachtet geblieben. In der Form, in der die Lösung hier gegeben wird, bietet es, wie sich zeigen wird, keine Schwierigkeit, diese Lücke auszufüllen.

Es sei nun in dem früher definierten Sinne das Potential des einen Magnets (des Stabs), bezogen auf ein fest mit ihm verbundenes Polarkoordinatensystem

$$\Pi = \sum_{1}^{\infty}{}_n \sum_{0}^{n}{}_m \; r^{-n-1} P_m^n(\cos\sigma)\,(M_{nm}\cos m\tau + N_{nm}\sin m\tau)$$

und das des andern (der Nadel) entsprechend

$$\Pi' = \sum_{1}^{\infty}{}_r \sum_{0}^{r}{}_s \; r'^{-r-1} P_s^r(\cos\sigma')\,(M'_{rs}\cos s\tau' + N'_{rs}\sin s\tau')$$

Die Anfangspunkte der beiden Systeme seien O und O', die positiven Achsenrichtungen OX und $O'X'$. Die letztgenannte Linie sei zur Drehungsachse $O'Z'$ senkrecht, also horizontal, während OX und $O'O$ mit der Vertikalen die Winkel ψ und η bilden mögen. Die Projektion von $O'O$ auf die durch $O'X'$ gehende Horizontal-

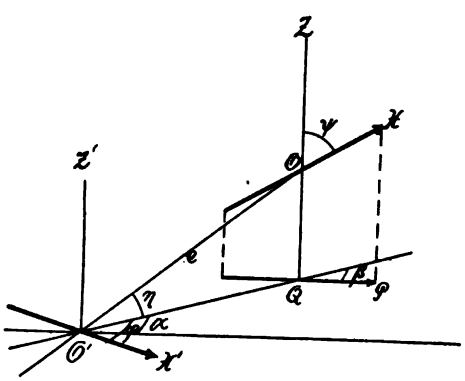


FIG. 1. *Nadel und Stab in beliebiger Lage.*

ebene heisse $O'Q$, diejenige der Stabachse QP, und es seien β und ϕ die Winkel von QP und $O'X'$ mit $O'Q$. Für die Anwendung auf erdmagnetische Beobachtungen werde noch der Winkel des magnetischen Meridians mit $O'Q$ eingeführt und α genannt. Die Entfernung $O'O$ endlich heisse e. Sind R und R' die Radien der

Digitized by Google

kleinsten Kugeln mit O und O' als Mittelpunkten, in deren Innenraum die Magnete vollständig enthalten sind, so muss $e > R + R'$ sein, damit die folgenden Reihenentwickelungen konvergieren.[20]

Mit diesen Bezeichnungen wird nun das gegenseitige Potential V der beiden Magnete aufeinander

$$(40)\quad V = \sum_1^\infty{}_n \sum_0^n{}_m \sum_1^\infty{}_r \sum_0^r{}_s \; (-1)^{n+m+s} \, e^{-n-r-1} \sum_0^n{}_p \sum_0^r{}_q \; (C^{nr}_{pq} \, G^n_{mp} \, J^r_{sq} + D^{nr}_{pq} \, H^n_{mp} \, K^r_{sq})$$

Die hierin auftretenden Grössen sind durch die nachstehenden Gleichungen definiert:

$$(41)\quad \left\{\begin{aligned} C^{nr}_{pq} &= \tfrac{1}{2} f^{nr}_{pq} \left[(-1)^q P^{n+r,\,p-q}(\cos\eta) + P^{n+r,\,p+q}(\cos\eta)\right] \\ D^{nr}_{pq} &= \tfrac{1}{2} f^{nr}_{pq} \left[(-1)^q P^{n+r,\,p-q}(\cos\eta) - P^{n+r,\,p+q}(\cos\eta)\right] \\ \text{mit} \qquad f^{nr}_{pq} &= 1.3.5. \ldots (2n + 2r - 1) \times \\ &\qquad \sqrt{\epsilon_p \, \epsilon_q : (n+p)! \, (n-p)! \, (r+q)! \, (r-q)!} \\ G^n_{mp} &= M_{nm} \, A^n_{mp}(\psi) \cos p\beta + N_{nm} \, B^n_{mp}(\psi) \sin p\beta \\ H^n_{mp} &= M_{nm} \, A^n_{mp}(\psi) \sin p\beta - N_{nm} \, B^n_{mp}(\psi) \cos p\beta \\ J^r_{sq} &= M'_{rs} \, A^r_{sq}\left(\frac{\pi}{2}\right) \cos q\phi + N'_{rs} \, B^r_{sq}\left(\frac{\pi}{2}\right) \sin q\phi \\ K^r_{sq} &= M'_{rs} \, A^r_{sq}\left(\frac{\pi}{2}\right) \sin q\phi - N'_{rs} \, B^r_{sq}\left(\frac{\pi}{2}\right) \cos q\phi \end{aligned}\right.$$

Unter $A(\psi)$ und $B(\psi)$ sind gewisse ganze Funktionen von $\cos\psi$ und $\sin\psi$ verstanden, die bei der Transformation der Kugelfunktionen auf eine neue Achse auftreten, deren Richtung mit derjenigen der alten den Winkel ψ bildet.[21] In dem für das Folgende vorzugsweise wichtigen Falle, dass einer der beiden unteren Indizes verschwindet, wird B zu Null, während A in eine Kugelfunktion übergeht. Es ist allgemein

$$A^n_{op}(\psi) = A^n_{po}(\psi) = P^n_p(\cos\psi) \qquad B^n_{op}(\psi) = B^n_{po}(\psi) = 0$$

[20] Die hier gewählte Bezeichnung schliesst sich möglichst eng an diejenige von B ö r g e n an. Es entsprechen einander:

hier:	α	ϕ	β	ψ	e	$\sin\eta$	$\cos\eta$
bei (Börgen):	α	$\alpha-\phi$	$\alpha-\beta$	ψ	e	$a:e$	$f:e$

[21] A d. S c h m i d t : Formeln zur Transformation der Kugelfunktionen bei linearer Aenderung des Koordinatensystems. *Zeitschrift für Math. und Phys.* 44. Bd. 1899.

Digitized by Google

Die angegebene Lösung erfährt übrigens keine wesentliche Aenderung, wenn die Nadelachse $O'X'$ nicht senkrecht auf der Drehungsachse $O'Z'$ ist. Bildet sie mit dieser den beliebig anzunehmenden Winkel χ, so hat man nur χ an Stelle von $\frac{\pi}{2}$ in den für J und K angegebenen Ausdrücken zu setzen. Der spezielle Wert $\frac{\pi}{2}$ führt nur insofern eine Vereinfachung herbei, als von den je zwei zusammengehörigen Werten von A und B stets einer verschwindet. Es ist nämlich

$$A^r_{sq}\left(\frac{\pi}{2}\right)=0 \qquad \text{wenn } (r+s+q) \text{ ungerade, und}$$

$$B^r_{sq}\left(\frac{\pi}{2}\right)=0 \qquad \text{wenn } (r+s+q) \text{ gerade ist.}$$

Setzt man die in den vier letzten der Gleichungen (41) angegebenen Ausdrücke in (40) ein, so erhält man

$$(42)\quad V=\sum_1^{\infty n}\sum_0^{n m}\sum_1^{\infty r}\sum_0^{r s} e^{-n-r-1}\,[M_{n m}\,M'_{r s}\,\Phi^{nr}_{ms}(\eta,\psi,\beta,\phi) + \ldots + \ldots + \ldots]$$

worin die durch Punkte angedeuteten Glieder dem ersten analog gebildet sind, aus dem sie durch Vertauschung von M mit N einerseits, M' mit N' andererseits entstehen. Der abgekürzt geschriebene Koeffizient des Produkts der beiden Parameter hat die Bedeutung

$$(43)\quad \Phi^{nr}_{ms}(\eta,\psi,\beta,\phi)=(-1)^{n+m+s}\sum_0^n{}_p\sum_0^r{}_q\; A^n_{mp}(\psi)\,A^r_{sq}\left(\frac{\pi}{2}\right)\times \left[C^{nr}_{pq}\cos p\beta\cos q\phi + D^{nr}_{pq}\sin p\beta\sin q\phi\right]$$

Ganz ähnlich sind natürlich die entsprechenden Koeffizienten der drei zugehörigen folgenden Glieder gebildet; es treten darin statt der A z. T. oder überall die B auf und die Funktionen von β und ϕ erscheinen in anderer Verknüpfung.

Der für Φ geltende Ausdruck kann auch in der Form

$$(44)\quad \Phi^{nr}_{ms}=(-1)^{n+m+s}\sum_0^n{}_p\sum_0^r{}_q\,\frac{1}{2}f^{nr}_{pq}\,A^n_{mp}(\psi)\,A^r_{sq}\left(\tfrac{\pi}{2}\right)\Big[P^{n+r,\,p+q}(\cos\eta)\cos(p\beta+q\phi)+(-1)^q P^{n+r,\,p-q}(\cos\eta)\cos(p\beta-q\phi)\Big]$$

Digitized by Google

geschrieben werden, die den Vorteil hat, in dem von η abhängigen Faktor nur unmittelbar verständliche Bezeichnungen zu verwenden und zugleich zum Ausdruck zu bringen, dass dieser Faktor nur von der Summe der Indizes n und r, nicht von diesen einzeln abhängt. In den abschliessenden Gebrauchsformeln wird man übrigens die Funktionen von $(p\beta + q\phi)$ und $(p\beta - q\phi)$ besser wieder in Produkte der Funktionen von $p\beta$ und $q\phi$ umwandeln.

Wie man sieht, ist der für V erhaltene Ausdruck sehr einfach und durchsichtig gebaut; in praktischer Hinsicht verwickelt wird er allein durch seinen grossen Umfang. Setzt man die Entwickelung von (42) bis zu den Gliedern ν-ter Ordnung fort, wie die mit $e^{-\nu-2}$ multiplizierten heissen mögen, so wird deren Anzahl gleich $\frac{1}{8}\nu(\nu+1)(\nu^2+9\nu+17)$, wovon auf die ν-te Ordnung allein $\frac{1}{8}\nu(2\nu^2+12\nu+13)$ kommen. Als einzelnes Glied ist dabei jedes Produkt zweier Parameter und des zugehörigen durch (43) definierten Koeffizienten gezählt, also ein selbst schon recht ausgedehntes Aggregat.

Die früher genannten Autoren sind in ihren (allerdings wesentlich spezialisierten) Entwickelungen meistens bis e^{-7}, vereinzelt bis e^{-9} gegangen; die letztgenannte Grenze noch zu überschreiten, dürfte schwerlich jemals nötig werden. In diesen zwei Fällen, d. h. für $\nu = 5$ und $\nu = 7$, enthält V nach dem Gesagten 435 und 1204 Glieder, deren explizite Darstellung freilich kaum noch durchführbar sein würde.

Eine ausserordentliche Vereinfachung tritt nun ein, wenn beide Magnete, der Stab wie die Nadel, achsensymmetrisch sind. Dann verschwinden sämtliche Parameter mit Ausnahme der M_{n0} und M'_{r0}, die in diesem Falle dem Früheren zufolge einfach durch M_n und M'_r bezeichnet werden sollen. Die Anzahl der Glieder ν-ter Ordnung ist dann nur noch ν, diejenige der ersten ν Ordnungen insgesamt $\frac{1}{2}\nu(\nu+1)$, also 15 für $\nu = 5$ und 28 für $\nu = 7$.

Nimmt man die Magnete darüber hinaus als regulär an, d. h. geht man zu dem bisher allein allgemein behandelten Falle über, so sind, mit ν als einer dann notwendig ungeraden Zahl, die entsprechenden Zahlen $\frac{1}{2}(\nu+1)$ und $\frac{1}{8}(\nu+1)(\nu+3)$. Letztere, die Gesamtzahl bis zur ν-ten Ordnung, beträgt 6 für $\nu = 5$ und 10 für $\nu = 7$. Die bis $\nu = 5$ reichende 6-gliedrige Formel (übrigens nicht für V selbst, sondern für das daraus abgeleitete Drehungsmoment) ist die einzige, die bisher vollständig entwickelt wurde; sie findet sich in der eingangs erwähnten Abhandlung von B ö r g e n (vgl. Anm. 2), wo sie nicht weniger als 2 Quartseiten füllt.

Eine weitere Vereinfachung ergibt sich, wenn man die gegen-

Digitized by Google

seitige Lage der beiden Magnete spezialisiert. Dadurch werden die hier als Φ bezeichneten Koeffizienten, auf deren explizite Darstellung ja die ganze Aufgabe hinauskommt, wesentlich einfacher. Die wichtigste, weil bei den meisten Anwendungen zutreffende Annahme in dieser Hinsicht ist die, dass die Achsen der beiden Magnete in einer und derselben Ebene, und zwar in der Drehungsebene der Nadel (nach der hier benutzten Ausdrucksweise also in der Horizontalebene) liegen, was sich durch die Ansätze $\eta = \frac{\pi}{2}$, $\psi = \frac{\pi}{2}$ ausdrückt. Besondere Annahmen über die Winkel β und ϕ führen dann endlich auf die Hauptlagen.

Für alle diese mehr oder minder spezialisierten Fälle soll nun im Folgenden die Rechnung bis zur Aufstellung unmittelbar anwendbarer Schlussformeln durchgeführt werden.

Achsensymmetrische Magnete in beliebiger Lage.

Die Formel für das Potential reduziert sich, wenn Stab und Nadel rings um ihre Achsen gleichmässig magnetisiert sind, auf

$$V = \sum_1^\infty{}_n \sum_1^\infty{}_r \, e^{-n-r-1} M_n M'_r \Phi_{00}^{nr} (\eta, \psi', \beta, \phi) \tag{45}$$

mit

$$\Phi_{00}^{nr} = (-1)^n \sum_0^n{}_p \sum_0^r{}_q \frac{1}{2} f_{pq}^{nr} P_p^n (\cos\psi) P_q^r (0) \Big[P^{n+r,\, p+q} (\cos\eta) \cos(p\beta + q\phi) + (-1)^q P^{n+r,\, p-q} (\cos\eta) \cos(p\beta - q\phi) \Big] \tag{46}$$

Es empfiehlt sich, in diesem Falle überall die von Gauss benutzten Funktionen einzuführen, was vermöge der Beziehung

$$P_p^n (\cos\psi) = 1.3.5 \ldots (2n-1) \sqrt{\epsilon_p : (n+p)!\,(n-p)!}\; P^{np} (\cos\psi)$$
$$\epsilon_0 = 1, \; \epsilon_1 = \epsilon_2 = \ldots = 2$$

geschieht. Setzt man gleichzeitig für $P_q^r (0)$ seinen Wert[22]

$$P_q^r (0) = (-1)^{\frac{r-q}{2}} \frac{\sqrt{\epsilon_q (r+q)!\,(r-q)!}}{(r+q)!!\,(r-q)!!} \quad \text{für } r-q \equiv 0 \mod 2$$
$$P_q^r (0) = 0 \quad \text{für } r-q \equiv 1 \mod 2$$

ein, so entsteht in Verbindung mit $(-1)^n \cdot \frac{1}{2} f_{pq}^{nr}$ der Zahlenfaktor

[22] Ich benutze hier die sehr zweckmässige, von A. Schuster eingeführte Bezeichnung

$$(2n-1)!! = 1.3.5 \ldots (2n-1) \qquad (2n)!! = 2.4.6 \ldots 2n$$

Dabei ist unter $(-1)!!$ und $0!!$ der Wert 1 zu verstehen.

Digitized by Google

$$(47)\quad j_{pq}^{nr} = (-1)^{n+\frac{r-q}{2}}\,\epsilon_p\,\frac{(2n+2r-1)!!\,(2n-1)!!}{(n+p)!\,(n-p)!\,(r+q)!!\,(r-q)!!}$$
$$= (-1)^k\,\frac{\epsilon_p}{2}\binom{2n}{n-p}\binom{r}{k}\,j_{nr}^{nr}$$

$$\text{mit}\quad j_{nr}^{nr} = j_{rn}^{rn} = -2\,\frac{(2n+2r-1)!!}{(2n)!!\,(2r)!!}\quad\text{und}\quad k=\frac{r-q}{2}$$

Setzt man zur weiteren Vereinfachung

$$(48)\quad (np\,;rq) = P^{np}(\cos\psi)\Big[P^{n+r,\,p+q}(\cos\eta)\cos(p\beta+q\phi) + (-1)^q\,P^{n+r,\,p-q}(\cos\eta)\cos(p\beta-q\phi)\Big]$$

also speziell

$$(n0\,;rq) = 2P^{n0}(\cos\psi)\,P^{n+r,\,q}(\cos\eta)\cos q\phi$$

so erhält man an Stelle von (46)

$$(49)\qquad \Phi_{oo}^{nr} = \sum_{0}^{n}{}_p\ \sum_{0}^{r}{}_q\ j_{pq}^{nr}\,(np\,;rq)$$

als Faktor von $e^{-n-r-1}\,M_n\,M'_r$ in V. Für q sind darin nur diejenigen Werte einzusetzen, die $(r-q)$ zu einer geraden Zahl machen. Ausserdem brauchen die Glieder, in denen q den Wert 0 hat, nicht berücksichtigt zu werden, da sie ϕ nicht enthalten. (Sie würden nur dann in Betracht kommen, wenn auch die Einwirkung der Nadel auf den Stab bestimmt werden sollte.)

Die Ausdrücke $(np;\,rq)$ sind vollständig bekannt und lassen sich mit Hilfe einer Tafel der Kugelfunktionen[23] ohne weiteres niederschreiben; es genügt also zur Aufstellung des Potentials die Auswertung der durch (47) definierten Koeffizienten. Zur Veranschaulichung seien die ersten Glieder der Entwickelung, die sich so ergibt, hingeschrieben, und zwar der formellen Vollständigkeit halber mit Einschluss derjenigen, in denen q gleich 0 ist:

$$(50)\quad V = -\frac{3}{2}\,M_1\,M_1'\,e^{-3}\,[(10;11)+(11;11)]$$
$$+\frac{15}{8}\,M_2\,M_1'\,e^{-4}\,[3\,(20;11)+4\,(21;11)+(22;11)]$$
$$+\frac{15}{8}\,M_1\,M_2'\,e^{-4}\,[(10;20)-(10;22)+(11;20)-(11;22)]$$
$$-\frac{105}{32}\,M_2\,M_2'\,e^{-5}\,[3\,(20;20)-3\,(20;22)+4\,(21;20)$$
$$-4\,(21;22)+(22;20)-(22;22)]+\ldots.$$

[23] Vgl. Anm. 5). Zur Ableitung von $P^{nm}(c)$ aus $P^{no}(c)$ dient die bequeme Rekursionsformel

$$P^{n,\,m+1}(c) = \frac{s^{m+1}}{n-m}\,\frac{d}{dc}\left(\frac{P^{n,\,m}(c)}{s^m}\right)\qquad s=\sqrt{1-c^2}\qquad m=0,1,2\ldots.n-1$$

Digitized by Google

Anhangsweise füge ich hier das Ergebnis für den wichtigsten Fall nicht vollkommener Achsensymmetrie an. Es ist dies der schon in den früheren Abschnitten immer wiederkehrende Fall flacher, aber im übrigen regelmässiger Magnete, bei denen ausser den Parametern M_{n0} noch M_{22} einen von Null verschiedenen Wert besitzt. Unter Benutzung der von mir seinerzeit (vgl. die in Anm. 21 angeführte Arbeit, S. 336, 337) abgeleiteten Ausdrücke

$$A^1_{00}(\psi) = \cos\psi \qquad A^1_{01}(\psi) = \sin\psi$$

$$A^2_{20}(\psi) = \frac{1}{2}\sqrt{15}\sin^2\psi\cos\psi \qquad A^2_{21}(\psi) = -\frac{1}{4}\sqrt{10}\sin\psi\,(3\cos^2\psi - 1)$$

$$A^2_{22}(\psi) = \frac{1}{2}(3\cos^3\psi - \cos\psi) \qquad A^2_{23}(\psi) = \frac{1}{4}\sqrt{6}\sin\psi\,(\cos^2\psi + 1)$$

findet sich nach Formel (44) als Faktor von $M_{32}\,M_1'\,e^{-5}$:

$$\begin{aligned}(51a)\quad -\frac{7}{16}\sqrt{15}\Big\{ & 20\sin^2\psi\cos\psi\,P^{41}\cos\phi \\ & - 5\sin\psi\,(3\cos^2\psi - 1)\left[P^{42}\cos(\beta+\phi) - P^{40}\cos(\beta-\phi)\right] \\ & + 2\,(3\cos^3\psi - \cos\psi)\left[P^{43}\cos(2\beta+\phi) - P^{41}\cos(2\beta-\phi)\right] \\ & + \sin\psi\,(\cos^2\psi + 1)\left[P^{44}\cos(3\beta+\phi) - P^{42}\cos(3\beta-\phi)\right]\Big\}\end{aligned}$$

und als Faktor von $M_1\,M_{32}'\,e^{-5}$.

$$\begin{aligned}(51b)\quad -\frac{7}{16}\sqrt{15}\Big\{ & 2\cos\psi\,[5\,P^{41}\cos\phi + P^{43}\cos 3\phi] \\ & + \sin\psi\,[5\,P^{42}\cos(\beta+\phi) - 5\,P^{40}\cos(\beta-\phi) \\ & + P^{44}\cos(\beta+3\phi) - P^{42}\cos(\beta-3\phi)\,]\Big\}\end{aligned}$$

Unter $P^n{}_m$ ist hierin überall $P^n{}_m(\cos\eta)$ zu verstehen.

Reguläre Magnete in beliebiger Lage.

Dieser Fall zeigt gegenüber dem vorigen keine wesentliche Aenderung; es fallen nur bei ihm alle M_n und M'_r mit geradem Index fort, so dass rein äusserlich der Umfang des Potentialausdrucks stark verringert wird. Wegen der grossen praktischen Wichtigkeit indessen, die gerade diesem Falle zukommt, empfiehlt es sich, die dafür geltende Formel, wie sie sich unmittelbar aus (47) und (49) ergibt, ausführlich hinzuschreiben. Bis zu den Gliedern 5. Ordnung einschliesslich (in dem früher angegebenen Sinne) lautet sie:

Digitized by Google

$$
\begin{aligned}
(52)\quad V = &-\frac{3}{2} M_1 M_1' e^{-3} [\,(10;11) + (11;11)\,] \\
&-\frac{35}{16} M_3 M_1' e^{-5} [\,10\,(30;11) + 15\,(31;11) + 6\,(32;11) \\
&\qquad + (33;11)\,] \\
&+\frac{35}{16} M_1 M_3' e^{-5} [3\,(10;31) - (10;33) + 3\,(11;31) - (11;33)] \\
&-\frac{693}{256} M_5 M_1' e^{-7} [\,126\,(50;11) + 210\,(51;11) \\
&\qquad + 120\,(52;11) + 45\,(53;11) + 10\,(54;11) \\
&\qquad + (55;11)\,] \\
&+\frac{1155}{128} M_3 M_3' e^{-7} [30\,(30;31) - 10\,(30;33) \\
&\qquad + 45\,(31;31) - 15\,(31;33) + 18\,(32;31) \\
&\qquad - 6\,(32;33) + 3\,(33;31) - (33;33)\,] \\
&-\frac{693}{256} M_1 M_5' e^{-7} [10\,(10;51) - 5\,(10;53) + (10;55) \\
&\qquad + 10\,(11;51) - 5\,(11;53) + (11;55)\,] \\
&-\frac{6435}{2048} M_7 M_1' e^{-9} [1716\,(70;11) + 3003\,(71;11) \\
&\qquad + 2002\,(72;11) + 1001\,(73;11) + 364\,(74;11) \\
&\qquad + 91\,(75;11) + 14\,(76;11) + 77;11)\,] \\
&+\frac{45045}{2048} M_5 M_3' e^{-9} [378\,(50;31) - 126\,(50;33) + 630\,(51;31) \\
&\qquad - 210\,(51;33) + 360\,(52;31) - 120\,(52;33) \\
&\qquad + 135\,(53;31) - 45\,(53;33) + 30\,(54;31) \\
&\qquad - 10\,(54;33) + 3\,(55;31) - (55;33)\,] \\
&-\frac{45045}{2048} M_3 M_5' e^{-9} [100\,(30;51) - 50\,(30;53) + 10\,(30;55) \\
&\qquad + 150\,(31;51) - 75\,(31;53) + 15\,(31;55) \\
&\qquad + 60\,(32;51) - 30\,(32;53) + 6\,(32;55) \\
&\qquad + 10\,(33;51) - 5\,(33;53) + (33;55)\,] \\
&+\frac{6435}{2048} M_1 M_7' e^{-9} [35\,(10;71) - 21\,(10;73) + 7\,(10;75) \\
&\qquad - (10;77) + 35\,(11;71) - 21\,(11;73) + 7\,(11;75) \\
&\qquad - (11;77)\,] - \ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

Das von dem Stabe auf die Nadel ausgeübte Drehungsmoment ist

$$X = -\frac{\partial V}{\partial \phi}$$

und ergibt sich also, wenn

Digitized by Google

$$(52a)\quad [n\,p;r\,q] = -\frac{\partial}{\partial\phi}(n\,p;r\,q) = q\,P^{n\,p}(\cos\psi)\Big[P^{n+r,\,p+q}(\cos\eta)\sin(p\,\beta+q\,\phi) - (-1)^q\,P^{n+r,\,p-q}(\cos\eta)\sin(p\,\beta-q\,\phi)\Big]$$

ist, ohne weiteres, indem man in (52) überall $(np;\,rq)$ durch $[np;\,rq]$ ersetzt. Der so für X erhaltene Ausdruck ist sachlich mit demjenigen identisch, den Börgen in seiner ersten Abhandlung (vgl. Anm. 2) auf S. 3—5 mitteilt.[24] Formell unterscheidet er sich (von der Schreibweise abgesehen) von jenem wie überhaupt von allen früher abgeleiteten in einem nicht unwesentlichen Punkte. Die bisherigen Formeln sind, was sich aus ihrer Herleitung durch die Binomialreihe erklärt, nach Potenzen der trigonometrischen Funktionen der auftretenden Winkel, besonders des Richtungswinkels der Nadel, entwickelt. In der hier gefundenen Formel schreiten die Reihen dagegen nach Funktionen der Vielfachen dieser Winkel fort. (Das gilt zunächst in Bezug auf ϕ und β, für die es auch vorwiegend von Bedeutung ist. Aber auch die Funktionen von ψ und η lassen sich ohne weiteres mittels bekannter Formeln in dieser Gestalt angeben.) Die letztere Form ist in mehrfacher Hinsicht zweckmässiger als die erstere; sie besitzt, wie man a priori wissen kann, kleinere Koeffizienten, ist für die numerische Auswertung bequemer, und die Ableitung des Drehungsmoments aus dem Potential gestaltet sich bei ihr einfacher und übersichtlicher. Demgegenüber ist es einigermassen verwunderlich, dass anscheinend niemand daran gedacht hat, die früher aufgestellten Formeln nachträglich in die bessere Form umzuwandeln, sondern dass man immer bei dem durch die Herleitung unmittelbar gegebenen Ausdruck stehen geblieben ist.

[24] In seiner zweiten, in Anm. 1) zitierten Abhandlung gibt Börgen Hilfsgrössen K_{nr} an, aus denen sich, allerdings durch eine etwas mühsame Rechnung, das Drehungsmoment X (und damit natürlich auch rückwärts das Potential V) bis zu den Gliedern 7. Ordnung ableiten lässt. Die von ihm in der Zusammenstellung auf S. 183 angewandte Form der Darstellung ist aber geeignet, irrezuführen. Die K_{nr} erscheinen dort als einfach gebaute Funktionen gewisser Grössen, A, B, B', die durch

$$A = e^{-2}\Big[x^2+x_1^2+2\,e\,x\,(\cos\eta\cos\psi+\sin\eta\sin\psi\cos\beta)\Big]$$

$$B = 2\,e^{-2}\,x_1\Big[e\sin\eta\cos\phi+x\sin\psi\cos(\phi-\beta)\Big]$$

$$B' = -2\,e^{-2}\,x_1\Big[e\sin\eta\sin\phi+x\sin\psi\sin(\phi-\beta)\Big]$$

definiert sind. Die angeschriebenen Gleichungen sind aber nicht buchstäblich aufzufassen, sondern besitzen (was sich aus der Ableitung ergibt, jedoch nirgends gesagt ist) nur einen symbolischen Sinn. Tatsächlich ist nämlich K_{nr} nicht gleich dem ganzen damit gleich gesetzten Ausdruck, sondern gleich dem Koeffizienten von $x^n\,x_1^r$ in der Potenzreihe, die sich ergibt, wenn man diesen Ausdruck nach x und x_1 entwickelt.

Digitized by Google

Reguläre Magnete in speziellen Lagen.

Die wichtigsten Besonderheiten der gegenseitigen Lage von Stab und Nadel sind durch die Werte 0 und $\frac{\pi}{2}$ der Winkel ψ und η charakterisiert. Um die allgemeine Formel auf diese Fälle anzuwenden, hat man in ihr nur die Werte von $P^{nm}(\cos 0)$ und $P^{nm}(\cos \frac{\pi}{2})$, d. h. $P^{nm}(1)$ und $P^{nm}(0)$ einzusetzen. Es ist

$$P^{nm}(1) = 0 \text{ für } m > 0 \qquad P^{n0}(1) = \frac{n!}{(2n-1)!!}$$

$$P^{nm}(0) = 0 \text{ für } n-m \equiv 1 \bmod 2, \text{ und für } n-m \equiv 0 \bmod 2$$

$$P^{nm}(0) = (-1)^{\frac{n-m}{2}} \frac{(n+m-1)!!\ (n-m-1)!!}{(2n-1)!!}$$

Es sind 4 Fälle möglich:

1) $\psi = 0$, d. h. der Stab steht vertikal (die Nadel der Einfachheit halber immer als horizontal angenommen). In diesem Falle, der bei B ö r g e n unter II aufgeführt ist, allerdings nur für $\phi = 0$, bleiben in (52) nur die Glieder $(n\ 0; rq)$ erhalten, und es ist allgemein

$$(n\ 0; r\ q) = \frac{2n!}{(2n-1)!!} P^{n+r,\,q}(\cos \eta) \cos q\,\phi$$

2) $\eta = 0$, d. h. die Mitte des Stabs liegt in der Drehungsachse der Nadel. Hier bleiben nur die Funktionen $P^{n+r,\,p-q}(\cos \eta)$ übrig, in denen $p = q$ ist, und in (52) verschwinden daher alle $(np; rq)$ ausser

$$(n\,p; r\,p) = (-1)^p \frac{(n+r)!}{(2n+2r-1)!!} P^{np}(\cos \psi) \cos p\,(\beta - \phi)$$

3) $\psi = \frac{\pi}{2}$, d. h. der Stab liegt horizontal. (Fall *IB* bei B ö r g e n.) Hier fallen in (52) diejenigen Glieder fort, in denen $(n-p)$ ungerade, d. h. in denen p gerade ist. (Für n kommen ja, da es sich um reguläre Magnete handelt, nur ungerade Werte in Betracht.)

4) $\eta = \frac{\pi}{2}$, d. h. die Mitte des Stabs liegt in derselben Horizontalebene, wie die Nadel. (Fall III bei B ö r g e n, abgesehen von dort gemachten weiteren Einschränkungen.) Hier verschwinden gleichfalls sämtliche Glieder, in denen p gerade ist. (Man beachte, dass $r-q$ stets gerade und hier also q ungerade ist.)

Nun können ψ und η gleichzeitig spezialisiert werden. Von den 4 möglichen Kombinationen (1, 2), (1, 4), (3, 2), (3, 4) liefern die

Digitized by Google

beiden ersten offenbar identisch Null für X. Die folgende, die man, wie früher erwähnt, als dritte Hauptlage bezeichnen könnte, und die im Doppelkompass Anwendung findet, führt zu einer einfachen Entwickelung, deren erste Glieder mit $\Delta = \beta - \phi$ so lauten (vgl. Börgen, I*Bf*) und Bidlingmaier[25], S. 17, 18):

$$(53) \quad V = M_1 M_1' e^{-3} \cos\Delta - \frac{3}{2}(M_3 M_1' + M_1 M_3') e^{-5} \cos\Delta$$
$$+ \frac{15}{8}(M_5 M_1' + M_1 M_5') e^{-7} \cos\Delta$$
$$+ \frac{5}{8} M_3 M_3' e^{-7} (9\cos\Delta + \cos 3\Delta)$$
$$- \frac{35}{16}(M_7 M_1' + M_1 M_7') e^{-9} \cos\Delta$$
$$- \frac{35}{16}(M_5 M_3' + M_3 M_5') e^{-9} (6\cos\Delta + \cos 3\Delta)$$

Weitaus am wichtigsten ist die vierte Kombination (Fall I*A* bei Börgen), die deshalb im Folgenden eingehender behandelt werden soll.

Stab und Nadel in derselben Horizontalebene.

Wenn sowohl ψ wie η ein rechter Winkel ist, der Stab also in der die Nadel enthaltenden Horizontalebene liegt, so bleiben, wie bemerkt, bei regulären Magneten nur diejenigen Glieder übrig, in denen sämtliche 4 Indizes ungerade Zahlen sind. Durch Einführung der oben angegebenen Werte der $P(0)$ in die allgemeinen Formeln erhält man in diesem Falle als Faktor von $\cos(p\beta + q\phi)$:

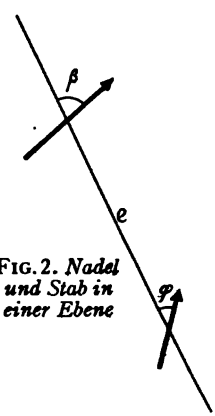


FIG. 2. *Nadel und Stab in einer Ebene*

$$S_{pq} = -2\frac{(n+p+r+q-1)!!\,(n-p+r-q-1)!!}{(n+p)!!\,(n-p)!!\,(r+q)!!\,(r-q)!!}$$

und als Faktor von $\cos(p\beta - q\phi)$:

$$D_{pq} = -2\frac{(n+p+r-q-1)!!\,(n-p+r+q-1)!!}{(n+p)!!\,(n-p)!!\,(r+q)!!\,(r-q)!!}$$

Für die weiteren Anwendungen empfiehlt sich die Umformung in:

$$(S_{pq} + D_{pq})\cos p\beta \cos q\phi - (S_{pq} - D_{pq})\sin p\beta \sin q\phi$$

[25] FR. BIDLINGMAIER: Der Doppelkompass, seine Theorie und Praxis. S.-A. aus "*Deutsche Südpolar-Expedition 1901-1903.*" Bd. V. Erdmagnetismus I. Berlin. (1907).

Digitized by Google

Schreibt man dann noch zur Abkürzung:

$$\cos\beta = c = c_1 \qquad \cos p\beta = c_p \qquad \cos\phi = \gamma = \gamma_1 \qquad \cos q\phi = \gamma_q$$
$$\sin\beta = s = s_1 \qquad \sin p\beta = s_p \qquad \sin\phi = \sigma = \sigma_1 \qquad \sin q\phi = \sigma_q$$

so lautet das Ergebnis:

$$\begin{aligned}(54)\quad V = {} & M_1 M_1' e^{-3}(-2c_1\gamma_1 + s_1\sigma_1) + M_3 M_1' e^{-5}\left(-\frac{3}{2}c_1\gamma_1\right.\\ & \left. + \frac{3}{8}s_1\sigma_1 - \frac{5}{2}c_3\gamma_1 + \frac{15}{8}s_3\sigma_1\right) + M_1 M_3' e^{-5}(\ldots)\\ & + M_5 M_1' e^{-7}\left(-\frac{45}{32}c_1\gamma_1 + \frac{15}{64}s_1\sigma_1 - \frac{105}{64}c_3\gamma_1 + \frac{105}{128}s_3\sigma_1\right.\\ & \left. - \frac{189}{64}c_5\gamma_1 + \frac{315}{128}s_5\sigma_1\right) + M_3 M_3' e^{-7}\left(-\frac{135}{32}c_1\gamma_1\right.\\ & \left. + \frac{45}{64}s_1\sigma_1 - \frac{105}{32}c_1\gamma_3 + \frac{105}{64}s_1\sigma_3 - \frac{105}{32}c_3\gamma_1 + \frac{105}{64}s_3\sigma_1\right.\\ & \left. - \frac{295}{32}c_3\gamma_3 + \frac{565}{64}s_3\sigma_3\right) + M_1 M_5' e^{-7}(\ldots)\\ & + M_7 M_1' e^{-9}\left(-\frac{175}{128}c_1\gamma_1 + \frac{175}{1024}s_1\sigma_1 - \frac{189}{128}c_3\gamma_1\right.\\ & \left. + \frac{567}{1024}s_3\sigma_1 - \frac{231}{128}c_5\gamma_1 + \frac{1155}{1024}s_5\sigma_1 - \frac{429}{128}c_7\gamma_1\right.\\ & \left. + \frac{3003}{1024}s_7\sigma_1\right) + M_5 M_3' e^{-9}\left(-\frac{525}{64}c_1\gamma_1 + \frac{525}{512}s_1\sigma_1\right.\\ & \left. - \frac{315}{64}c_1\gamma_3 + \frac{945}{512}s_1\sigma_3 - \frac{945}{128}c_3\gamma_1 + \frac{2835}{1024}s_3\sigma_1 - \frac{1015}{128}c_3\gamma_3\right.\\ & \left. + \frac{6895}{1024}s_3\sigma_3 - \frac{693}{128}c_5\gamma_1 + \frac{3465}{1024}s_5\sigma_1 - \frac{2835}{128}c_5\gamma_3\right.\\ & \left. + \frac{22365}{1024}s_5\sigma_3\right) + M_3 M_5' e^{-9}(\ldots) + M_1 M_7' e^{-9}(\ldots) + \ldots\end{aligned}$$

Die Faktoren von $M_\mu M'_\nu$ und $M_\nu M'_\mu$ gehen in einander über, wenn β mit ϕ, also c mit γ und s mit σ vertauscht wird; es ist deshalb immer nur der eine von beiden angegeben worden.

Das auf die Nadel ausgeübte Drehungsmoment X, bei dem natürlich diese Symmetrie wegfällt, lässt sich ohne weiteres aus der vorstehenden Formel ablesen; wegen der grossen praktischen Bedeutung gerade dieser Grösse mögen indessen wenigstens die ersten, am häufigsten gebrauchten Glieder der Entwickelung hingeschrieben werden. Es ist

Digitized by Google

$$(55)\quad X = -M_1 M_1' e^{-3}\,(2c_1\sigma_1 + s_1\gamma_1)$$
$$- M_3 M_1' e^{-5}\,(\tfrac{3}{2}c_1\sigma_1 + \tfrac{3}{8}s_1\gamma_1 + \tfrac{5}{2}c_3\sigma_1 + \tfrac{15}{8}s_3\gamma_1)$$
$$- M_1 M_3' e^{-5}\,(\tfrac{3}{2}c_1\sigma_1 + \tfrac{3}{8}s_1\gamma_1 + \tfrac{15}{2}c_1\sigma_3 + \tfrac{45}{8}s_1\gamma_3)$$
$$- M_5 M_1' e^{-7}\,(\tfrac{45}{32}c_1\sigma_1 + \tfrac{15}{64}s_1\gamma_1 + \tfrac{105}{64}c_3\sigma_1 + \tfrac{105}{128}s_3\gamma_1 + \tfrac{189}{64}c_5\sigma_1 + \tfrac{315}{128}s_5\gamma_1)$$
$$- M_3 M_3' e^{-7}\,(\tfrac{135}{32}c_1\sigma_1 + \tfrac{45}{64}s_1\gamma_1 + \tfrac{315}{32}c_1\sigma_3 + \tfrac{315}{64}s_1\gamma_3 + \tfrac{105}{32}c_3\sigma_1 + \tfrac{105}{64}s_3\gamma_1 + \tfrac{885}{32}c_3\sigma_3 + \tfrac{1695}{64}s_3\gamma_3)$$
$$- M_1 M_5' e^{-7}\,(\tfrac{45}{32}c_1\sigma_1 + \tfrac{15}{64}s_1\gamma_1 + \tfrac{315}{64}c_1\sigma_3 + \tfrac{315}{128}s_1\gamma_3 + \tfrac{945}{64}c_1\sigma_5 + \tfrac{1575}{128}s_1\gamma_5) - \ldots\ldots$$

Es ist nicht ohne weiteres möglich, diese Resultate mit den von anderen Forschern bisher schon für den vorliegenden Fall abgeleiteten Formeln zu vergleichen. Wie bereits erwähnt, haben diese stets (von der Verschiedenheit der Bezeichnung[26] ganz abge-

[26] Was die Bezeichnung betrifft, so entsprechen einander:

hier:	α	ϕ	β	$\alpha-\phi$	$\beta-\phi$	e	c	γ	$\cos(\beta-\phi)$
bei Fritsche:	θ	$\theta-v$	$\theta-w$	v	$v-w$	R	A	B	C
bei Leyst:		$\pi-\phi_1$	$\pi-\phi_2$	$-\psi$	Δ	e	$-\cos\phi_2$	$-\cos\phi_1$	$\cos\Delta$

Es ist dazu noch zu bemerken, dass beide Autoren nicht das Drehungsmoment angeben, sondern eine Grösse, die sich um einen konstanten Faktor davon unterscheidet. In die hier benutzte Schreibweise übersetzt ist der von Fritsche angegebene Wert gleich $-e^2X: M_1M_1'$ und der von Leyst angegebene $X: M_1'$. Börgen dagegen schreibt das Drehungsmoment, genauer gesagt $(-X)$, an. Der letztere weicht auch insofern von den beiden anderen ab, als er allein die Magnete zwar als regulär, aber im übrigen beliebig voraussetzt, während sich diese auf den Fall einer speziellen Verteilung der Magnetisierung beschränken. Fritsche, der sich in seiner Herleitung der Hauptsache nach an eine Arbeit von Kowalsky anschliesst, setzt die Dichte des freien Magnetismus proportional einer Potenz des Abstandes von der Mitte des Magnets, Leyst nimmt schematische Magnete an. Indessen schadet diese Spezialisierung nichts, da man von ihren Formeln ohne weiteres zu dem allgemeinen Ausdruck übergehen kann, wie er hier und bei Börgen auftritt. Es genügt dazu,

Bei Fritsche:	M	m	Mpa^2	$mq\delta^2a^2$	$\frac{Mpa^4}{2-p}$	$\frac{mq\delta^4a^4}{2-q}$	$\frac{Mpa^6}{3-2p}$	$\frac{mq\delta^6a^6}{3-2q}$
Bei Leyst:	M_3	M_1	r_2^2	r_1^2	r_2^4	r_1^4	r_2^6	r_1^6
durch:	M_1	M_1'	M_3	M_3'	M_5	M_5'	M_7	M_7'

zu ersetzen.

Digitized by Google

sehen) nach Potenzen der trigonometrischen Funktionen entwickelt; ausserdem führen sie alle bis auf C h w o l s o n , soweit sie die Formeln überhaupt bis zu einem gebrauchsfertigen Ausdruck durchrechnen, den Winkel $\Delta = \beta - \phi$ zwischen den Achsen der beiden Magnete in die Darstellung ein. Um die Vergleichung zu erleichtern, transformiere ich mit Hilfe der bekannten Beziehungen

$$c_3 = 4c^3 - 3c \qquad c_5 = 16c^5 - 20c^3 + 5c \qquad c_7 = 64c^7 - 112c^5 + 56c^3 - 7c$$
$$s_3 = 4c^2 s - s \qquad s_5 = 16c^4 s - 12c^2 s + s \qquad s_7 = 64c^6 s - 80c^4 s + 24c^2 s - s$$

die oben für V und X gefundenen Ausdrücke auf Potenzreihen. Es wird: das Potential

$$\begin{aligned}(56)\quad V = {} & M_1 M_1' e^{-3}(-2c\gamma + s\sigma) + M_3 M_1' e^{-5}(6c\gamma - \tfrac{3}{2}s\sigma \\ & - 10c^3\gamma + \tfrac{15}{2}c^2 s\sigma) + M_1 M_3' e^{-5}(\ldots.) \\ & + M_5 M_1' e^{-7}(-\tfrac{45}{4}c\gamma + \tfrac{15}{8}s\sigma + \tfrac{105}{2}c^3\gamma - \tfrac{105}{4}c^2 s\sigma \\ & - \tfrac{189}{4}c^5\gamma + \tfrac{315}{4}c^4 s\sigma) \\ & + M_3 M_3' e^{-7}(-\tfrac{135}{2}c\gamma + \tfrac{25}{4}s\sigma + \tfrac{195}{2}c^3\gamma - \tfrac{115}{4}c^2 s\sigma \\ & - \tfrac{115}{4}s\gamma^2\sigma + \tfrac{195}{2}c\gamma^3 - \tfrac{295}{2}c^3\gamma^3 + \tfrac{565}{4}c^2 s\gamma^2\sigma) \\ & + M_1 M_5' e^{-7}(\ldots.) + M_7 M_1' e^{-9}(\tfrac{35}{2}c\gamma - \tfrac{35}{16}s\sigma - \tfrac{315}{2}c^3\gamma \\ & + \tfrac{945}{16}c^2 s\sigma + \tfrac{693}{2}c^5\gamma - \tfrac{3465}{16}c^4 s\sigma - \tfrac{429}{2}c^7\gamma + \tfrac{3003}{16}c^6 s\sigma) \\ & + M_5 M_3' e^{-9}(\tfrac{525}{2}c\gamma - \tfrac{245}{16}s\sigma - 1155c^3\gamma + \tfrac{1645}{8}c^2 s\sigma \\ & + \tfrac{1953}{2}c^5\gamma - \tfrac{4725}{16}c^4 s\sigma - \tfrac{735}{2}c\gamma^3 + \tfrac{1085}{16}s\gamma^2\sigma \\ & + 1645c^3\gamma^3 - \tfrac{7525}{8}c^2 s\gamma^2\sigma - \tfrac{2835}{2}c^5\gamma^3 + \tfrac{22365}{16}c^4 s\gamma^2\sigma) \\ & + M_3 M_5' e^{-9}(\ldots.) + M_1 M_7' e^{-9}(\ldots.) + \ldots\ldots\end{aligned}$$

Digitized by Google

und das Drehungsmoment

$$
\begin{aligned}
(57)\quad X = &- M_1 M_1' e^{-3} (2 c\sigma + s\gamma) \\
&+ M_3 M_1' e^{-5} (6 c\sigma + \frac{3}{2} s\gamma - 10 c^3\sigma - \frac{15}{2} c^2 s\gamma) \\
&+ M_1 M_3' e^{-5} (6 c\sigma + \frac{33}{2} s\gamma - 30 c\gamma^2\sigma - \frac{45}{2} s\gamma^3) \\
&- M_5 M_1' e^{-7} (\frac{45}{4} c\sigma + \frac{15}{8} s\gamma - \frac{105}{2} c^3\sigma - \frac{105}{4} c^2 s\gamma \\
&\qquad + \frac{189}{4} c^5\sigma + \frac{315}{8} c^4 s\gamma) \\
&- M_3 M_3' e^{-7} (\frac{135}{2} c\sigma + \frac{255}{4} s\gamma - \frac{585}{2} c\gamma^2\sigma - \frac{345}{4} s\gamma^3 \\
&\qquad - \frac{195}{2} c^3\sigma - \frac{1245}{4} c^2 s\gamma + \frac{885}{2} c^3\gamma^2\sigma + \frac{1695}{4} c^2 s\gamma^3) \\
&- M_1 M_5' e^{-7} (\frac{45}{4} c\sigma + \frac{435}{8} s\gamma - \frac{315}{2} c\gamma^2\sigma - \frac{945}{4} s\gamma^3 \\
&\qquad + \frac{945}{4} c\gamma^4\sigma + \frac{1575}{8} s\gamma^5) + \ldots .
\end{aligned}
$$

Am einfachsten gestaltet sich die Vergleichung von (56) mit Chwolson's bis e^{-7} gehendem Ausdruck (auf S. 4 der in Anm. 3 genannten Arbeit), der, abgesehen von der Bezeichnung, nur in der Anordnung und Zusammenfassung der Glieder von dem hier gefundenen abweicht. (Das an einer Stelle darin vorkommende $\cos(\theta - \theta')$, d. i. $\cos(\beta - \phi)$, kann als blosse gelegentliche Abkürzung für $(c\gamma + s\sigma)$ betrachtet werden.)

Die anderen genannten Autoren führen dagegen, wie bemerkt, $\Delta = \beta - \phi$ grundsätzlich als Variable ein, so dass einige Umrechnung nötig ist, um die Identität ihrer Ausdrücke mit den vorstehenden Resultaten zu erkennen. Es mag genügen, zur Veranschaulichung die ersten Glieder ihrer Formeln, in die Bezeichnungsweise der vorliegenden Arbeit umgeschrieben, mitzuteilen. Sie lauten:

$$
\begin{aligned}
(58)\quad V = &\; M_1 M_1' e^{-3} (\cos\Delta - 3 c\gamma) \\
&+ M_3 M_1' e^{-5} (-\frac{3}{2}\cos\Delta + \frac{15}{2} c^2\cos\Delta - \frac{35}{2} c^3\gamma + \frac{15}{2} c\gamma) \\
&+ M_1 M_3' e^{-5} (-\frac{3}{2}\cos\Delta + \frac{15}{2}\gamma^2\cos\Delta - \frac{35}{2} c\gamma^3 + \frac{15}{2} c\gamma) \\
&+ \ldots
\end{aligned}
$$

Digitized by Google

und

$$(59)\quad X = -M_1 M_1' e^{-3}(\sin\Delta + 3c\sigma) + M_2 M_1' e^{-5}\left(\frac{3}{2}\sin\Delta - \frac{15}{2}c^2\sin\Delta + \frac{35}{2}c^3\sigma - \frac{15}{2}c\sigma\right) + M_1 M_2' e^{-5}\left(\frac{3}{2}\sin\Delta - \frac{15}{2}\gamma^2\sin\Delta + 15\gamma\sigma\cos\Delta - \frac{105}{2}c\gamma^2\sigma + \frac{15}{2}c\sigma\right) + \ldots$$

Anhangsweise füge ich noch die wichtigsten Zusatzglieder in der Entwickelung des Potentials für nicht vollkommen reguläre Magnete hinzu. Ich beschränke mich dabei auf die von M_2, M_2' und M_{22}, M_{22}' abhängigen Terme, die mit keiner höheren Potenz als e^{-5} multipliziert sind. Das genügt in allen praktisch vorkommenden Fällen, da es sich dabei nur um Grössen handelt, die gelegentlich als Korrektionen auftreten können. Für den Fall allgemeiner Lage von Stab und Nadel sind diese Glieder in den Formeln (50), (51a) und (51b) angegeben. Setzt man darin gemäss den Bedingungen des vorliegenden Spezialfalles $\psi = \frac{\pi}{2}$ und $\eta = \frac{\pi}{2}$, so findet man sofort als entsprechende Korrektion von V [27]

$$(60)\quad \partial V = +\frac{3}{4}M_2 M_1' e^{-4}(\gamma_1 + 3c_2\gamma_1 - 2s_2\sigma_1) - \frac{3}{4}M_1 M_2' e^{-4}(c_1 + 3c_1\gamma_2 - 2s_1\sigma_2) + \frac{3}{16}M_2 M_2' e^{-5}(3 + 5c_2 + 5\gamma_2 + 19c_2\gamma_2 - 16s_2\sigma_2) + \ldots - \frac{\sqrt{15}}{8}M_{22} M_1' e^{-5}(4c_1\gamma_1 - s_1\sigma_1 - 4c_3\gamma_1 + 3s_3\sigma_1) - \frac{\sqrt{15}}{8}M_1 M_{22}' e^{-5}(4c_1\gamma_1 - s_1\sigma_1 - 4c_1\gamma_3 + 3s_1\sigma_3) + \ldots$$

[27] Man beachte, dass beiden Glieder mit den Faktoren $M_2 M_1'$ und $M_1 M_2$ entgegengesetztes Vorzeichen haben. Der Grund dieser im ersten Augenblick vielleicht auffallenden Asymmetrie, die übrigens schon in der Fundamentalformel (40) im Exponenten von (—1) hervortritt, ist der, dass bei der Definition der Winkel β und ϕ die Beziehung auf den Stab und die Nadel nicht dieselbe ist. Vertauscht man die Bedeutung der beiden, so gehen β und ϕ in $(\pi + \beta)$ und $(\pi + \phi)$ über.

Digitized by Google

und als Korrektion von X:

$$(61)\quad \partial X = +\frac{3}{4} M_2 M_1' e^{-4} (\sigma_1 + 3 c_2 \sigma_1 + 2 s_2 \gamma_1)$$
$$-\frac{3}{2} M_1 M_2' c^{-4} (3 c_1 \sigma_2 + 2 s_1 \gamma_2)$$
$$+\frac{3}{8} M_2 M_2' e^{-5} (5 \sigma_2 + 19 c_2 \sigma_2 + 16 s_2 \gamma_2) + \ldots$$
$$-\frac{\sqrt{15}}{8} M_{22} M_1' e^{-5} (4 c_1 \sigma_1 + s_1 \gamma_1 - 4 c_3 \sigma_1 - 3 s_3 \gamma_1)$$
$$-\frac{\sqrt{15}}{8} M_1 M_{22}' e^{-5} (4 c_1 \sigma_1 + s_1 \gamma_1 - 12 c_1 \sigma_3 - 9 s_1 \gamma_3) + \ldots$$

Der zweite Teil der in (60) angegebenen Korrektion lässt sich mit dem von Chwolson für Magnete mit 8 Polpunkten abgeleiteten Resultat vergleichen, das sich auf S. 17 seiner in Anm. 3 genannten Arbeit findet. Der entsprechende Anteil des dort in Formel 35 auftretenden Ausdrucks lautet, teilweise bereits umgeschrieben, folgendermassen:

$$(62)\quad -\frac{3}{2} m m_1 e^{-5} \{ (y^2+y_1^2+z^2+z_1^2) (s\sigma - 4c\gamma) + 5 (y^2\sigma^2+y_1^2 s^2)$$
$$(6 c\gamma - s\sigma) - 10 (y^2 \gamma\sigma - y_1^2 c s) (c\sigma - s\gamma) \}$$

mit $m = 8\mu x$ und $m_1 = 8\mu_1 x_1$. Dabei sind μ, x, y, z bei der Nadel und μ_1, x_1, y_1, z_1 bei dem Stabe dasjenige, was hier an einer früheren Stelle $\bar{\mu}$, a, b, c genannt wurde (vgl. S. 197).

Der vorstehende, in Bezug auf die beiden Magnete symmetrische Ausdruck lässt sich in den folgenden umwandeln, von dem es genügt, den die Nadel betreffenden Teil anzuschreiben:

$$(63)\quad -\frac{3}{2} m m_1 e^{-5} \{ (y^2 + z^2) (6 c\gamma - \frac{3}{2} s\sigma - 10 c\gamma^3 + \frac{15}{2}\gamma^2 s\sigma)$$
$$+\frac{5}{2} (y^2 - z^2) (4 c\gamma\sigma^2 - s\sigma + 3\gamma^2 s\sigma) + \ldots . \}$$

Der Faktor von $-\frac{3}{2} m m_1 (y^2 + z^2)$ hierin stimmt, wie man sieht, mit demjenigen von $M_1 M_3'$ in (56) überein, was dort, als auf lineare Magnete bezüglich, an Stelle von $m_1 m x^2$ bei Chwolson steht. Der durch Zusammenfassung entstehende Wert $m[x^2 - \frac{3}{2}(y^2+z^2)]$, als dessen Faktor $m_1 e^{-5}(6c\gamma - \frac{3}{2} s\sigma - 10c\gamma^3 + \frac{15}{2}\gamma^2 s\sigma)$ auftritt, ist nun nach früheren Ausführungen (vgl. Formel 18) gerade das für den hier angenommenen Magnet geltende M_{30}' oder kurz M_3'. Entsprechendes, wie hier bei der Nadel, ergibt sich bei dem Stabe. Damit gelangt man wieder auf den Ausdruck von Formel (56) zurück, dem als Korrektion nur noch der zweite Teil

Digitized by Google

von (61) nebst dem entsprechenden Betrage für den Stab hinzuzufügen ist. Beachtet man nun, dass $\frac{15}{4} m (y^2 - z^2)$ nach (18) gleich $\frac{1}{2} \sqrt{15}\, M_{32}'$ und der trigonometrische Faktor gleich $\frac{1}{4}(4 c_1 \gamma_1 - s_1 \sigma_1 - 4 c_1 \gamma_3 + 3 s_1 \sigma_3)$ ist, so erkennt man die Uebereinstimmung mit der in (60) auftretenden Korrektions-Grösse. Nur ist letztere von allgemeinerer Bedeutung, da sie für jeden Magnet und nicht nur für den schematisierten Fall von Magneten mit 8 Polpunkten gilt. Uebrigens hat C h w o l s o n selbst sein Resultat noch verallgemeinert (s. § 4 seiner Arbeit), indem er es auf alle Magnete mit 3 Symmetrieebenen ausdehnte.

Besondere Lagen.

Die erste Gaussische Hauptlage ist durch $\beta = \pi$ charakterisiert.[28] Setzt man dies in (55) ein, so erhält man den folgenden Ausdruck für das Drehungsmoment, das ich der Kürze halber allein anschreiben will:

$$
\begin{aligned}
(64)\quad X = {} & 2 M_1 M_1' e^{-3} \sigma_1 + 4 M_3 M_1' e^{-5} \sigma_1 + \frac{3}{2} M_1 M_3' e^{-5} (\sigma_1 + 5 \sigma_3) \\
& + 6 M_5 M_1' e^{-7} \sigma_1 + \frac{15}{2} M_3 M_3' e^{-7} (\sigma_1 + 5 \sigma_3) \\
& + \frac{45}{32} M_1 M_5' e^{-7} (\sigma_1 + \frac{7}{2} \sigma_3 + \frac{21}{2} \sigma_5) + 8 M_7 M_1' e^{-9} \sigma_1 \\
& + \frac{21}{2} M_5 M_3' e^{-9} (\sigma_1 + 5 \sigma_3) + \frac{105}{8} M_3 M_5' e^{-9} (\sigma_1 + \frac{7}{2} \sigma_3 \\
& \qquad + \frac{21}{2} \sigma_5) \\
& + \frac{175}{128} M_1 M_7' e^{-9} (\sigma_1 + \frac{81}{25} \sigma_3 + \frac{33}{5} \sigma_5 + \frac{429}{25} \sigma_7) + \ldots .
\end{aligned}
$$

Der Ausdruck für das Potential lässt sich hieraus ohne weiteres ablesen; man hat nur überall σ_n durch $\frac{1}{n} \gamma_n$ zu ersetzen. Wie der unmittelbare Anblick zeigt, sind die darin auftretenden trigonometrischen Ausdrücke bis auf Zahlenfaktoren mit den Kugelfunktionen von ϕ identisch. Einen besonderen Beweis dafür der sich natürlich auch mit den hier gegebenen Hilfsmitteln erbringen liesse, hat C h w o l s o n in seiner mehrfach erwähnten Arbeit (S. 5) gege-

[28] Es läge eigentlich näher, $\beta = 0$ und weiterhin bei der 2. Hauptlage $\beta = \frac{\pi}{2}$ zu setzen. Die hier getroffene Wahl war durch den Wunsch bestimmt, den Hauptgliedern der speziellen Formeln, wie üblich, das positive Vorzeichen zu geben.

Digitized by Google

ben Setzt man abkürzend P_0^n statt $P_0^n(\cos\phi)$, so ergibt sich für das Potential:

$$(65)\quad V = 2M_1' e^{-3} P_0^1 (M_1 + 2M_3 e^{-2} + 3M_5 e^{-4} + 4M_7 e^{-6}) + 4M_3' e^{-5} P_0^3 (M_1 + 5M_3 e^{-2} + 7M_5 e^{-4}) + 6M_5' e^{-7} P_0^5 (M_1 + \tfrac{28}{3} M_3 e^{-2}) + 8M_7' e^{-9} P_0^7 M_1 + \ldots$$

Die in (64) gegebene Form des Wertes von X empfiehlt sich bei allgemeinen Betrachtungen, sowie insbesondere zur Unter-

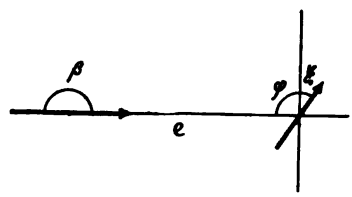


FIG. 3. *Erste Hauptlage.*

suchung der Schwingungen, die die Nadel unter dem Einflusse des Stabs ausführt. Bei der Behandlung von Ablenkungsbeobachtungen ist es dagegen zweckmässiger und auch allgemein üblich, die Ablenkung nicht durch den Winkel ϕ, sondern durch $\zeta = \phi - \frac{\pi}{2}$ zu messen, dessen Nullwert der ersten Lamontschen Hauptlage entspricht. Es ist dazu nur $\sigma_1 = \cos\zeta$, $\sigma_3 = -\cos 3\zeta$ u. s. w. einzusetzen. Ausserdem pflegt man den Faktor $\cos\zeta$ abzusondern. Benutzt man zu diesem Zwecke die Beziehungen $\cos 3\zeta : \cos\zeta = 2\cos 2\zeta - 1$, $\cos 5\zeta : \cos\zeta = 2\cos 4\zeta - 2\cos 2\zeta + 1$ u. s. w., so erhält man für $X : \cos\zeta$ wieder eine nach Funktionen der Vielfachen von ζ fortschreitende Reihe, deren Anführung der Kürze halber unterbleiben mag. Wählt man andererseits die Beziehungen $\cos 3\zeta : \cos\zeta = 1 - 4\tau^2$, $\cos 5\zeta : \cos\zeta = 1 - 12\tau^2 + 16\tau^4$, $\cos 7\zeta : \cos\zeta = 1 - 24\tau^2 + 80\tau^4 - 64\tau^6$, worin τ für $\sin\zeta$ geschrieben ist, so folgt die bekannte Formel:

$$(66)\quad X = \cos\zeta \Big\{ 2M_1 M_1' e^{-3} + 4M_3 M_1' e^{-5} - 6M_1 M_3' e^{-5}(1-5\tau^2) + 6M_5 M_1' e^{-7} - 30M_3 M_3' e^{-7}(1-5\tau^2) + \tfrac{45}{4} M_1 M_5' e^{-7}(1-14\tau^2+21\tau^4) + 8M_7 M_1' e^{-9} - 42M_5 M_3' e^{-9}(1-5\tau^2) + 105M_3 M_5' e^{-9}(1-14\tau^2+21\tau^4) - \tfrac{35}{2} M_1 M_7' e^{-9}(1-27\tau^2+99\tau^4-\tfrac{429}{5}\tau^6) + \ldots \Big\}.$$

Digitized by Google

die sich in etwas anderer Anordnung bei Fritsche (S. 26), in wesentlich gleicher Form wie hier bei Börgen (*Terr. Magn.* I, S. 186 mit dem ungenauen Koeffizienten 105 statt 99 im vorletzten Gliede) und bei Leyst (S. 102) findet.

Die Hauptglieder der bei unvollkommen regulären Magneten anzubringenden Korrektion sind nach (61) für $\beta = \pi$ die folgenden:

$$(67)\quad \partial X = 3\,M_2\,M_1'\,e^{-4}\,\sigma_1 + \frac{9}{2}\,M_1\,M_2'\,e^{-4}\,\sigma_2 + 9\,M_2\,M_2'\,e^{-5}\,\sigma_2 + \frac{\sqrt{15}}{2}\,M_1\,M_{22}'\,(\sigma_1 - 3\sigma_3) + \ldots$$

$$= \cos\zeta\left\{3\,M_2\,M_1'\,e^{-4} - 9\,M_1\,M_2'\,e^{-4}\,\tau - 18\,M_2\,M_2'\,e^{-5}\,\tau + 2\sqrt{15}\,M_1\,M_{22}'\,e^{-5}\,(1 - 3\tau^2) + \ldots.\right\}$$

Wie man sieht, verschwindet der Faktor von $M_{22}\,M'_1$ für jeden Wert von τ, ein praktisch nicht unwichtiges Resultat, das sich noch verallgemeinern lässt.

Der Vollständigkeit halber sei schliesslich noch die dem speziellen Werte $\zeta = 0$ entsprechende Formel für die erste Lamont'sche Hauptlage, die man aus dem Vorhergehenden ablesen kann, besonders hingeschrieben, und zwar sogleich mit Einschluss der Korrektionsglieder:

$$(68)\quad X = 2\,M_1\,M_1'\,e^{-3} + 3\,M_3\,M_1'\,e^{-4} + e^{-5}\left(4\,M_3\,M_1' - 6\,M_1\left(M_3' - \frac{\sqrt{15}}{3}\,M_{22}'\right)\right) + e^{-7}\,(6\,M_5\,M_1' - 30\,M_3\,M_3' + \frac{45}{4}\,M_1\,M_5') + e^{-9}\,(8\,M_7\,M_1' - 42\,M_5\,M_3' + 105\,M_3\,M_5' - \frac{35}{2}\,M_1\,M_7') + \ldots.$$

Die zweite Gaussische Hauptlage entspricht dem Ansatze $\beta = \frac{3\pi}{2}$. Dieser liefert für das Drehungsmoment, das wieder allein angegeben werden soll:

$$(69)\quad X = M_1\,M_1'\,e^{-3}\,\gamma_1 - \frac{3}{2}\,M_3\,M_1'\,e^{-5}\,\gamma_1 + \frac{3}{8}\,M_1\,M_3'\,e^{-5}\,(\gamma_1 + 15\,\gamma_3) + \frac{15}{5}\,M_5\,M_1'\,e^{-7}\,\gamma_1 - \frac{15}{16}\,M_3\,M_3'\,e^{-7}\,(\gamma_1 + 23\,\gamma_3) + \frac{15}{64}\,M_1\,M_5'\,e^{-7}\,(\gamma_1 + \frac{21}{2}\,\gamma_3 + \frac{105}{2}\,\gamma_5) - \frac{35}{16}\,M_7\,M_1'\,e^{-9}\,\gamma_1 + \frac{105}{64}\,M_5\,M_3'\,e^{-9}\,(\gamma_1 + 31\,\gamma_3) - \frac{105}{128}\,M_3\,M_5'\,e^{-9}\,(\gamma_1 + \frac{29}{2}\,\gamma_3 + \frac{225}{2}\,\gamma_5) + \frac{175}{1024}\,M_1\,M_7'\,e^{-9}\,(\gamma_1 + \frac{243}{25}\,\gamma_3 + 33\,\gamma_5 + \frac{3003}{25}\,\gamma_7) + \ldots$$

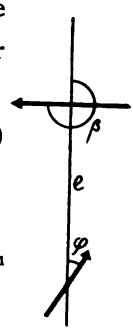


FIG. 4. *Zweite Hauptlage.*

Digitized by Google

oder in der üblichen, durch Absonderung des Faktors γ enstehenden Form

$$(70)\quad X=\cos\phi\Big\{M_1M_1'e^{-3}-\frac{3}{2}M_3M_1'e^{-5}+6M_1M_3'e^{-5}\Big(1-\frac{15}{4}\sigma^2\Big)+\frac{15}{8}M_5M_1'e^{-7}-\frac{45}{2}M_3M_3'e^{-7}\Big(1-\frac{23}{6}\sigma^2\Big)$$
$$+15M_1M_5'e^{-7}\Big(1-\frac{21}{2}\sigma^2+\frac{105}{8}\sigma^4\Big)$$
$$-\frac{35}{16}M_7M_1'e^{-9}+\frac{105}{2}M_5M_3'e^{-9}\Big(1-\frac{31}{8}\sigma^2\Big)$$
$$-105M_3M_5'e^{-9}\Big(1-11\sigma^2+\frac{225}{16}\sigma^4\Big)$$
$$+28M_1M_3'e^{-9}\Big(1-\frac{81}{4}\sigma^2+\frac{495}{8}\sigma^4-\frac{3003}{64}\sigma^6\Big)+\ldots\Big\}$$

Der zweite Ausdruck lässt sich wiederum durch Vergleichung mit den a. a. O. zu findenden Resultaten der drei genannten Autoren verifizieren, die die Entwickelung gleichfalls bis e^{-9} ausgedehnt haben.

Die Hauptglieder der Korrektion, die bei nicht vollkommen regulären Magneten hinzuzufügen ist, werden im vorliegenden Falle:

$$(71)\quad \partial X=-\frac{3}{2}M_2M_1'e^{-4}\sigma_1+3M_1M_2'e^{-4}\gamma_2-\frac{21}{4}M_2M_2'e^{-5}\sigma_2$$
$$+\frac{\sqrt{15}}{2}M_{22}M_1'e^{-5}\gamma_1+\frac{\sqrt{15}}{8}M_1M_{22}'e^{-5}(\gamma_1-9\gamma_2)$$
$$=-\frac{3}{2}M_2M_1'e^{-4}\sigma+3M_1M_2'e^{-4}(1-2\sigma^2)+\ldots$$
$$+\cos\phi\Big\{-\frac{21}{2}M_2M_2'e^{-5}\sigma+\frac{\sqrt{15}}{2}M_{22}M_1'e^{-5}$$
$$-\sqrt{15}M_1M_{22}e^{-5}\Big(1-\frac{9}{2}\sigma^2\Big)+\ldots.\Big\}$$

Für die durch $\phi=0$ definierte zweite Lamont'sche Hauptlage folgt:

$$(72)\quad X=M_1M_1'e^{-3}+3M_1M_2'e^{-4}+e^{-5}\Big(-\frac{3}{2}\Big(M_3-\frac{\sqrt{15}}{3}M_{22}\Big)M_1'$$
$$+6M_1\Big(M_3'-\frac{\sqrt{15}}{6}M_{22}'\Big)\Big)$$
$$+e^{-7}\Big(\frac{15}{8}M_5M_1'-\frac{45}{2}M_3M_3'+15M_1M_5'\Big)$$
$$+e^{-9}\Big(-\frac{35}{16}M_7M_1'+\frac{105}{2}M_5M_3'-105M_3M_5'+28M_1M_7'\Big)+\ldots$$

Digitized by Google

Von besonderen Lagen verdient noch diejenige Beachtung, bei der die Achsen der beiden Magnete zu einander senkrecht sind, während die Mittelpunkte beliebig liegen. Als spezielle Fälle sind darin sowohl die Lamont'schen Hauptlagen als auch die von Börgen zur Bestimmung des Polabstands empfohlene Hauptlage ohne Ablenkung enthalten.

Diese Lage ist allgemein durch die Bedingung $\beta = \frac{\pi}{2} + \phi$ charakterisiert. Bei ihrer doch nur sekundären Bedeutung mag es genügen, die zugehörige Entwickelung für das Drehungsmoment X bis e^{-7} hinzuschreiben, wie sie sich durch die Substitution $c = -\sigma$, $s = \gamma$ aus (57) ergibt. Drückt man noch alles durch sin ϕ aus, so erhält man den Ausdruck in der Form, in der ihn auch Leyst (S. 117) angibt (doch mit umgekehrtem Vorzeichen, da a. a. O. $\beta = \phi - \frac{\pi}{2}$ ist)

$$(73)\quad X = -M_1 M_1' e^{-3}(1-3\sigma^2) + M_3 M_1' e^{-5}(\tfrac{3}{2} - 15\sigma^2 + \tfrac{35}{2}\sigma^4) + M_1 M_3' e^{-5}(-6 + \tfrac{105}{2}\sigma^2 - \tfrac{105}{2}\sigma^4) - M_5 M_1' e^{-7}(\tfrac{15}{8} - \tfrac{315}{8}\sigma^2 + \tfrac{945}{8}\sigma^4 - \tfrac{693}{8}\sigma^6) - M_3 M_3' e^{-7}(-\tfrac{45}{4} + \tfrac{1785}{4}\sigma^2 - 1260\sigma^4 + \tfrac{3465}{4}\sigma^6) - M_1 M_5' e^{-7}(15 - \tfrac{525}{2}\sigma^2 + \tfrac{5355}{8}\sigma^4 - \tfrac{3465}{8}\sigma^6) + \ldots$$

Die aus (61) hervorgehende Korrektion für unvollkommen reguläre Magnete wird:

$$(74)\quad \partial X = -\frac{9}{2} M_2 M_1' e^{-4}\sigma(1 - \tfrac{5}{3}\sigma^2) - 3 M_1 M_2' e^{-4}\gamma(1 - 5\sigma^2) - \frac{45}{2} M_2 M_2' e^{-5}\gamma\sigma(1 - \tfrac{7}{3}\sigma^2) + \ldots - \frac{\sqrt{15}}{2} M_{22} M_1' e^{-5}(1 - 8\sigma^2 + 7\sigma^4) + \sqrt{15}\, M_1 M_{22}' e^{-5}(1 - \tfrac{19}{2}\sigma^2 + \tfrac{21}{2}\sigma^4)$$

Soll das Drehungsmoment verschwinden, so muss annähernd $1 - 3\sigma^2 = 0$ sein. Es empfiehlt sich deshalb, die Entwickelung für diesen Fall durch die Substitution $1 - 3\sigma^2 = \epsilon$ umzuformen. Man findet so unter Hinzufügung der Korrektionsglieder:

Digitized by Google

$$(75)\quad X = -M_1 M_1' e^{-3}\epsilon - 2 M_2 M_1' e^{-4}\sigma\left(1+\frac{5}{4}\epsilon\right)$$
$$+ 2 M_1 M_2' e^{-4}\gamma\left(1-\frac{5}{2}\epsilon\right) - 5 M_2 M_2' e^{-5}\gamma\sigma\left(1+\frac{7}{2}\epsilon\right)$$
$$-\frac{14}{9} M_3 M_1' e^{-5}\left(1-\frac{5}{7}\epsilon-\frac{5}{4}\epsilon^2\right)+\frac{17}{3} M_1 M_3' e^{-5}\left(1-\right.$$
$$\left.\frac{35}{34}\epsilon-\frac{35}{34}\epsilon^2\right)+\frac{4}{9}\sqrt{15}\, M_{32} M_1' e^{-5}\left(1-\frac{5}{4}\epsilon-\frac{7}{8}\epsilon^2\right)$$
$$-\sqrt{15}\, M_1 M_{32}' e^{-5}\left(1-\frac{5}{6}\epsilon-\frac{7}{6}\epsilon^2\right)$$
$$+\frac{4}{3} M_5 M_1' e^{-7}\left(1+\frac{21}{8}\epsilon-\frac{21}{8}\epsilon^2-\frac{77}{32}\epsilon^3\right)$$
$$-\frac{55}{3} M_3 M_3' e^{-7}\left(1+\frac{21}{11}\epsilon-\frac{105}{44}\epsilon^2-\frac{7}{4}\epsilon^3\right)$$
$$+\frac{85}{6} M_1 M_5' e^{-7}\left(1+\frac{63}{68}\epsilon-\frac{63}{34}\epsilon^2-\frac{77}{68}\epsilon^3\right)+\ldots$$

Für $\epsilon = -2$ erhält man die erste, für $\epsilon = 1$ die zweite Lamont'sche Hauptlage, letztere mit umgekehrter Stabrichtung wie bei Formel (72). Für die Lage ohne Ablenkung, d. h. für $X = 0$, wird in erster Näherung

$$M_1 M_1' e^{-3}\epsilon = -\frac{14}{9} M_3 M_1' e^{-5} + \frac{17}{3} M_1 M_3' e^{-5}$$

oder, wenn $2l$ und $2l'$ die Poldistanzen von Stab und Nadel sind,

$$\epsilon = \left(-\frac{14}{9} l^2 + \frac{17}{3} l'^2\right) : e^2$$

Zum Uebergang auf die Schreibweise von Börgen hat man $\sigma = h : e$ zu setzen. Es ist also genähert $e^2 - 3h^2 = -\frac{14}{9} l^2 + \frac{17}{3} l'^2$.

(*Schluss folgt.*)

Digitized by Google

ALIBAG MAGNETIC OBSERVATORY.

By N. A. F. Moos.

Early in 1900 the city of Bombay finally decided to employ electricity as motive power for its street tram service. Magnetic work, hence, which had run uninterruptedly for over fifty years by then at the Bombay Observatory, was thus threatened by the proposed introduction of the electric traction, and prompt action in the interests of the Observatory seemed essentially urgent. Efforts to find some *modus vivendi* which might allow this work to be continued at the Observatory without serious vitiation from the disturbing effects of the lines, having proved abortive, no alternative seemed possible but to remove the magnetic work to some suitable site in the neighborhood. It became imperative under the circumstances to make provision without delay for the establishment of the new Observatory at some protected site elsewhere, which might allow comparative observations for a fairly long period to be made at the old and new sites before the conversion of the ordinary tram-lines into electric traction was completed, and enable, if possible, the continuity of the Bombay record to be preserved.

The necessary sanction and orders were graciously issued by Government and a suitable site, one least open to objection, from several recommended, was selected some eighteen miles away from Bombay. It was far away, enough to be beyond the disturbing effects of the electric lines in Bombay, but sufficiently near, presumably, to have similar magnetic conditions. The geographical position of Alibag also was otherwise favorable. There was no reasonable probability, at least in the near future, of a railway service (eventually driven by electric motive power) extending southwards from the Bombay line along the coast through Alibag, or passing by it near enough to affect the magnetic work and compel the removal of the Observatory again to another site in a few decades.

The site was secured in 1902. The construction of the buildings, the description of which follows, was completed in 1903, and comparative observations commenced in April, 1904. After securing an undisturbed duplicate record extending over a period of two years, the magnetic work at Colaba was stopped on 31st March, 1905, fortunately before the electric tram service was completed and had commenced its operations. Disturbed records at Colaba were subsequently also secured for a few days when the electric tram-lines were working in Bombay. The comparative results of absolute values of the three elements *H, D,* and *I,* at the two sites show that at Alibag *H* stands

Digitized by Google

lower by 510 γ, D (easterly) stands higher by 53'.5, and I stands higher by 1° 4'.7 than at Colaba (Bombay), which are the finally adopted differences. The variations of H, D, and V were secured by a set of Watson's magnetographs at Alibag, and Kew type magnetographs at Colaba, and the results show, as anticipated, that the variation phenomena were precisely the same at the two sites, agreeing to within a γ or two in the intensity variations, and 0'.1 in the declination variation.

Alibag is a small town on the mainland, lying across the harbor about eighteen miles south-southeast of Bombay. The site of the Observatory is located on the foreshore, facing southwest, with a fair extent of sandy beach stretching before it. The ground, unlike Colaba, is flat and low, lying about 15 feet above the mean sea-level. The situation of the Observatory is otherwise fairly similar to that at Colaba. The rocks which abound on the surrounding coasts are traps, as at Colaba. The chain of small hills on the north and northeastwards about four miles away from Alibag practically rises in small mounds and rocky slopes, which gather into a more or less central range of hills running northwest to southeast, of which Kankeshwar, Sakarghad, and Ramdharan, about 1,200 feet high, form the highest peaks.

The geographical position of the Observatory is: Latitude, 18° 38'.3 N.; longitude, 72° 52'.4 E. The Observatory grounds, consisting of about seven acres, are self-contained, being bounded on three sides by roads and on the southwest side by the sea. The Observatory consists, besides the Director's quarters, assistants' residence, and office and photographic room, of two buildings: (1) the magnetometer office, where absolute observations are made and a set of eye-reading variometers is located, and (2) a magnetograph building, in which two sets of variation instruments (by Watson and Eschenhagen) have been installed. Both buildings have been constructed absolutely of nonmagnetic materials. All the fittings are of either copper or brass duly tested before their use in the buildings, and the building materials throughout are nonmagnetic, the stone used being a pure sandstone from Porebunder in Kathiawar.

The magnetometer building, a plan and section of which is given in Fig. 1, consists of a basement room and a tower, with two floors. In the basement room is mounted the transit instrument for the determination of time locally. For this purpose the flat roof is so constructed as to allow of a portion of it to be opened for star observations. In the same room are mounted, also on isolated pillars, the magnetographs (Kew type) which have been in action at Colaba for thirty-five years, but which are used here as simple eye-reading variometers only (observed five times a day). In the tower-room on the second floor are mounted the observatory standard instruments—magnetometer No. 7, by Cooke, Schulze's inductor, and dip circle. On the first floor

Digitized by Google

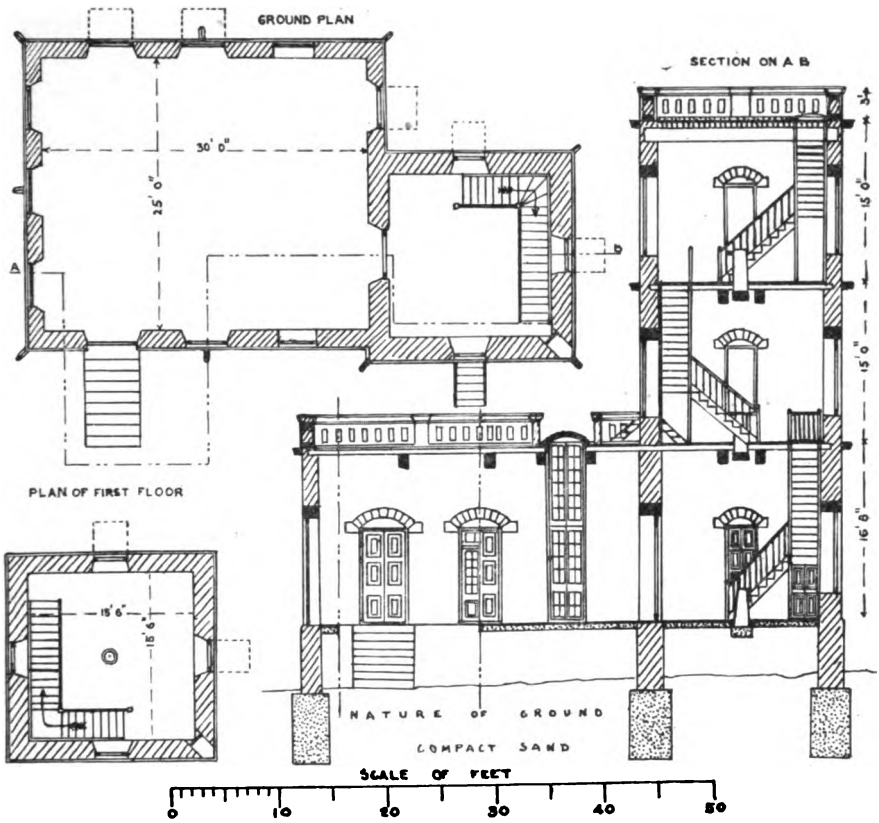


FIG. 1. *Magnetometer Building.*

auxiliary observations of H and D are taken with magnetometer No. 3, by the Cambridge Scientific Instrument Company. On the ground floor magnetometer No. 137, by Dover, of Charlton, is used for deflection experiments, which serve as a check on the deflection experiments of magnetographs for standardizing the curves. The reference mark used for azimuth observations is the Kennery lighthouse, situated some five miles away from Alibag, which can also be seen from Colaba and has served the same purpose there for the last fifty years. This mark, the azimuth of which is 137° 53′ 25″ W. of S., can be seen from any of the three tower-rooms through specially constructed slit windows which are provided in the corners of the tower-rooms, as the building is oriented N-S and E-W (*vide* plate No. II, this Journal, June, 1912).

The recording variation instruments are mounted in an *overground* magnetograph building, the plan and section of which are seen in Fig. 2

Digitized by Google

(*vide* plate II, *idem*). The inner room is 25 x 30 feet. It is surrounded by walls 3 feet thick of Porebunder stone. Access to the room is obtained by a single entrance centrally placed in the west wall. This is reached by a narrow passage all round the room, entrance to which from outside is gained from the east side. The passage is formed by the walls of the inner room and a wooden sawdust compartment three feet thick. An air space 3 feet wide is left all round the wooden compartment, and the whole is then surrounded by the outer wall of the building, 3 feet thick, of Porebunder stone. The sawdust compartment, it will be seen from the section given, is so designed that it completely encircles the inner magnetograph chamber, passing over the top of this room below the flagstones of the flat roof above, and underneath the flagstones of the floor of the chamber. Suitably designed ventilators are provided both for the ingress and egress of the air at suitable levels. The thermograph curves show that the design has proved successful in keeping the temperature uniform inside the room. The daily changes are practically nil, while the seasonal range is from 6° F. to 8° F. The temperature curve in the Alibag magnetograph room, which is an overground building, it will be seen from the charts, compares most favorably with that of the Colaba room, which is an underground building.

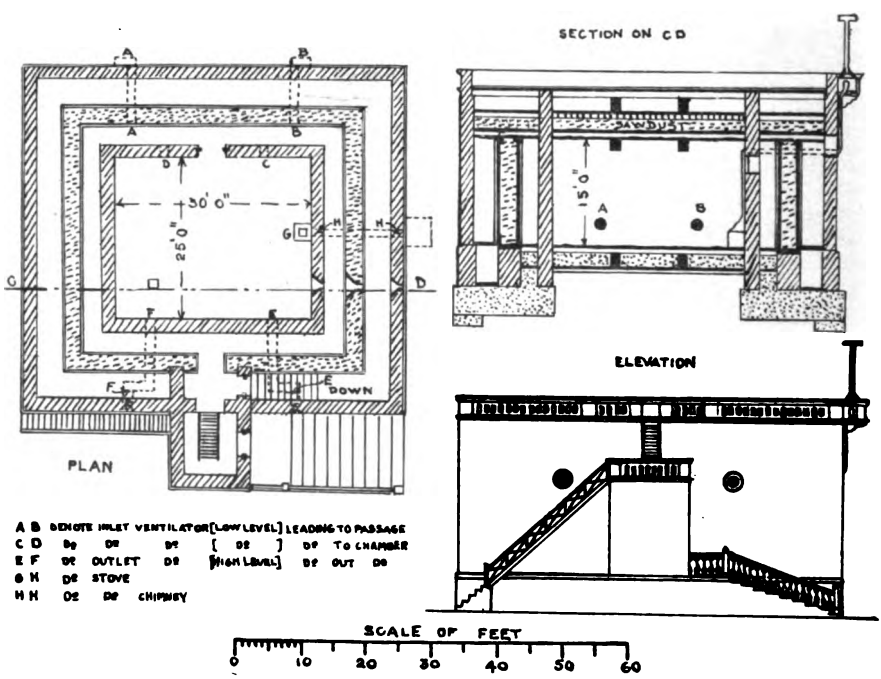

SECTION ON C D
30' 0"
25'0"
15'0"
PLAN
ELEVATION
DOWN
A B DENOTE INLET VENTILATOR [LOW LEVEL] LEADING TO PASSAGE
C D " " " " ["] " TO CHAMBER
E F " OUTLET " [HIGH LEVEL] " OUT "
G H " STOVE
H H " CHIMNEY
SCALE OF FEET
0 10 20 30 40 50 60

FIG. 2. *Magnetograph Building.*

Digitized by Google

Isolated pillars are provided in the magnetograph room for the various instruments, which come up through the wooden compartment and flagstones of the floor. As already remarked elsewhere, the room contains the Watson's recording magnetographs for *H*, *D*, and *V*, and Eschenhagen's variometers for *H* and *V*. The latter are used as eye-reading instruments for control over the Watson's records, simultaneous eye-readings of both being secured five times a day.

The routine work of the Observatory consists of:

1. Absolute observations of *H* made every Wednesday by Cooke's magnetometer on the second floor of the magnetometer tower, and every Monday by the Cambridge Scientific Instrument Company's magnetometer No. 3 on the first floor; of *D* every Thursday by both these instruments on their respective pillars; of *I* by the inductor every day in the week (except on Sundays) on the second floor, and also by the dip circle in the same room on every Saturday.

2. Eye observations of the instruments specified below, which are taken daily (Sundays included) at the five hours, 6, 10, 14, 16, and 22:

(*a*) Declination Variometer and Magnetograph;

(*b*) Vertical Force Variometers (Kew and Eschenhagen) and Watson's Magnetograph;

(*c*) Horizontal Force Variometers (Kew and Eschenhagen) and Watson's Magnetograph;

(*d*) Thermometers of the Variation Instruments and Thermograph in the Magnetograph room.

3. Deflection experiments with the magnet of Magnetometer No. 3, the moment of which is known, as deflector. These are made at 100 cm. distance on the Horizontal Force Variometers, and at 75 cm. distance on the Vertical Force Variometers, once every fortnight. Similar deflection experiments, with the magnet of Magnetometer No. 137 as deflector, are made once a month on the Magnetographs, as check work on the fortnightly work; and corresponding deflection experiments at the same distances are also made every month by the same deflector upon a magnet appropriately suspended over the pillar on the ground floor room of the tower.

4. Transit observations of stars for the determination of time; these are made regularly twice a week.

All other work, including the tabulation of the magnetograms, the reduction of the observations, and special investigations, are carried on at Colaba (Bombay), the routine work there generally consisting of the hourly tabulations and the determination of the diurnal variations and daily means (absolute values) of all days, as also of quiet days selected by the Astronomer Royal and by the International Commission of Terrestrial Magnetism, and of quiet days selected locally.

Digitized by Google

MAGNETIC REACTIONS PRODUCED BY A COPPER DISK ROTATING BETWEEN THE POLES OF A MAGNET.[1]

W. G. CADY AND F. G. BENEDICT.

[Abstract.]

For measuring the amount of work produced by a human being, a "bicycle ergometer" is in use at the Nutrition Laboratory of the Carnegie Institution of Washington. A description is given of this instrument, and of its calibration inside of a large calorimeter. It is found that the amount of heat expended per revolution of the pedals, instead of increasing proportionately with the speed, reaches a maximum and then decreases. Over the limited range of speeds commonly employed, the heat per revolution can be taken as practically a constant quantity. The present investigation was undertaken primarily in order to investigate this peculiarity in the calibrations.

The theory of the eddy currents induced in the copper disk of the bicycle ergometer is discussed, and equations are derived showing the relations between impressed flux, counter-flux due to the eddy currents, speed, and rate of production heat.

To test these relations, the field between the poles of the magnet was explored by means of a bismuth spiral. From the observations, curves were drawn indicating the magnetic field at different points in the air gap and at different speeds. These curves show clearly the distortion of field produced by the rotation of the disk, as well as a pronounced weakening of the field as the speed increases.

By measuring the areas of the curves it is possible to test the validity of the equations derived from theory. These are found to be roughly satisfied, and in particular it is seen that a curve can be drawn, based on magnetic data alone, having the same general form as the actual calibration curve of the bicycle ergometer. This proves conclusively that the peculiar form of the calibration curves is due to the demagnetizing effect of the eddy currents in the disk.

The magnetic field due to the eddy currents in the disk was partially explored with a compass. The distribution of current lines in the disk was also investigated, and an estimate made of the total intensity of the eddy currents. The diminution in total magnetic flux caused by the rotation of the disk was measured with a ballistic galvanometer.

[1] Part III of "A Bicycle Ergometer with an Electric Brake," by F. G. BENEDICT and W. G. CADY, Publ. No. 167, Carnegie Institution of Washington, 1912. *Cf, Phys. ZS.* 13 Jahrg, 1912, 920–930.

Digitized by Google

The weakening of permanent magnets when they are moved relatively to conducting masses is discussed, particularly with reference to damped astatic needles and to the permanent magnets of recording electric meters.

The general principles to be considered in the design of electric brakes and other electric dampening devices are summarized, with special reference to the problem of minimizing the demagnetizing effect of the eddy currents.

Nutrition Laboratory of the
Carnegie Institution of Washington,
Boston, Massachusetts.

PRINCIPAL MAGNETIC STORMS RECORDED AT THE CHELTENHAM MAGNETIC OBSERVATORY.

JULY 1 TO SEPTEMBER 30, 1912.

Latitude 38° 44'.0N; Longitude 76° 50'.5, or 5h 07m.4 W. of Greenwich.

GREENWICH MEAN TIME		RANGE		
Beginning	Ending	D (Declination)	H (Hor'l Int.)	Z (Vert'l Int.)
Day h m	Day h	′	γ	γ
August 5, 14 13	August 5, 15	24.4	141	99
Sept. 17, 13 44	Sept. 18, 15	13.9	89	39
Sept. 24, 0 32	Sept. 24, 12	23.5	85	63

GEORGE HARTNELL, *Observer-in-Charge.*

O. H. TITTMANN, *Superintendent,*
Coast and Geodetic Survey.

NOTES

9. Personalia. Prof. V. Bjerknes, formerly of the University of Christiania, has accepted the newly established professorship in geophysics at the University of Leipzig. *Prof. F. Bidlingmaier* has been chosen to succeed *Prof. J. Messerschmitt,* deceased, as director of the Magnetic and Seismological Observatory at Munich. *Mr. B. D. Evans,* computer at the Royal Observatory, Greenwich, has accepted the position as first assistant at the Hongkong Observatory, which is now under the directorship of *Mr. T. F. Claxton.*

10. Lectures on Terrestrial Magnetism, 1912-13. Information has

Digitized by Google

been received regarding the following lectures on terrestrial magnetism and allied topics: *Adolf Schmidt* at the University of Berlin, winter semester; *W. Trabert* at the University of Vienna, winter semester; *L. A. Bauer,* Johns Hopkins University, spring, 1913, and Halley lecture at the University of Oxford, May, 1913.

11. Marine Earth Inductor. An earth inductor for use at sea, designed by *Messrs. N. E. Dorsey* and *J. A. Fleming* and constructed in the workshop of the Department of Terrestrial Magnetism, was installed on the *Carnegie* at Tahiti in September. A cablegram received from *Mr. Peters,* in command of the vessel, announcing arrival at Coronel, Chile, on November 24th, stated that the marine earth inductor had proved wholly successful; there will thus result increased accuracy in ocean dip observations. An article on the theory of the earth inductor, by *Dr. N. E. Dorsey,* appears in the March, 1913, issue.

12. Total Solar Eclipse, October 10, 1912. Besides special magnetic observations by observers of the Department of Terrestrial Magnetism on the day of this eclipse, co-operation was secured from various institutions in North and South America. Furthermore, *Dr. Walter Knoche,* Director of the Instituto Central Meteorológico y Geofisico de Chile, proceeded, under the joint auspices of his Institute and the Carnegie Institution of Washington, to a station in the belt of totality, in Southeastern Brazil, and made atmospheric electric and allied observations.

13. Eminent Geographers at Washington, D. C., October 13, 1912. Through the courtesy of the National Geographic Society we are enabled to reproduce in this issue (Plate III) a photograph of the group of eminent geographers from various parts of the globe who participated in the Transcontinental Excursion of 1912, conducted, under the auspices of the American Geographical Society of New York, by *Professor W. M. Davis.* Among those interested in the topics of this Journal were: Prof. E. Brückner (Vienna), Prof. George Chisholm (Edinburgh), Prof. Erich von Drygalski (Munich), Prof. Fritz Jaeger (Berlin), Prof. Emmanuel de Margerie (Paris), Prof. Joseph Partsch (Leipzig), Prof. Eugen Oberhummer (Vienna), General Jules de Schokalsky (St. Petersburg), Messrs. Beckit and Ogilvie of Oxford, Prof. G. W. von Zahn (Jena), and others. Prof. von Drygalski is in the lower right hand corner of the picture and Prof. Partsch is standing on his right.

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google